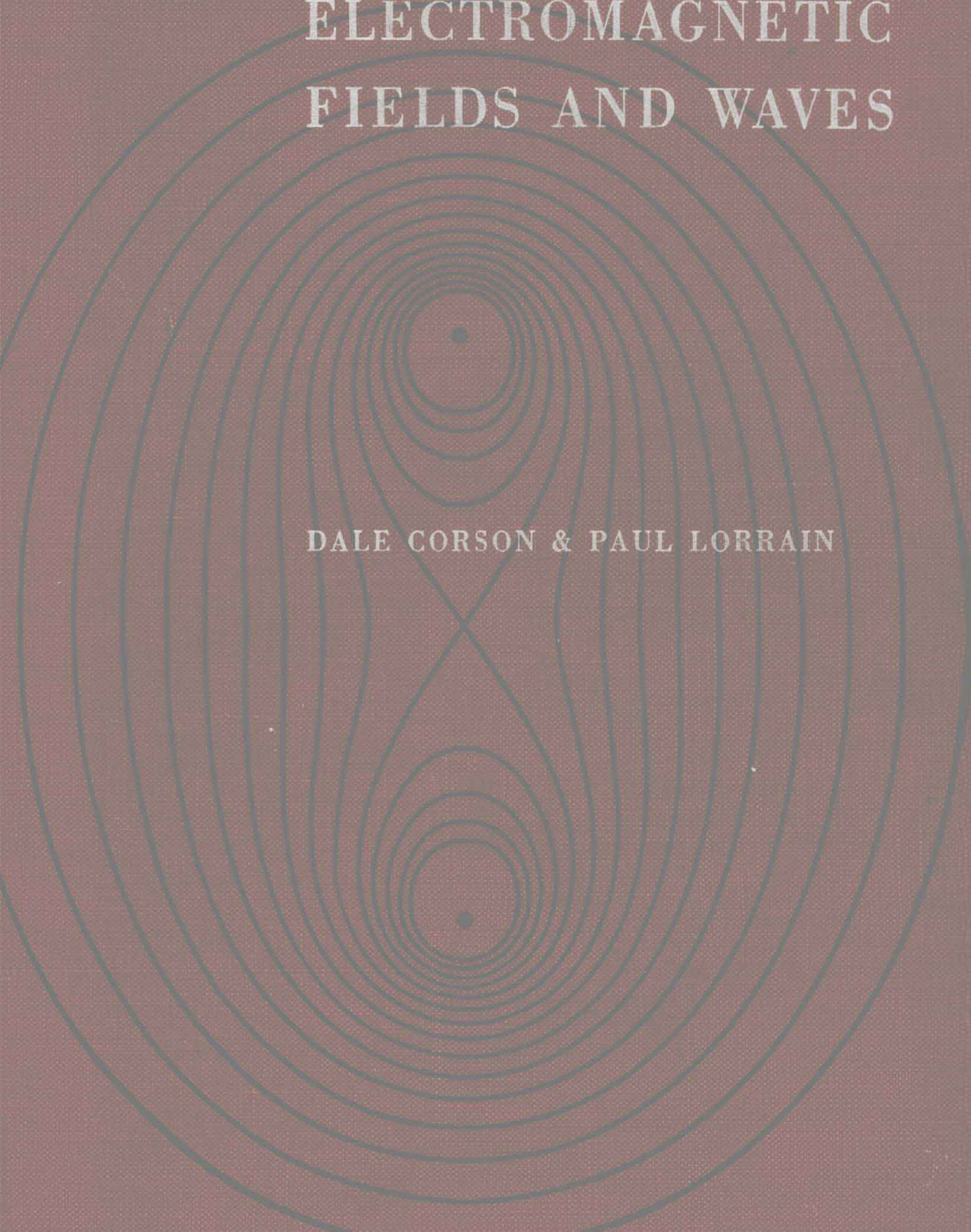
INTRODUCTION TO ELECTROMAGNETIC FIELDS AND WAVES



DALE CORSON & PAUL LORRAIN

INTRODUCTION TO ELECTROMAGNETIC FIELDS AND WAVES

Dale R. Corson CORNELL UNIVERSITY

Paul Lorrain UNIVERSITY OF MONTREAL

W. H. FREEMAN AND COMPANY

SAN FRANCISCO AND LONDON

Library of Congress Catalog Card Number: 62-14193

Printed in the United States of America (*C5*)

