

SECOND EDITION

MUGRIDGE

INTERMEDIATE

A L G E B R A

A Intermediate Algebra

Second Edition



SAUNDERS COLLEGE PUBLISHING

HARCOURT BRACE COLLEGE PUBLISHERS

Fort Worth Philadelphia San Diego New York Orlando Austin San Antonio
Toronto Montreal London Sydney Tokyo

Copyright © 1994, 1991 by Saunders College Publishing

All rights reserved. No part of this publication may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopy, recording, or any information storage and retrieval system, without permission in writing from the publisher.

Requests for permission to make copies of any part of the work should be mailed to Permissions Department, Harcourt Brace College Publishers, 8th Floor, Orlando, Florida 32887.

Text Typeface: 10/12 Times Roman
Compositor: York Graphic Services, Inc.
Acquisitions Editor: Deirdre Lynch
Developmental Editor: Ellen Newman
Managing Editor: Carol Field
Production Management: York Production Services
Manager of Art and Design: Carol Bleistine
Art and Design Coordinator: Christine Schueler
Text Designer: York Production Services
Cover Designer: Lawrence R. Didona
Text Artwork: York Graphic Services, Inc.
Director of EDP: Tim Frelick
Production Manager: Carol Florence
Marketing Manager: Monica Wilson

Cover Credit: W. Cody/Westlight

Printed in the United States of America

INTERMEDIATE ALGEBRA, second edition

ISBN: 0-03-072943-2

Library of Congress Catalog Card Number: 93-42142

6 7 8 048 9 8 7 6 5

Preface

This book is intended for students who have a knowledge of elementary algebra and need to cover intermediate algebra before taking other college mathematics courses. Each concept is introduced by example before the definition is given, and the student is encouraged to become an active participant. A complete pedagogical system, designed to motivate the student and make mathematics more accessible, includes the following features:

Key Features

Chapter Overviews	Each chapter begins with an Overview to introduce the student to the material and demonstrate its relevance to the real world.
Objectives	Each section contains Objectives to help the student focus on skills to be learned in the chapter.
Learning Advantages	These hints occur in Chapters 1 through 8 to give students additional help; they immediately follow the objectives of each section. For more detailed explanations on studying algebra, I recommend the pamphlet <i>How to Study Mathematics</i> by James Margenau and Michael Sentlowitz (The National Council of Teachers of Mathematics, Inc., Reston, Virginia, 1977) and <i>Mathematics at Work in Society: Opening Career Doors</i> by James R. Choike (Mathematical Society of America, Washington D.C., 1988), from which some of the Learning Advantages are adapted.
Examples with Solutions	The text provides approximately 550 examples with complete, worked-out solutions; explanations are highlighted in blue print. Some of the examples contain

steps that can be done mentally once the student has achieved a certain level of proficiency (although it may not be possible at first); a blue dashed box calls the student’s attention to this portion of the equation. A yellow triangle indicates the end of each solution.

Common Error Boxes

Wherever appropriate, students are shown errors that are commonly made. The correct approach is then illustrated and explained.

Strategies

After developing a mathematical technique, such as factoring a trinomial, a summary of the technique is given as a strategy. This will provide the student with a reference when doing problems from the exercise set.

Graphing Calculator Crossovers

Many of the examples are followed by problems specifically designed to be solved with a graphing calculator. These problems are similar to the examples and illustrate the role of the graphing calculator in solving problems. Graphing Calculator Crossovers are set off by a symbol for easy reference and may be used at the instructor’s discretion.

Warming Up

These true-false questions, which precede each set of exercises, can be used as a classroom activity.

Exercises

Graded by level of difficulty, the exercises (approximately 6,500) are carefully constructed so that there is odd-even pairing. In addition, each exercise problem is related to an example from the section. The numbers used in the examples and exercises have been carefully selected so that the student becomes confident in using fractions and decimals in addition to the integers. The numbers, however, do not become so complicated as to “turn off” the student or teacher. All problems have been thoroughly reviewed for accuracy.

Some of the exercises are specifically designed to be solved with either a calculator or a graphing calculator. They may be assigned at the instructor’s discretion and are marked with a symbol for identification. There are also problems in each section called **Say It In Words**, which require students to express mathematics in written form.

