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El-Kébir Boukas

Stochastic Switching Systems

Analysis and Design

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Control Engineering

This book is dedicated to
My wife Saida,
my daughters Imane and Ibtissama,
my son Anas,
and my mother for their love and support

Preface

The success of any control system depends on the precision of the model (nonlinear or linear) of the plant to be controlled. This model can be obtained using physical laws or identification techniques. Practicing control engineers can use models to synthesize the appropriate controller to guarantee the required performances.

For the nonlinear case, the techniques are few and in general hard to apply. However, if we linearize the nonlinear model to get a linear one in the state-space representation, for instance, we can find in the literature many techniques that can be used to get the controller that guarantees the desired performances. There are now many controllers that we can design for linear systems, such as the famous PID (proportional, integral, and derivative) controller, the $\mathscr{H}_2$ controller, the $\mathscr{H}_\infty$ controller, the state feedback controller, the output feedback controller, and the observer-based output feedback controller. The linear model we will design for our dynamical system will be locally valid and, to prevent performance degradations, uncertainties will be introduced to describe the neglected dynamics or any other phenomena, such as aging.

In the literature we can find different types of uncertainties, among them the norm bounded, the polytopic, and the linear fractional transformation. Nowadays, there are interesting results for the analysis and design of the class of linear systems with or without uncertainties. We are also able to control systems with some special nonlinearities, like saturation, using different types of controllers such as the state feedback controller and the output feedback controller. The last two decades we have brought new control design tools that can be used to design control systems that meet the required specifications.

In practical systems, the state vector is often not available for feedback for practical reasons such as, the nonavailability of the appropriate sensor to measure the components of the state vector or limitations in the budget. Therefore the design of an appropriate filter is required to estimate the state vector that can be used for control purposes. Many techniques can be used to estimate the state vector, including $\mathscr{H}_2$ filtering and $\mathscr{H}_\infty$ filtering.

In practice, some industrial systems, such as those with abrupt changes in their dynamics, can not be appropriately described by the famous linear time-invariant state-space representation. Such systems can be adequately described by the class of stochastic switching systems called piecewise deterministic systems or jump systems, which have two components in the state vector. The first component of this state vector takes values in $\mathbb{R}^n$ and evolves continuously in time, it represents the classical state vector generally used in modern control theory. The second takes values in a finite set and switches in a random manner between the finite number of states. This switching is represented by a continuous-time Markov process taking values in a finite space. The state vector of the class of piecewise deterministic systems is usually denoted by $(x(t), r(t))$. This class of systems has been successfully used to model different practical systems such as manufacturing systems, communications systems, aerospace systems, power systems, and economics systems.

This book gives up-to-date approaches for the analysis and design of control systems for the class of piecewise deterministic systems with or without uncertainties in the system matrices and/or in the transition probability rate matrix. This book can be used as a textbook for graduate-level engineering courses or as a reference for practicing control engineers and researchers in control engineering. Prerequisites to this book are elementary courses on mathematics, matrix theory, probability, optimization techniques, and control system theory.

We are deeply indebted to our colleagues P. Shi, V. Dragan, S. Al-Amer, A. Benzaouia, H. Liu and O. L. V. Costa for reading the manuscript, in full or in part, and making corrections and suggestions. We would also like to thank students J. Raouf and V. Remillard for their help in solving some of the examples in the book.

The draft of this book was completed in April 2004. We added new results that are related to the topics covered by this book as we became aware of them through journals and conference proceedings. However, because of the rapid developments of the subjects, it is possible that we inadvertently omitted some results and references. We apologize to any author or reader who feels that we have not given credit where it is due.

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Montréal, Canada
April 25th, 2005

Contents

1 Introduction

This chapter introduces the class of stochastic switching systems we discuss in this book by giving the motivation for studying it. After giving some practical systems, it also defines the problems we deal with. The contents of this book can be viewed as an extension of the class of linear time-invariant systems studied extensively in the last few decades. As will be shown by some examples, this class of systems is more general since it allows the modeling of systems with some abrupt changes in the state equation that cannot be described using the class of linear time-invariant systems. In this volume we concentrate mainly on the linear case, which has been extensively studied and reported in the literature. References [52, 12, 45, 51] and the references therein are particularly noted. But we would like to advise the reader that nonlinear models have also been introduced; we again refer the reader to [12, 45, 52] and the references therein.