INTRODUCTION TO ELECTROMAGNETIC FIELDS AND WAVES

A SERIES OF BOOKS IN PHYSICS

Editors: HENRY M. FOLEY AND
MALVIN A. RUDERMAN

Concepts of Classical Optics
John Strong

Thermophysics
Allen L. King

Introduction to Electromagnetic Fields and Waves
Dale R. Corson and Paul Lorrain

Modern Quantum Theory
Behram Kurşunoğlu

On the Interaction Between Atomic Nuclei and Electrons (*A Golden Gate Edition*)
H. B. G. Casimir

X-Ray Diffraction in Crystals, Imperfect Crystals, and Amorphous Bodies
A. Guinier

Symmetry Principles at High Energy: Coral Gables Conference
Behram Kurşunoğlu and Arnold Perlmutter, Editors

Spacetime Physics
Edwin F. Taylor and John Archibald Wheeler

PREFACE

This introduction to electromagnetic theory is intended for students having a good background in elementary electricity and in differential and integral calculus. It should also be useful for scientists and engineers who wish to review the basic concepts and methods of the subject. It has been taught for many years by the authors both at Cornell University and at the University of Montreal at the advanced undergraduate level.

To reduce the mathematical requirements, we have included discussions of various topics in mathematics which are essential to a proper understanding of the text. For example, the first chapter deals with vector analysis in Cartesian and in other coordinate systems. There is also an appendix on the technique which involves replacing $\cos \omega t$ by $\exp (j\omega t)$ for dealing with periodic phenomena, and another on wave propagation. The solution of partial differential equations by the separation of variables and the solution of the Legendre differential equation are discussed in Chapter 4. The mathematical level may be adjusted to a considerable extent either by delaying or omitting various sections, as indicated in footnotes.

Our aim has been to give the student a good grasp of the basic concepts and methods. This meant covering fewer subjects more thoroughly. For example, we have dealt with the solution of Laplace's equation in rectangular and spherical coordinates, but not in cylindrical coordinates. For the same reason, the discussion of electromagnetic waves in the second half of the book is concerned with only three types of media: dielectrics, good conductors, and low-pressure ionized gases.

The discussion has been kept as systematic and as thorough as possible, and hazy "physical" arguments have been avoided. We have also stressed the internal logic of the subject, and we have clearly stated all assumptions. This approach should accelerate the learning process and help the student gain self-confidence. In most cases we have illustrated the theory by means of examples which are accompanied by figures showing the main characteristics of the fields.

The illustrations have all been designed and executed with exceptional care, in the hope that they would give the student a much better grasp of the subject than would the usual blackboard-type sketch. Wherever possible, the illustrations are *quantitative*, and three-dimensional objects or phenomena are represented as such. In particular, fields such as those shown in Figures 4-9 and 13-10 are quantitative.

Short summaries are provided at the end of each chapter to give a general view of the subject matter, and, throughout the book, new results are tied in with previously acquired results so as to consolidate, as well as to extend, the reader's knowledge.

The problems form an essential part of the book. Many are designed to help the student extend the theory expounded in the text. They should be done with care.

A few words are in order concerning the units and the notation used in this book. As to the units, we have used exclusively the rationalized m.k.s., or meter-kilogram-second system. A table is provided in Appendix C for converting these units into the various c.g.s. systems. The notation used is that suggested by the Commission on Symbols, Units, and Nomenclature of the International Union of Pure and Applied Physics, with the following three exceptions. We have used the term "dielectric coefficient" and the symbol K_e, instead of the relative permittivity ϵ/ϵ_0. We have also used K_m instead of μ/μ_0, but have called it the *relative* permeability, as suggested by the Commission. Thus ϵ has been replaced by $K_e\epsilon_0$ and μ by $K_m\mu_0$, which has the advantage of stressing the effects of the properties of the medium. In the latter part of the book, however, from Chapter 10 on, this consideration is pedagogically less important and we have used ϵ and μ so as to simplify the notation. Finally, we have used the operators ∇, $\nabla\cdot$, and $\nabla\times$ instead of grad, div, and curl, respectively. The ∇ notation is pedagogically much preferable to the other for Cartesian coordinates. For example, a student recognizes immediately that $\nabla\cdot\nabla V = \nabla^2 V$ and that $\nabla\cdot\nabla\times\mathbf{A} = 0$ (the two top rows of the determinant are identical), whereas div grad V and div curl $\mathbf{A}$ seem quite meaningless. The student is made to realize clearly that the operator ∇ can be defined only in Cartesian coordinates.