Beginning with Chapter 2, each exercise set contains **Review Exercises** to help the student understand and remember concepts studied in earlier sections. Each review exercise set covers background material essential to the following section. The **Enrichment Exercises** at the end of each section are challenging problems that the more capable students will be able to solve. Answers to the odd-numbered exercises, Review Exercises, and all of the Enrichment Exercises are given in the Answer Section following the text.

Applications Each chapter begins with a real-world photograph and an application that poses a problem to be solved later in the chapter. Whenever appropriate, word problems have been included throughout the text so that the student constantly practices translating word phrases and statements into mathematical equations. The word problems are realistic and are culled from applications in geometry, physics, business, economics, history, and psychology. The structure of problem solving is emphasized throughout the text, beginning with the simple conversion of word phrases into mathematical expressions. Many concepts introduced in early chapters are repeated using different applications.

Team Project To emphasize cooperative learning skills, each chapter includes a research project that encourages students to work together.

Chapter Summary and Review Each chapter concludes with a comprehensive review which includes definitions and strategies learned in the chapter. All terms are keyed to the section number for easy reference. Marginal examples are included to further illustrate the concepts reviewed.

Review Exercise Set These exercises are included at the end of the chapter and represent all types of problems to ensure that the student has attained a level of proficiency and is comfortable proceeding to the next chapter. All exercises are keyed to sections so the student can refer to the text for assistance. Answers to all of these exercises can be found in the back of the book.

Chapter Test Each chapter concludes with a Chapter Test that contains representative problems from each section.

Pedagogical Use of Color This text uses color in the figures and to highlight the various pedagogical features throughout the text. Multiple colors are useful, for example, to distinguish between two lines that are graphed simultaneously, or to highlight important statements. The complete color system is described in more detail on page xvii.

Changes in the Second Edition

A number of changes and improvements have been made in preparing the new edition of this text. Many of these changes are in response to comments and suggestions offered by users and reviewers of the manuscript. The following represent the major changes in the second edition:

1. Approximately 30 percent of the exercises have been changed, for a total of almost 2,000 new problems. We have also added problems in each section that require students to express mathematics in written form; this section is called **Say It In Words**. All of the exercises have been thoroughly reviewed for accuracy.

2. Each exercise set is now preceded by **Warming Up**, a group of true–false questions that can be used as a classroom activity.
3. New **Graphing Calculator Crossovers**, in appropriate chapters, contain explanations and examples specifically designed to be solved with a graphing calculator to enhance the graphing of functions and conic sections. They are optional and may be assigned at the instructor’s discretion. The **Calculator Crossovers** from the first edition have been moved to the end-of-section exercises, which also contain exercises for the graphing calculator; each type of calculator exercise is marked with its own symbol for easy reference.
4. **Mental Math** symbols have been deleted. A blue dashed box now indicates the portion of the equation that can be done mentally once the student has achieved a certain level of proficiency.
5. Each chapter now begins with a **Connection**, a real-world application and photograph posing a problem that is solved later in the chapter. For example, Chapter 3 is introduced with a study of commerce in colonial America, a topic being researched using mathematics by historian Dr. Christine Styrna Devine.
6. A team project has been added to each chapter to emphasize cooperative learning skills.
7. Coverage of linear equations and their graphs was moved from Chapter 7 to Chapter 3 to give students earlier exposure to real-world applications involving linear equations in two variables. A new section has been added on linear functions (Section 3.5).
8. The section on variation, Section 6.9, has been moved to Chapter 6, Roots and Radicals.
9. New material has been added in Section 1.1 on the universal set and complement and in Section 1.5 on multiplying a number by -1 to simplify expressions.
10. Some sections have been streamlined and combined (Section 2.1 was combined with Section 2.2, and Section 6.1 was combined with Section 6.2 to form new Section 7.1).
11. Coverage of factoring has been expanded to include more explanation of factoring by grouping in Section 4.4 and the substitution method of factoring in Section 4.5.

