

Fourth Edition

Fundamental
Mechanics
of Fluids

I.G. Currie

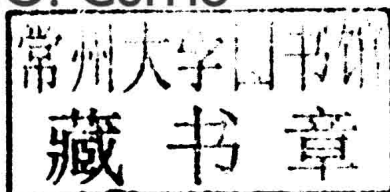


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Fourth Edition

Fundamental Mechanics of Fluids

To my wife Cathie, our daughter Karen, and our sons David and Brian

Preface

This book covers the fundamental mechanics of fluids as they are treated at the senior level and at the introductory graduate level. Many excellent books exist that treat special areas of fluid mechanics such as ideal-fluid flow or boundary-layer theory. However, there are very few books at this level that sacrifice an in-depth study of one of these special areas of fluid mechanics for a briefer treatment of a broader area of the fundamentals of fluid mechanics. This situation exists despite the fact that many institutions of higher learning offer a broad, fundamental course to a wide spectrum of their students before offering more advanced specialized courses to those who are specializing in fluid mechanics. This book is intended to remedy this situation.

The book is divided into five parts. Part I, "Governing Equations," deals with the derivation of the basic conservation laws, flow kinematics, and some basic theorems of fluid mechanics. Part II is titled "Ideal-Fluid Flow," and it covers two-dimensional potential flows, three-dimensional potential flows, and surface waves. Part III, "Viscous Flows of Incompressible Fluids," contains chapters on exact solutions, low-Reynolds-number approximations, boundary-layer theory, and buoyancy-driven flows. Part IV of the book is titled "Compressible Flow of Inviscid Fluids," and this part contains chapters that deal with shock waves, one-dimensional flows, and multidimensional flows. Finally, Part V, which is titled "Methods of Mathematical Analysis," presents a summary of some of the commonly used methods of analysis as used in this book as well as many others. Appendixes are also included that summarize vectors, tensors, the governing equations in the common coordinate systems, Fourier series, and thermodynamics.

The treatment of the material is such as to emphasize the phenomena associated with the various properties of fluids while providing techniques for solving specific classes of fluid-flow problems. The treatment is not geared to any one discipline, and it may readily be studied by physicists and chemists as well as by engineers from various branches. Since the book is intended for teaching purposes, phrases such as "it can be shown that" and similar clichés that cause many hours of effort for many students have been avoided. In order to aid the teaching process, several problems are included at the end of each of the 14 chapters. These problems serve to illustrate points brought out in the text and to extend the material covered in the text.

Most of the material contained in this book can be covered in about 50 lecture hours. For more extensive courses, the material contained here may be completely covered and even augmented. Parts II, III, and IV are essentially independent so that they may be interchanged or any one or more of them may be omitted. This permits a high degree of teaching flexibility and allows the instructor to include or substitute material that is not covered in

the text. Such additional material may include free convection, density stratification, hydrodynamic stability, and turbulence with applications to pollution, meteorology, etc. These topics are not included here, not because they do not involve fundamentals, but rather because I set up a priority of what I consider to be the basic fundamentals.

For the fourth edition, Chapter 14, which is titled "Some Useful Methods of Analysis," has been added. The problems in all chapters have also been reviewed, and some of them have been revised in order to clarify and/or extend the questions. Some new problems have also been included, bringing the total number to 140.

Many people are to be thanked for their direct or indirect contributions to this text. I had the privilege of taking lectures from F. E. Marble, C. B. Millikan, and P. G. Saffman, and some of the style and methods of these great scholars are probably evident in the following pages. Thanks are also due to the many instructors and students who have used the book as a text and who have pointed out errors and/or ambiguities in the material in the earlier versions of this text.

I. G. Currie

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