



Series on Mathematics Education Vol. **6**

Edited by

Fan Lianghuo • Wong Ngai-Ying
Cai Jinfa • Li Shiqi

HOW CHINESE TEACH MATHEMATICS

Perspectives from Insiders



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Focusing on Chinese Mathematics Teaching, Teachers and Teacher Education: An Introduction

Since our first book, *How Chinese Learn Mathematics: Perspectives from Insiders* (hereafter called “*Insiders 1*”), was published in 2004, we as editors have received much encouraging feedback from educational researchers and practitioners, as well as general readers, from different countries. We have also gladly noticed that *Insiders 1* has been reviewed in leading research journals and other publication avenues in a number of countries including the US, UK, Germany, Singapore, and China.¹ In particular, soon after the publication of the book, we began to receive encouragement from our publisher and our advisors, as well as from some readers to publish a new book, or *Insiders 2*, with a particular focus on Chinese mathematics teaching.

Meanwhile, the last decade has continuously witnessed a growing interest in Chinese education and, in particular, Chinese mathematics education. This growth of interest is arguably related to the fact that, in

¹ For example, see Bishop, A. J. (2005). Review of “*How Chinese Learn Mathematics: Perspectives from Insiders*” [in Chinese]. *Journal of Mathematics Education*, 14(2), 100-102; Xie, L., & Zhao, X. (2006). How do we teach mathematics: Perspectives from *How Chinese Learn Mathematics?* [in Chinese]. *Hunan Education*, Issue No. 6, 44-45; Bishop, A. J. (2006). How Chinese learn mathematics perspectives from insiders (book review). *Asia Pacific Journal of Education*, 26(1), 127-129; Star, J. R., & Chang, K. -L. (2008). Looking inside Chinese mathematics education: A review of “*How Chinese learn mathematics: Perspectives from insiders*”. *Journal for Research in Mathematics Education*, 39(2), 213-216; and Jones, K. (2008). Book review—Windows on mathematics education research in mainland China: A thematic review. *Research in Mathematics Education*, 10(1), 107-113. Also see S. Lancaster’s (2005) online review on the Mathematical Association of American website at <http://www.maa.org/>.

the Programme for International Student Assessment (PISA) 2009 and 2012 assessments, the Chinese participating students from Shanghai schools, were the very best performers.² As a matter of fact, the Shanghai students' average scores in mathematics were 600 and 613 in PISA 2009 and 2012 respectively, while the average scores of the second best performers, Singapore students, were 562 and 573, with the international average for each assessment being 500. Although there exist different interpretations and views about the top performance of Shanghai students, the gaps measured using the average scores in these two large-scale assessments between the Chinese students and their counterparts from other countries are too large to be ignored. It is also notable that, after PISA 2009, Tucker and his colleagues published a book entitled *Surpassing Shanghai: An Agenda for American Education Built on the World's Leading Systems* (Tucker, 2011),³ and after PISA 2012, the UK government announced, in March 2014, its intention to recruit about 60 Shanghai mathematics teachers to work in English state schools to raise the standards of mathematics and close the gap between the two countries (Parton, 2014).⁴ It seems apparent that the interest in Chinese mathematics education will continue for many years, and in terms of academic research, many issues about Chinese mathematics education and Chinese students' performance need to be thoroughly examined.

The present book, or *Insiders 2*, is not only a continuation of our first book, *Insiders 1*, which focused on the learning side of Chinese mathematics education, but also in a sense a response, from a research perspective, to the on-going interest and scholarly discourse about Chinese mathematics teaching and learning.

² PISA is organized by the Organisation for Economic Co-operation and Development (OECD) starting from 2000 on a 3-year cycle. PISA 2009 is the first time that students from the Chinese mainland took part in the assessment (see: www.oecd.org/pisa). So far Chinese students from the mainland have not participated in another international comparative study, the Trends in Mathematics and Science Study (TIMSS).

³ Tucker, M. S. (Ed.). (2011). *Surpassing Shanghai: An agenda for American education built on the world's leading systems*. Cambridge, MA: Harvard Education Publishing Group.

⁴ Paton, G. (2014, March 14). Chinese teachers sent into English schools to boost results. *The Daily Telegraph*. Retrieved from <http://www.telegraph.co.uk>

The focus of this book is on the teaching side of Chinese mathematics education. More specifically, it is about teaching, teachers, and teacher education and professional development relating to Chinese mathematics education.

The book is organized into three main sections. In Section 1, **Historical and Contemporary Perspectives**, we start with Chapter 1 by DAI and CHEUNG, which aims to provide readers with a broad and historical perspective on traditional mathematical teaching in ancient China by examining the key values, thoughts, and approaches as documented in ancient Chinese mathematics texts. In contrast, Chapter 2 by FAN, MIAO and MOK examines contemporary international research and presents an up-to-date review on how modern Chinese mathematics teachers teach and pursue their pre-service training and in-service professional development, in which the crucial role of the Teaching Research Group in schools and Teaching Research Office at different government levels is worth particular attention.