An Overview of the Book

Chapter 1: Operations and Variables

The main thrust of this book is to enable students to develop algebraic skills to be used to solve word problems. This is the primary aim of each chapter; we start with developing a skill and then use it to solve word problems. In addition, many geometric problems are included. A chapter-by-chapter overview follows:

Since the foundation of algebra is arithmetic, it is reviewed comprehensively in this chapter, starting with addition, subtraction, multiplication, and division of fractions. Next, we review basic symbols and order of operations. The set of

real numbers is examined along with the subsets of natural, whole, rational, and irrational numbers. Then, we review the four basic operations on real numbers: addition, subtraction, multiplication, and division. In Section 1.4, we deal with the properties of real numbers. We study variables and variable expressions in the final section.

Chapter 2: Linear Equations and Inequalities

In this chapter, algebraic expressions are used to construct linear equations and inequalities. In Section 2.1, we investigate properties of equality that are used to solve linear equations. In Section 2.2, we solve word problems that require solving linear equations. Next, we develop formulas with associated applications from geometry and trigonometry (area and volume formulas of regions and solids) in Section 2.3. We also develop formulas from business, such as simple interest and cost-revenue-profit. Section 2.4 deals with solving inequalities, and in the final section, we examine equations and inequalities with absolute value.

Chapter 3: Linear Equations in Two Variables and Linear Functions

In this chapter, we investigate the connection between analytic geometry and its applications, beginning by plotting points in a plane and then developing linear equations and their graphs. In Section 3.2, we find the slope of a line given two points on the line. We also find the slope of a line from its graph. The difference between zero slope and undefined slope is explained, and the geometric meaning of positive and negative slope is discussed. In the next section, we find the equation of a line given (a) the slope and y-intercept, (b) the slope and a point on the line, or (c) two points on the line. Section 3.4 discusses graphing linear inequalities in two variables. Linear functions are discussed in Section 3.5. Section 3.6 includes many business and economic applications of linear functions, such as linear depreciation, linear cost-revenue-profit, and linear supply and demand equations.

Chapter 4: Polynomials and Factoring

In the first section, we study multiplication properties with exponents and simplify expressions using properties of exponents. In the next two sections, we introduce the algebra of polynomials, which is followed by four sections on factoring. Coverage includes factoring of monomials; general trinomials; and special polynomials such as the difference of two squares, perfect square trinomials, and the sum of the difference of two cubes. We also develop factoring by grouping and state a strategy for general factoring. These techniques are then applied to solving quadratic equations that are factorable, which, in turn, leads to applications of quadratic equations. The applications include using the Pythagorean Theorem, solving geometric problems, and understanding applications to business that involve quadratic equations.

Chapter 5: Rational Expressions

In the first section, we define rational expressions, evaluate them, and determine where a rational expression is defined. In Section 5.2, we develop the techniques of dividing polynomials, which leads to the algebra of rational expres-

sions in Sections 5.3 and 5.4. Next, we discuss complex fractions, where we develop methods to simplify complex fractions into rational expressions. We then solve equations containing rational expressions and introduce formulas that involve rational expressions such as the total resistance formula.

Chapter 6: Roots and Radicals

We introduce square roots and higher roots in Section 6.1 and connect these concepts with rational exponents in Section 6.2. In the next four sections, we deal with the algebra of radical expressions and simplifying radical expressions. Radical equations and word problems using radical equations are solved in Section 6.7. We conclude this chapter by defining a complex number in terms of i and covering the algebra of complex numbers. The final section deals with variation and applications in such fields as physics.

Chapter 7: Quadratic Equations

One of the main goals of this book is developing techniques for solving equations. In this chapter, we return to the problem of solving the general quadratic equation $ax^2 + bx + c = 0$, where the trinomial may not be factorable. We begin with a discussion of the square root method to solve quadratic equations and then develop the completing of the square method. The quadratic formula is discussed in Section 7.3 and then used in applications of quadratic equations. Next, we investigate quadratic and rational inequalities. In the final section, we solve quadratic equations using complex numbers and solve equations that are quadratic in form.