The exponential function for periodic phenomena can be chosen to be either exp $(j\omega t)$ or exp $(-j\omega t)$, since the real part of both of these functions is $\cos\omega t$. We have chosen the positive exponent. This, we believe, is essential at this level; otherwise, in circuit theory, an impedance Z becomes equal to $R - jX$ instead of the conventional $R + jX$.

We have also used extensively the radian length $ƛ = \lambda/2\pi$ instead of the wave length λ. This considerably simplifies the calculations. The situation here is similar to that with respect to the frequency f. In both cases, the quantity which has an intuitively obvious meaning, namely, frequency and wave length, is *not* the one which enters into the calculations, but rather $2\pi f = \omega$ and $\lambda/2\pi = \lambda\!\!\!^{-}$.

The book starts with Coulomb's law and ends with the electromagnetic field of a moving charge. As indicated below, there are many possible ways of utilizing only part of this material. The first chapter deals with vector analysis in Cartesian, orthogonal curvilinear, cylindrical, and spherical coordinates. The following three chapters then cover electrostatic fields, first in a vacuum, and then in dielectrics. These are investigated at length from the "molecular" point of view. Chapters 5 and 6 deal, respectively, with the magnetic fields associated with constant and with variable currents. The first is based on the force between two current-carrying circuits, whereas the second is based on the Lorentz force on a charged particle moving in a magnetic field.

Chapter 7 contains a discussion of magnetic materials which parallels to a certain extent that of Chapter 3 on dielectrics. At this point, all four of Maxwell's equations have been found, and Chapter 8 is devoted to a short discussion of these equations, as well as to some new material which follows directly from them.

Chapters 9 to 14 are all based on the Maxwell equations. The first subject discussed is the propagation of plane electromagnetic waves, in a vacuum, and then in dielectrics, in good conductors, and in low-pressure ionized gases. Chapter 11 then discusses at length the phenomena of reflection and refraction at the interface between two dielectrics and between a dielectric and a good conductor. The reflection in a low-pressure ionized gas such as the ionosphere is also discussed at some length. This chapter should make the student thoroughly familiar with the use of Maxwell's equations. The next chapter is concerned with guided waves. It covers some general considerations and includes a discussion of two relatively simple cases, that of the coaxial line and that of the *TE* wave in a rectangular wave guide.

Finally, the last two chapters deal with the radiation of electromagnetic fields. They are based entirely on the electromagnetic potentials V and $\mathbf{A}$, which, in turn, follow from Maxwell's equations. Chapter 13 covers some general considerations and then discusses electric and magnetic dipole and quadrupole radiation. Chapter 14 contains a simplified discussion of the fields of moving charges using only elementary methods. This chapter should provide a useful background for a course in relativity.

Many will be of the opinion that the book contains more material than can be discussed thoroughly in the time available in class. There are, however, many ways in which parts of the book may be omitted without losing continuity. For example, a relatively elementary course could be limited to the first eight chapters leading up to and including a discussion of Maxwell's equations. Or, for a more advanced course, the first seven chapters could either be omitted or reviewed rapidly by making use of the summaries provided at the end of each one. Chapters 3 and 7, on the molecular approach to dielectrics and magnetic materials, could be omitted if necessary. This would limit the discussion to the macroscopic theory and the Maxwellian point of view. It would not be advisable, however, to omit Chapter 3 and not Chapter 7, since the latter rests rather heavily on Chapter 3. Chapter 11, "Reflection and Refraction," could be treated briefly; it is not essential to Chapter 12, "Guided Waves." Chapter 12 is instructive both because of the insight it provides into electromagnetic wave propagation and because of its engineering implications, but none of its results are required for the following chapters. Chapter 13, "Radiation of Electromagnetic Waves," is of course fundamental, but it should not be studied without having a thorough grasp of Chapters 8 and 9. Chapter 13 is necessary for a proper understanding of Chapter 14, "Electromagnetic Field of a Moving Charge."

It is a pleasure indeed to acknowledge the help of the many persons who cooperated in producing this book. We are indebted to Mr. Gilles Cliche, who was responsible for the numerical calculations required for the many quantitative figures. The $K\lambda$ surface (Figure 13-10) was drawn at his suggestion. Mr. Gaétan Marchand also took part in the calculations and helped in the preparation of the manuscript. All the illustrations were designed and partly executed by Mr. Paul Carrière. The cooperation of Miss Evanell Towne in putting them in their final form was much appreciated. The typing was ably done by Miss Huguette Boileau, Mrs. Judith Barnes, Mrs. Marjorie Kinsman, Mrs. Thérèse Fournier, and Mrs. Yolande LeCavalier. Last but not least, we are indebted to the many students, both at Cornell University and at the University of Montreal, who provided the main incentive and many stimulating discussions.