Section 2, **Understanding the Chinese Ways of Teaching Mathematics**, contains 12 chapters, investigating a wide range of issues at both the macro- and micro- levels on how Chinese mathematics teachers teach mathematics. These investigations were undertaken by different researchers in different regions.

Both Chapters 3 and 4 involve some new theoretical models and frameworks to analyze and understand the Chinese ways of teaching mathematics. Chapter 3 by HUANG Rongjin, MILLER and TZUR presents a hybrid model consisting of a tripartite theoretical lens and hence provides a fine-grained examination of learning opportunities created via Chinese classroom instruction, particularly the general features of teaching with variation. Chapter 4 by WANG, CAI and HWANG presents another careful analysis of a model Chinese mathematics lesson and explores the discourse strategies the teacher used to achieve instructional coherence and more generally the features of classroom instruction in China (mainland), for which the authors also put forward a framework for examining instructional coherence.

Chapters 5 through 10 provide readers with an in-depth look into how Chinese mathematics teachers teach a variety of specific mathematics

topics. In Chapter 5, HUANG Hsin-Mei examines 12 instructional cases collected from elementary schools in Taiwan using videotaping and interviews and provides a portrait of how teachers conducted lessons and what they were concerned about when teaching length, area, and volume measurements, which the author intends to help in understanding such classroom practices. In Chapter 6, FANG looks at how an experienced secondary teacher in Shanghai explained student homework on geometric proofs and uncovers the hidden dimensions of mathematics teaching in Shanghai classrooms mediated through homework practice. The chapter sheds new light on the role and potential of homework in mathematics instruction.

Chapter 7 by LI Titus reports on primary students' ability in solving time interval problems in Macau, Hong Kong and the Netherlands, and reveals, through interviews and lesson observation, how time interval calculations are taught in each place. Chapter 8 by YANG Der-Ching, CHEN Pei-Chieh, TSAI Yi Fang and HSIEH reports how number sense was taught using interactive multimedia in a primary classroom in Taiwan and shows the differences in students' use of number sense strategies before and after the instruction. The results suggest that interactive multimedia can be an effective tool both in helping children develop number sense and in promoting children's motivation for learning. In Chapter 9, DING, JONES and ZHANG Dianzhou present a case study and analyze how an expert teacher in Shanghai used the "Shen Tou" ("permeation") method to teach theorems in geometry to an eighth-grade mathematics class, in which two key features of the instruction were identified: one is the complex learning support structure and the other the repetition and accumulation of practices of hierarchically-ordered skills and gradual understanding of the systematical connections of knowledge within the multiple-layered teaching procedures. In Chapter 10, HUANG Xingfeng, YANG and LI Shiqi examine three experienced Chinese teachers' teaching of the use of letters to represent numbers based on the new curriculum standards in four dimensions (strands), i.e., knowledge and skills, mathematical thinking, solving problem, and affect and attitude.

In contrast to the previous chapters, Chapters 11 to 14 focus more on general teaching approaches in Chinese mathematics classrooms. In

Chapter 11, XU and ZHU Guangtian look into two cases of project-based instruction, a relatively new development in Chinese mathematics teaching practices, and examine multiple aspects of students' engagement in the project-based classroom environment. In Chapter 12, LEE Yuan-Shun and LIN Fou-Lai investigate the teaching behaviors of Taiwanese mathematics teachers using a large-scale video survey of the fourth-grade classrooms across Taiwan, and reveal the general features of their mathematics teaching. Similarly, in Chapter 13, MA and ZHAO also examine the features of exemplary lessons in the Chinese mainland under the curriculum reform by studying 13 such lessons in elementary mathematics. The results show that these exemplary lessons not only practiced the advocated ideas of the new current reform, but also embodied some elements that might reflect the stable characteristics of Chinese mathematics classrooms. Finally, in Chapter 14, GU, YANG Yudong and HE present the Qingpu experiment, a very well-known mathematics teaching reform in the Chinese mainland, and examine its impact on the eighth-grade students' learning in mathematics.

Section 3, **Chinese Mathematics Teachers, Teacher Education and Teacher Professional Development**, comprises seven chapters, focusing on issues about Chinese mathematics teachers' knowledge, beliefs, and their professional development.