Chapter 8: Systems of Equations

In this chapter, we continue to examine the connection between analytic geometry and its applications. We show how a system of two equations in two variables enables us to study more complex applications. Our study of systems of equations begins by finding solutions using three methods: the graphing method, the elimination method, and the substitution method. In Section 8.3, we develop techniques to solve a system of linear equations in three variables, which is followed by the development of Cramer's Rule and matrix methods for solving systems of equations in Sections 8.4 and 8.5. In the final section, we solve systems of linear inequalities.

Chapter 9: Conic Sections

This chapter focuses on four conic sections in Sections 9.1 through 9.4: parabolas, circles, ellipses, and hyperbolas. In these discussions, our aim is to obtain the standard form equations for these conic sections. Nonlinear systems of equations and inequalities are covered in the last two sections of this chapter.

Chapter 10: More Functions

There are many real-world examples of dependence of one quantity upon another; here we develop the mathematical basis to describe the concept of dependency. Beginning with the review of relation and function in Section 10.1, we show how to determine when a relation is also a function. Function notation

and the algebra of functions is introduced in Section 10.2, where the student learns to form the quotient $[f(x + h) - f(x)]/h$ and to simplify the result. Section 10.3 develops linear, quadratic, polynomial, and exponential functions, and in the last section of the chapter the student learns the concept of one-to-one and how to find an equation for the inverse function when it exists.

Chapter 11: Exponential and Logarithmic Functions

We introduce the exponential function to the base e and logarithmic functions that lead to an ever greater variety of applications. In Section 11.1, the exponential function $f(x) = e^x$ is introduced along with a plethora of applications such as exponential growth, compound interest, and growth and decay equations. The logarithmic functions are then developed along with the technique of solving the equation $y = \log_b x$ for y , x , or b . The next section deals with the properties of logarithms, which are used to solve logarithmic equations. The chapter concludes with a section that involves solving exponential equations by using either a calculator or the tables in the appendix.

Chapter 12: Sequences and Series

This chapter focuses on a special kind of function—the sequence. The first section introduces the sequence notation as well as the technique for finding the n th term of a sequence. Section 12.2 then introduces the concept of series along with the sigma notation to write a series. Arithmetic and geometric sequences are discussed in the next section, showing the student how to find the sum of the first n terms of an arithmetic series, the general term of a geometric sequence, and the sum of the first n terms of a geometric series. Section 12.4 develops the binomial theorem along with Pascal's triangle to find the binomial coefficients.

State Requirements

Particular attention has been paid to the testing requirements for various states (e.g., TASP, ELM, CLAST, etc.) Please see pages xviii–xxi for additional information on how this text meets the requirements for your state.

Ancillary Package

The following supplements are available to the student to accompany this text:*

Student Solutions Manual and Study Guide (Linda Holden, Indiana University)

Contains step-by-step solutions (every other odd-numbered problem) in the exercise sets in addition to providing the student with a short summary of the important concepts in each chapter.

*For the instructor, we have an Instructor's Manual with solutions to all the exercises; Prepared Tests with six tests for each chapter, as well as midterm and final examinations and a Diagnostic test; a Computerized Test Bank for the IBM and Macintosh computers; and a Printed Test Bank containing tests generated from the Computerized Test Bank.

Videotapes

A complete set of videotapes (18 hours), free to adopters, has been created and scripted by the author. Keyed to the text, the videotapes explain, using computer graphics, examples with corresponding practice problems. The student can participate by stopping the tape to work the practice problems on his or her own and then checking the solutions by continuing the tape. The tapes are available in VHS format and include a list of topics and the amount of time spent on each section.

MathCue Tutorial Software

An interactive software package keyed to every section in the text, MathCue enables students to practice their skills and correct weak areas. Exercises offer the option of partial solutions or step-by-step, annotated solutions for missed problems. Available for IBM and Macintosh.

MathCue Solution Finder Software

This expert system approach allows students to enter problems, receive help, and check their answers as if they were working with a tutor. Software tracks students' progress and refers them to sections of the text for help. Available for IBM and Macintosh.