1.1 Overview

Linear time-invariant systems have been and continue to be the engine of control theory development. They have been successfully used to model different industrial systems. Most running industrial plants are designed based on the theory of such a class of systems.

Systems with nonlinear behavior are generally linearized around an operating point; the theory of linear systems is then used for the analysis and design. Sometimes, when the nonlinearities are critical, it is preferable to use a nonlinear model for the analysis and design.

Nowadays there are interesting results on such a class of linear systems that can be used to analyze and design control systems. Among the problems that have been successfully solved are the stability problem, the stabilization problem, the filtering problem, and their robustness. Controllers such as the state feedback and the dynamic output feedback (or the special observer-based output control) are usually used in the stabilization problems. Various design approaches that use the algebraic Riccati equation (ARE), linear matrix inequalities (LMIs), among others, have been developed. For more details on these results, we refer the reader to [31, 41, 59, 66, 12] and the references therein.

In practice, some industrial systems cannot be represented by the class of linear time-invariant model since the behavior of the state equation of these systems is random with some special features. As examples we mention those with abrupt changes and breakdowns of components. Such classes of dynamical systems can be adequately described by the class of stochastic switching systems or the class of piecewise deterministic systems, which is the subject of this book.

If we restrict ourselves to the continuous-time version, this class of systems was introduced by Krasovskii and Lidskii [48]. These two authors built the formalism of this class of systems and studied the optimal control problem. In 1969, Sworder [60] studied the jump linear regulator. In 1971, Wonham [63] extended the formalism of the class of systems to include Gaussian noise in the state equation and studied the stability problem and the jump linear quadratic optimization problem. In 1990, Mariton summarized the established results, including his results and those of other researchers in his book [52]. In 1990, Ji and Chizeck [44] studied the controllability, observability, stability, and stabilizability problems. They also considered the jump linear regulator by developing the coupled set of Riccati equations. In 1993 de Souza and Fragoso [33] studied the $\mathscr{H}_\infty$ control problem. In 1995, Boukas [9] studied the robust stability of this class of systems. In all these contributions, the results are stated in the form of Riccati equations for the optimization problem or Lyapunov equations for the stability problem.

In the last decade, with the introduction of LMIs in control theory, we have seen the use of this technique for some results on the class of piecewise deterministic systems. Most of the problems like stability, stabilization, $\mathscr{H}_\infty$ control, and filtering. have been tackled and LMI results have been reported in the literature.

Among the authors who contributed to the stability problem and/or its robustness are Wonham [63], Ji and Chizeck [44], Feng et al. [40], Boukas [9], Dragan and Morozan [34, 35], Shi et al. [57], Benjelloun and Boukas [6], Boukas and Liu [14, 11, 13, 10], Boukas and Shi [16] , Boukas and Yang [20, 19], Costa and Boukas [25], Costa and Fragoso [28, 27], and Kats and Martynyuk [45]. For more details on the recent review of the literature on this topic, we refer the reader to Boukas and Liu [12], Kats and Martynyuk [45], Mahmoud and Shi [51], and the references therein. The existing results are either in the form of Lyapunov equations or LMIs. The stabilization problem has also attracted many researchers and interesting results have been reported in the literature: Ji and Chizeck [44], Benjelloun et al. [8], Boukas and Liu [13, 15, 11], Boukas et al. [18], Cao and Lam [22], Shi and Boukas [56], de Souza and Fragoso [33], Ait-Rami and El-Ghaoui [1], Bao et al. [5], Dragan and Morozan [34, 35], Ezzine and Karvaoglu [39], Costa et al. [26]. For more details, we refer the reader to