June 1962

DALE R. CORSON

PAUL LORRAIN

CONTENTS

CHAPTER 1

Vectors

We shall discuss electric and magnetic phenomena in terms of the *fields* of electric charges and currents. For example, the force between two electric charges will be considered as being due to an interaction between one of the charges and the field of the other.

It is therefore essential that the student acquire at the very outset a thorough understanding of the mathematical methods required to deal with fields. This is the purpose of the present chapter on Vectors. It is important to note that the concept of field and the mathematics of vectors are essential not only to electromagnetic theory but also to most of present-day physics. We shall assume that the student is not familiar with vectors and that a thorough discussion is required.

Mathematically, a field is a function which describes a physical quantity at all points in space. In *scalar fields* this physical quantity is completely specified by a single number for each point. Temperature, density, and electrostatic potential are examples of scalar quantities which can vary from one point to another in space. For *vector fields* both a number and a direction are required. Wind velocity, gravitational force, and electric field intensity are examples of such vector quantities.

Vector quantities will be indicated by **boldface** type; lightface type will indicate either a scalar quantity or the magnitude of a vector quantity.

We shall follow the usual custom of using a *right-hand coordinate system* as in Figure 1-1: the positive z direction is the direction of advance of a right-hand screw rotated in the sense that turns the positive x-axis into the positive y-axis through a 90 degree angle.

1.1. Vector Algebra

A vector can be specified by its *components* along any three mutually perpendicular axes. In the Cartesian coordinate system of Figure 1-1, for example, the vector **A** has components A_x, A_y, and A_z.

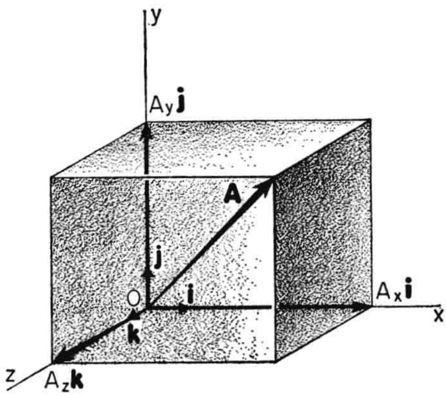


Figure 1-1

A vector **A** *and the three vectors* A_x**i**, A_y**j**, A_z**k**, *which, when placed end-to-end, are equivalent to* **A**.

The vector can be uniquely expressed in terms of its components through the use of *unit vectors* **i**, **j**, and **k**, which are defined as vectors of unit magnitude in the positive x, y, and z directions, respectively:

$$\mathbf{A} = \mathbf{i}A_x + \mathbf{j}A_y + \mathbf{k}A_z. \tag{1-1}$$

The vector **A** is the sum of three vectors of magnitude A_x, A_y, and A_z, parallel to the x-axis, y-axis, and z-axis, respectively. It is clear that the magnitude of **A** is given by

$$A = (A_x^2 + A_y^2 + A_z^2)^{1/2}. \tag{1-2}$$

The sum of two vectors is equal to the sum of their components:

$$\mathbf{A} + \mathbf{B} = \mathbf{i}(A_x + B_x) + \mathbf{j}(A_y + B_y) + \mathbf{k}(A_z + B_z). \tag{1-3}$$

Subtraction is simply addition with one of the vectors changed in sign:

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}) = \mathbf{i}(A_x - B_x) + \mathbf{j}(A_y - B_y) + \mathbf{k}(A_z - B_z). \tag{1-4}$$

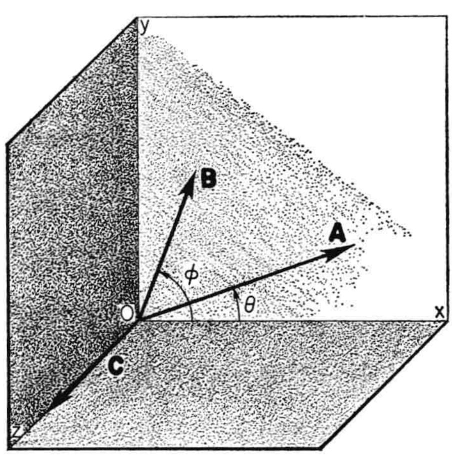


Figure 1-2

Two vectors **A** *and* **B** *in the xy-plane. The vector* **C** *is their vector product* **A** × **B**.