F/C Graph (George W. Bergeman, Northern Virginia Community College)

Available for both IBM and Macintosh, this software graphs functions and conic sections. It provides an easy-to-use, basic tool to help users visualize, investigate, and understand a variety of important topics from the text. In addition, computer exercises accompany F/C Graph to guide students' explorations. Students can zoom in and out, evaluate a function at a point, graph up to four functions simultaneously, find roots, intercepts, and intersections easily, graph circles, ellipses, parabolas, and hyperbolas, enter parameters via various standard forms of conic sections, and save and retrieve graphing set-ups via data files.

Acknowledgments

This text was prepared with the assistance of many professors who reviewed the manuscript throughout the course of its development. I wish to acknowledge the following people and express my appreciation for their suggestions, criticisms, and encouragement:

Carole Bergen, Mercy College

Robert Billups, Citrus College

Mary Chabot, Mt. San Antonio College

Ruth R. Daniel, Broward Community College, North Campus

Arthur P. Dull, Diablo Valley College

Sharon Edgmon, Bakersfield College

Rackham Goodlet, Howard University

Lois Higbie, Brookdale Community College

Linda Holden, Indiana University

Herbert Kasube, Bradley University

Jerome Krakowiak, Jackson Community College
Susan McClory, San Jose State University
Jane Morrison, Thornton Community College
Karla Neal, Louisiana State University
Bob Olsen, Jefferson Community College
John C. Rayko, University of Texas at San Antonio
Harvey E. Reynolds, Golden West College
Roberta J. Smit, University of Maryland at College Park
Howard Sorkin, Broward Community College
Kathleen Conway Stiehl, Southeast Missouri State University
Jan Vandever, South Dakota State University
June Oliverio-Wallace, Montgomery County Community College

I would also like to thank the following reviewers who were invaluable in ensuring the accuracy of the text: June Oliverio-Wallace, Linda Farish, and Ken Clarke. I would also like to thank Regina Brunner for her contribution to the text, and Melissa Handy and Robert Gemin for their excellent photographs.

Special thanks to the following people for their work on the various ancillary items that accompany *Intermediate Algebra, second edition*:

George W. Bergeman, Northern Virginia Community College (MathCue Tutorial Software, MathCue Solution Finder and F/C Graph)
Jacqueline Stone and Gabrielle Bedewi, University of Maryland (Prepared Tests)
Linda Holden, Indiana University (Student Solutions Manual and Study Guide)
Linda Farish, North Lake College, and **Ken Clarke**, Texas Christian University
Mary Chabot, Mt. San Antonio College (Test Questions for Computerized Test Bank)

I want to thank the dedicated staff at Saunders College Publishing for their tireless energy: Deirdre Lynch, Mathematics Editor, with her insightful analysis of the situation; Ellen Newman, Developmental Editor, who spent long hours, that frequently shortened the evening, on the book; and Susan Bogle, Project Editor with York Production Services, who guided the book to a successful conclusion with her expertise in converting the manuscript to a polished full-color book.

Larry R. Mugridge
Kutztown University

Jacqueline Stone and Gabrielle Bedewi, University of Maryland (Prepared Tests)

Linda Holden, Indiana University (Student Solutions Manual and Study Guide)

Linda Farish, North Lake College, and **Ken Clarke**, Texas Christian University

Mary Chabot, Mt. San Antonio College (Test Questions for Computerized Test Bank)

I want to thank the dedicated staff at Saunders College Publishing for their tireless energy: Deirdre Lynch, Mathematics Editor, with her insightful analysis of the situation; Ellen Newman, Developmental Editor, who spent long hours, that frequently shortened the evening, on the book; and Susan Bogle, Project Editor with York Production Services, who guided the book to a successful conclusion with her expertise in converting the manuscript to a polished full-color book.

Larry R. Mugridge
Kutztown University

Pedagogical Use of Color

The various colors in the text figures are used to improve clarity and understanding. Many figures with three-dimensional representations are shown in various colors to make them as realistic as possible. Color is used in those graphs where different lines are being plotted simultaneously and need to be distinguished.

In addition to the use of color in the figures, the pedagogical system in the text has been enhanced with color as well. We have used the following colors to distinguish the various pedagogical features:

PROPERTY

STRATEGY



CALCULATOR CROSSOVER

DEFINITION

COMMON ERROR

RULE

Table No. Table Title

Col. Head

Col. Head

How Text Meets State Requirements

TASP Skills

The following table lists the Texas TASP skills and their location in *Intermediate Algebra*.