Boukas and Liu [12] and Mahmoud and Shi [51] and the references therein. Among the stabilization techniques that were studied are the state feedback stabilization, output feedback stabilization, $\mathscr{H}_\infty$ state feedback stabilization, and $\mathscr{H}_\infty$ output feedback stabilization. Among the authors who tackled the state feedback stabilization are Ji and Chizeck [44], Ait-Rami and El-Ghaoui [1], Bao et al. [5], Benjelloun et al. [8], Boukas and Liu [15, 11], Boukas et al. [18], Costa and Boukas [25], Dragan and Morozan [34, 35], and the references therein. For the $\mathscr{H}_\infty$ stabilization we quote the work of Aliyu and Boukas [2, 3], Benjelloun et al. [7], Boukas and Liu [10, 13, 14], Boukas and Shi [17], Cao and Lam [22, 23], Cao et al. [24], Costa and Marques [30], Dragan et al. [36], and the references therein. The filtering problem has been studied by Boukas and Liu [11], Costa and Guerra [29], Dufour and Bertrand [37, 38], Liu et al. [50], Shi et al. [58], Wang et al. [62], Xu et al. [65], and the references therein.

Manufacturing systems, power systems, communications systems, and aerospace systems are some applications in which this class of systems has been used successfully to model industrial plants. In manufacturing systems, for instance, piecewise deterministic systems were used to model production planning and/or maintenance planning. Olsder and Suri [54] were the first to use the formalism in manufacturing systems and studied the production planning with failure-prone machines. After 1980, the model was extended by many authors and other optimization problems were considered. Among the authors who contributed to the field are Gershwin and his coauthors, Zhang and his coauthors, and Boukas and his coauthors. The books of Gershwin [42] and Sethi and Zhang [55] and the references therein summarize most of the contributions in this area up to 1994. In this direction of research, most of the authors are interested by developing production and/or maintenance planning. Their methodology, used to develop the production and/or maintenance policies is, in general, dynamic programming and some computation tools.

1.2 State-Space Representation

Mathematically a dynamical system can be interpreted as an operator that maps the inputs to outputs. More specifically, if the system represents an industrial plant P that has as inputs $u(t)$ and $w(t)$ and outputs $y(t)$ and $z(t)$, the relationship between these inputs and outputs is given by the following equation:

$$\begin{bmatrix} y(t) \\ z(t) \end{bmatrix} = P \begin{bmatrix} u(t) \\ w(t) \end{bmatrix}. \tag{1.1}$$

The vectors $y(t)$ and $z(t)$ are referred to, respectively, as the measured output and the controlled output. More often, the measured output $y(t)$ is used to design a control law $u(.)$ that, maps this output to an action that will give to the closed loop system the desired behavior for the controlled output $z(t)$, despite the presence of exogenous input $w(t)$. Mathematically, this is represented by

$$u(t) = K(y(t)). \tag{1.2}$$

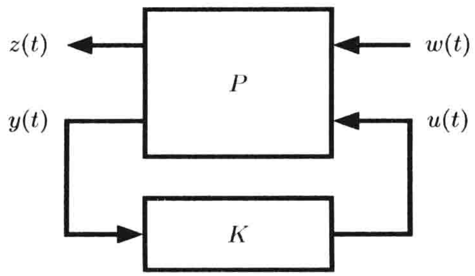

Fig. 1.1. Feedback system block diagram.

Control engineers often represent this operator when controlled in a closed loop by block diagram as illustrated in Figure 1.1. The inputs and the outputs are almost time varying and are linked by the following dynamics:

$$\begin{cases} \dot{x}(t) = f(x(t), u(t), w(t)), \\ y(t) = g(x(t), u(t)), \\ z(t) = h(x(t), u(t)), \end{cases} \tag{1.3}$$

where $x(t) \in \mathbb{R}^n$; $u(t) \in \mathbb{R}^m$; $y(t) \in \mathbb{R}^p$, $z(t) \in \mathbb{R}^q$, and $w(t) \in \mathbb{R}^s$ represent, respectively, the state vector, input vector, measured output vector, controlled output of the system at time t, and exogenous input that has to satisfy some conditions as it will be presented further, $f(.)$, $g(.)$, and $h(.)$ are given smooth vector-valued functions.

Remark 1. In (1.3) the functions $f(.)$, $g(.)$, and $h(.)$ are in general nonlinear in their arguments. The first equation is a differential equation that is referred to as the state equation and the second and the third are pure algebraic equations that represent, respectively, the output equations for $y(t)$ and $z(t)$.

This nonlinear model can always be linearized around the equilibrium point $(0, 0)$, which gives

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + B_w w(t), \\ y(t) = C_y x(t) + D_y u(t), \\ z(t) = C_z x(t) + D_z u(t), \end{cases} \tag{1.4}$$

where A, B, B_w, C_y, D_y, C_z, and D_z are appropriate constant matrices with appropriate dimensions.

