

Third Edition

ANALYTICAL FLUID DYNAMICS

George Emanuel



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ANALYTICAL FLUID DYNAMICS

Dedicated with love to my wife and companion, Lita, whose patience and support are beyond comprehension.

Preface

The objectives of this edition remain the same as in earlier versions. The analysis and formulation are provided for a variety of topics in inviscid and viscous fluid dynamics; it is hoped with analytical and physical insight. In part, this means formulating the appropriate equations and transforming them into a suitable form for the specific flow under scrutiny. The approach is applied to viscous boundary layers, shock waves, and numerous other flows, including asymmetric thrust nozzles and supersonic diffusers. Of special interest are the analytical process and the corresponding physical interpretation.

An in-depth presentation is favored compared to one that bypasses crucial or difficult details. Whenever pertinent, assumptions and limitations are addressed for the topic under discussion. Frequently, the reason why a particular topic deserves study is pointed out. For instance, a solution may be useful as a first, or initial, estimate for a computational fluid dynamics (CFD) calculation. Analytical solutions, such as those provided by the substitution principle, can be used to verify Euler codes. An analytical approach often yields insight and first estimates for parameters of interest. In this regard, some of the homework problems are designed to give the student practice in obtaining relevant solutions. My personal motivation, however, still remains the beauty and elegance of analytical fluid dynamics (AFD).

The need for a more flexible mathematical language in fluid dynamics, to cover its increasing complexity, has long been evident. Two of these “languages” are utilized in this book. They are vector and tensor analysis and what might be called transformation theory. The use of transformations in fluid dynamics is ubiquitous; matched asymptotic expansions are the mathematical backbone of boundary-layer theory. Transformations also play a major role, especially in inviscid theory. This category includes Jacobian theory, the substitution principle, the hodograph transformation, characteristic theory, and operator methods. These topics are introduced

either in a chapter or in an appendix, as is also the case for vector and tensor analysis.

A small monograph of mine, *Shock Wave Dynamics*, was published in late 2013 by CRC Press. This book was an outgrowth of Chapter 6 of the earlier editions of the current book. After publication, a considerable amount of new material was developed; shock wave material now encompasses Chapters 6 through 11. This new material partly motivated a decision to incorporate the monograph into its parent text. Further supporting this decision was the inclusion of additional novel, non-shock wave material, such as the thrust and lift analysis of an asymmetric nozzle and a supersonic diffuser analysis.

Earlier editions contained a number of special topics, including viscous dissipation, calorically imperfect gas flows, aerodynamic sweep, shock wave interference, unsteady one-dimensional flow, internal ballistics, force and momentum balance, the substitution principle, etc. Aside from these topics, new material covers rarefaction shock waves; a comprehensive treatment of flow property derivatives just downstream of an unsteady three-dimensional shock; shock-generated vorticity, triple points; an extended version of the Navier–Stokes equations, which is required for an ultrasonic bulk viscosity absorption measurement and for shock wave structure; shock-free supersonic diffusers; and the lift and thrust from an asymmetric nozzle.

Topics where future research is warranted are evident. Fluid dynamics, including the AFD specialty, is very much alive and growing. Consequently, not everything in this book is complete or, despite my best effort, necessarily correct. A variety of important topics are not discussed. These include turbulent flow, CFD, experimental methods, etc., which are major subjects in themselves.

I owe a debt of gratitude to the many friends who have contributed to this undertaking, especially past students and past and current colleagues. I am especially in debt to Gloria Madden for her superb typing of the manuscript.

Author

George Emanuel earned his PhD in aeronautical sciences from Stanford University, Stanford, California. Subsequently, he was employed at the Aerospace Corp., TRW, and Los Alamos National Laboratory as a research engineer. He spent the next 19 years as a professor in the school of Aerospace and Mechanical Engineering at the University of Oklahoma, Norman, Oklahoma, from which he is now emeritus. He is the author of *Gasdynamics: Theory and Applications* and *Advanced Classical Thermodynamics*, both with the AIAA Education Series, and *Analytical Fluid Dynamics, Second Edition, Solution of Ordinary Differential Equations by*

Continuous Groups, and *Shock Wave Dynamics* (CRC Press, 2000, 2000, and 2013, respectively). He is also the author of 4 chapters in 3 handbooks and the author or coauthor of more than 100 peer-reviewed articles in more than 20 different journals. Currently, he is the president of KSY Corp., which is involved in the research and development of chemical lasers. Through his company, he holds nine U.S. patents relating to the chemical oxygen-iodine laser and its applications. In 2001, he received the AIAA Plasmadynamics and Lasers Award for his contributions to chemical lasers.

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