<i>Description of TASP Skill</i>	<i>Chapter, Section</i>
Use number concepts and computation skills	1.1, 1.2, 1.3, 1.4
Solve word problems involving integers, fractions, or decimals (including percents, ratios, and proportions)	1.1, 1.2, 1.3, 1.4
Interpret information from a graph, table, or chart	3.1, 3.2, 3.3, 3.6
Graph numbers or number relationships	3.1, 3.2, 3.4
Solve one- and two-variable equations	2.1, 8.1, 8.4, 8.5
Solve word problems involving one and two variables	2.3, 3.6, 5.7, 5.4, 3.6, 8.2
Understand operations with algebraic expressions	1.5, 2.1, 2.2, 2.3, 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8, 5.1, 5.2, 5.3, 5.4, 5.5, 6.1, 6.2, 6.3, 6.4, 6.5, 6.6
Solve problems involving quadratic equations	4.8, 4.9, 7.1, 7.2, 7.3, 7.5
Solve problems involving geometric figures	2.3
Apply reasoning skills	2.3, 2.4, 4.9, 5.7, 7.3, 3.6, 8.2 All Enrichment Exercises

ELM Skills

The following table lists the California ELM skills and the location of those skills in *Intermediate Algebra*.

<i>Description of ELM Skill</i>	<i>Chapter, Section</i>
Whole numbers and their operations	1.2, 1.3
Fractions and their operations	1.1, 1.3
Exponentiation and square roots	1.2, 4.1, 6.1
Applications (percents and word problems)	2.7, 2.3
Simplification of a polynomial by grouping—one and two variables	4.2
Evaluation of a polynomial—one and two variables	4.2
Addition and subtraction of polynomials	4.2
Multiplication of a monomial with a polynomial	4.3
Multiplication of two binomials	4.3
Squaring a binomial	4.3
Division of polynomials with a monomial divisor—no remainder	5.2
Division of polynomials with a linear binomial divisor—no remainder	5.2
Factoring polynomials by finding common factors	4.4
Factoring a trinomial	4.5
Factoring a difference of squares	4.6
Simplification of a rational expression by cancellation of common factors—one and two variables	5.1
Evaluation of a rational expression	5.1
Addition and subtraction of rational expressions	5.4
Multiplication and division of rational expressions	5.3
Simplification of a compound rational expression	5.5
Definition of exponentiation with positive exponents	4.1
Laws of exponents with positive exponents	4.1
Simplification of an expression with positive exponents	4.1
Definition of exponentiation with integral exponents	4.1

<i>Description of ELM Skill</i>	<i>Chapter, Section</i>
Laws of exponents with integral exponents	4.1
Simplification of an expression with integral exponents	4.1
Scientific notation	4.1
Definition of radical sign	6.1
Simplification of products under a single radical	6.4
Addition and subtraction of radical expressions	6.5
Multiplication of radical expressions	6.6
Solution of a simple radical equation	6.7
Solution of a linear equation in one unknown with numerical coefficients	2.1
Solution of a linear equation in one unknown with literal coefficients	2.3
Solution of a simple equation in one unknown which is reducible to a linear equation	2.1
Solution of a linear inequality in one unknown with numerical coefficients	2.4
Solution of two linear equations in two unknowns with numerical coefficients—by substitution	8.1
Solution of two linear equations in two unknowns with numerical coefficients—by elimination	8.1
Solution of a quadratic equation from factored form	4.8
Solution of a quadratic equation by factoring	4.8
Graphing a point on the number line	1.2
Graphing linear inequalities in one unknown	2.4, 2.5
Graphing a point in the coordinate plane	3.1
Graphing a simple linear equation: $y = mx$, $y = b$, $x = b$	3.1
Reading data from a graph	3.2
Measurement formulas for perimeter and area of triangles, squares, rectangles, and parallelograms	2.3
Measurement formulas for circumference and area of circles	2.3
The Pythagorean Theorem	4.9