In general, this linearized model will never represent adequately the nonlinear dynamical system. The following model is used to take care of the uncertainties that may represent the neglected dynamics, for instance, and the effect of external random disturbances:

$$\begin{cases} dx(t) = \left[A + \Delta A(t)\right] x(t)dt + \left[B + \Delta B(t)\right] u(t)dt \\ \qquad\qquad +B_w w dt + \mathbb{W}_1 x(t) d\omega(t), \\ y(t) = \left[C_y + \Delta C_y(t)\right] x(t) + \left[D_y + \Delta D_y(t)\right] u(t) + \mathbb{W}_2 w(t), \\ z(t) = \left[C_z + \Delta C_z(t)\right] x(t) + \left[D_z + \Delta D_z(t)\right] u(t), \end{cases} \tag{1.5}$$

where the matrices A, B, B_w, C_y, D_y, C_z, and D_z keep the same meaning; as before; $\Delta A(t)$, $\Delta B(t)$, $\Delta C_y(t)$, $\Delta D_y(t)$, $\Delta C_z(t)$, and $\Delta D_z(t)$ represent, respectively, the uncertainties in the matrices A, B, C_y, D_y, C_z, and D_z; $\mathbb{W}_1$ and $\mathbb{W}_2$ are given matrices with appropriate dimensions; $\omega(t)$ and $w(t)$ are external disturbances that have some properties to be discussed later in this book.

Sometimes systems cannot be put in the previous form for physical reasons. These systems are referred to as singular systems. The state equation of such a class of systems is given by the following:

$$\begin{cases} Edx(t) = Ax(t)dt + Bu(t)dt + B_w w(t)dt \\ \qquad\qquad +\mathbb{W}_1 x(t) d\omega(t), \\ y(t) = C_y x(t) + D_y u(t) + \mathbb{W}_2 w(t), \\ z(t) = C_z x(t) + D_z u(t), \end{cases} \tag{1.6}$$

where the matrices A, B, B_w, C_y, D_y, C_z, D_z, $\mathbb{W}_1$, and $\mathbb{W}_2$ keep the same meaning as before and E is singular matrix that has a rank equal to n_E, which is less than n (the dimension of the system).

The uncertain model is given in a similar way to the regular one as follows:

$$\begin{cases} Edx(t) = \left[A + \Delta A(t)\right] x(t)dt + \left[B + \Delta B(t)\right] u(t)dt \\ \qquad\qquad +B_w w(t)dt + \mathbb{W}_1 x(t) d\omega(t), \\ y(t) = \left[C_y + \Delta C_y(t)\right] x(t) + \left[D_y + \Delta D_y(t)\right] u(t) + \mathbb{W}_2 w(t), \\ z(t) = \left[C_z + \Delta C_z(t)\right] x(t) + \left[D_z + \Delta D_z(t)\right] u(t), \end{cases} \tag{1.7}$$

where the different components keep the same meaning as before.

The models (1.4)–(1.7) have been extensively used to describe different type of systems. In the literature, we can find many references that deal with problems like stability, stabilizability, $\mathscr{H}_\infty$ control, filtering, and their robustness. For more information on these topics, we refer the reader to [31, 41, 59, 66, 12] and the references therein. Unfortunately these state equations cannot represent adequately some systems, such as those with abrupt changes. In the next section we will present a model that generalizes this one and that appropriately models the behavior of systems with breakdowns and abrupt changes in their dynamics.

1.3 Stochastic Switching Systems

Let us consider a simple system with the following dynamics:

$$\dot{x}(t) = a(t)x(t) + bu(t), x(0) = x_0, \tag{1.8}$$

where $x(t) \in \mathbb{R}$, $u(t) \in \mathbb{R}$, b is a given constant, and $a(t)$ is a Markov process that switches between two values a_1 and a_2 with the following transition rates matrix:

$$\Lambda = \begin{bmatrix} -p & p \\ q & -q \end{bmatrix},$$

where p and q are positive scalars.

The switches between the two modes are instantaneous and they occur randomly. Based on probability theory, we can find the steady-state probabilities that give how long the process $a(t)$ will spend in mode #1 and in mode #2, respectively. These two probabilities can be computed using the following relations:

$$\begin{aligned} \begin{bmatrix} \pi_1 & \pi_2 \end{bmatrix} \begin{bmatrix} -p & p \\ q & -q \end{bmatrix} &= 0, \\ \pi_1 + \pi_2 &= 1. \end{aligned}$$

The resolution of these equations gives

$$\begin{aligned} \pi_1 &= \frac{q}{p+q}, \\ \pi_2 &= \frac{p}{p+q}. \end{aligned}$$

When time t evolves, the state equation of the system will switch in random between the following two dynamics:

$$\begin{aligned} \dot{x}(t) &= a_1 x(t) + bu(t), \\ \dot{x}(t) &= a_2 x(t) + bu(t). \end{aligned}$$

This simple system belongs to the class of stochastic switching systems or piecewise deterministic systems. This class of systems is more general since it can be used to model practical systems with special features like breakdowns or abrupt changes in the parameters.

The question now is how to handle, for instance, the stability of such a system. Also, when the system with some appropriate scalars a_1, a_2, p, and q

is unstable, how can we design the appropriate controller that stochastically stabilizes the system? We can continue our list of problems until it is clear that the theory of LTI systems does not apply and some extensions are needed to handle the new problems raised.

Since the behavior of the system is stochastic, all the concepts should be stochastic. For the stability problem, the concept has been extended and two approaches are available. The first approach is due to Gihman and Skorohod [43]. The second one is due to Kushner [49] and is a direct extension of the Lyapunov approach that we will use extensively in the rest of this volume. Kushner's approach generalizes the Lyapunov approach to handle the stability of the class of systems we are dealing with here.

The class of piecewise deterministic systems is a switching class of systems that has two components in the state vector. The first component takes values in $\mathbb{R}^n$, evolves continuously in time, and represents the classical state vector that is usually used in the modern control theory. The second one takes values in a finite set and switches in a random manner between the finite number of states. This component is represented by a continuous-time Markov process. Usually the state vector of the class of piecewise deterministic systems is denoted by $(x(t), r(t))$. The evolution of this class of systems in time is comprised of two state equations, the switching and the continuous state equation described below.

- Switching: Let $\mathscr{S} = \{1, 2, \cdots, N\}$ be an index set. Let $\{r(t), t \geq 0\}$ be a continuous-time Markov process with right continuous trajectories taking values in $\mathscr{S}$ with the following stationary transition probabilities:

$$P\left[r(t+h) = j | r(t) = i\right] = \begin{cases} \lambda_{ij} h + o(h), & i \neq j, \\ 1 + \lambda_{ii} h + o(h), & \text{otherwise}, \end{cases} \tag{1.9}$$

  where $h > 0$; $\lim_{h \to 0} \frac{o(h)}{h} = 0$; and $\lambda_{ij} \geq 0$ is the transition probability rate from the mode i to the mode j at time t and $\lambda_{ii} = -\sum_{\substack{j=1,\\ j \neq i}}^{N} \lambda_{ij}$.

- Continuous state equation:

$$\begin{cases} dx(t) = A(r(t), t)x(t)dt + B(r(t), t)u(t)dt + B_w(r(t))w(t)dt \\ \qquad\qquad + \mathbb{W}_1(r(t))x(t)d\omega(t), x(0) = x_0, \\ y(t) = \left[C_y(r(t)) + \Delta C_y(r(t), t)\right] x(t) \\ \qquad\qquad + \left[D_y(r(t)) + \Delta D_y(r(t), t)\right] u(t) + \mathbb{W}_2(r(t))w(t), \\ z(t) = \left[C_z(r(t)) + \Delta C_z(r(t), t)\right] x(t) \\ \qquad\qquad + \left[D_z(r(t)) + \Delta D_z(r(t), t)\right] u(t), \end{cases} \tag{1.10}$$

  where $x(t) \in \mathbb{R}^n$ is the state vector at time t; $u(t) \in \mathbb{R}^p$ is the control at time t; $w(t) \in \mathbb{R}^m$ is an arbitrary external disturbance with norm-bounded energy or bounded average power; $\omega(t) \in \mathbb{R}$ is a standard Wiener process