In Chapter 15, ZHANG Qiaoping and WONG first provide a review of the literature on how beliefs and knowledge influence mathematics teachers' teaching with a particular focus on studies conducted in the Chinese regions and then introduce readers to a series of five studies with different research methods to address how beliefs and knowledge affect the teaching of mathematics in the Chinese context. In Chapter 16, FAN, ZHU Yan and TANG investigate more than 30 mathematics master teachers in seven different regions of China through questionnaires and interviews, and examine the reasons behind the success of those master teachers in their acclaimed teaching careers. The results reveal that the master teachers valued internal (personal) factors more than external ones in their professional growth. In Chapter 17, CHEN Qian and LEUNG investigate three Chinese teachers' mathematics beliefs that were espoused and enacted in the context of a constructivism-oriented curriculum reform, and reveal the different beliefs of these teachers.

Focusing on teacher professional development, Chapter 18 by YUAN and LI Xuhui use a case study method to explore how “Same Content Different Designs” (SCDD, or *Tong Ke Yi Gou* in Chinese), which has become popular as a new form of teacher professional activity in the Chinese mainland, has an impact on the professional development of prospective mathematics teachers. They propose a model that characterizes the key components and stages of SCDD activities. In Chapter 19, JIN, LU and ZHONG examine Chinese mathematics teachers’ perceptions of concept map, and their incorporation, after brief training, of concept map in mathematics teaching through lesson plans and practical teaching. Their study reveals the importance of operational training and professional development for teachers to adopt new tools and ideas in their teaching. In Chapter 20, LIN Pi-Jen and TSAI Wen-Huan present a study on how the use of research-based cases in a teacher training programme in Taiwan enhanced mathematics teachers’ awareness of and abilities in maintaining high-level cognitive demands of mathematical tasks in classroom practice. The final chapter of this book, by LEU, CHAN and WONG, takes an in-depth look at the relationships between religious beliefs and teaching among mathematics teachers in the Chinese mainland, Taiwan and Hong Kong through a comprehensive review of the related literature and two empirical studies they conducted. It is a must read for any readers who are interested in this very specialized and under-researched topic.

It has taken quite a long time to reach the completion of this book since 2010 when we started calling for contributions to the book after having reached official agreement with the publisher in early 2010. For a project of this magnitude, it would be impossible without much support and help along the way from many people, and for this we wish to offer here our deep appreciation.

First, we would like to thank all the contributors who submitted their proposals and later initial manuscripts, and all the authors of accepted chapters, who we think have made great efforts in writing and revising their chapters.

Second, all the contributed manuscripts have gone through blind peer review by at least two and, in some cases, three or four colleagues, who are all university-based academics and/or hold doctoral degrees. We are

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This book intends to present another concerted effort from an international group of researchers from different parts of the world on the study of Chinese mathematics education. Nevertheless, it remains clear, or in a sense has even become clearer, to us that given the complex nature of teaching and learning, particularly in connection to the large variety of social, economic, cultural and even religious backgrounds of Chinese schools, students and teachers, many issues remain to be further studied in this area. Like our first book, *Insiders 1*, we hope that this present book, *Insiders 2*, will also make a meaningful contribution to the advancement of research in Chinese mathematics education, and hence more generally to that in the international mathematics education. We continue to welcome exchanges and feedback from colleagues and general readers as well.

FAN Lianghuo
WONG Ngai-Ying
CAI Jinfa
LI Shiqi

July 2014

Contents

Focusing on Chinese Mathematics Teaching, Teachers and Teacher Education: An Introduction	vii
Section 1 Historical and Contemporary Perspectives	
Chapter 1 The Wisdom of Traditional Mathematical Teaching in China	3
<i>DAI Qin, Inner Mongolia Normal University</i>	
<i>CHEUNG Ka Luen, Hong Kong Institute of Education</i>	
Chapter 2 How Chinese Teachers Teach Mathematics and Pursue Professional Development: Perspectives from Contemporary International Research	43
<i>FAN Lianghuo, The University of Southampton</i>	
<i>MIAO Zhenzhen, The University of Southampton</i>	
<i>MOK Ah Chee Ida, The University of Hong Kong</i>	
Section 2 Understanding the Chinese Ways of Teaching Mathematics	
Chapter 3 Mathematics Teaching in a Chinese Classroom: A Hybrid-Model Analysis of Opportunities for Students' Learning	73
<i>HUANG Rongjin, Middle Tennessee State University</i>	
<i>MILLER L. Diane, Middle Tennessee State University</i>	
<i>TZUR Ron, The University of Colorado Denver</i>	
Chapter 4 Achieving Coherence in the Mathematics Classroom: Toward a Framework for Examining Instructional Coherence	111
<i>WANG Tao, The University of Tulsa</i>	
<i>CAI Jinfa, The University of Delaware</i>	
<i>HWANG Stephen, The University of Delaware</i>	
Chapter 5 Elementary School Teachers' Instruction in Measurement: Cases of Classroom Teaching of Spatial Measurement in Taiwan	149
<i>HUANG Hsin-Mei E., University of Taipei</i>	

Chapter 6	Pedagogical and Curriculum Potentials of Homework: A Case Study about Geometric Proofs in Shanghai <i>FANG Yanping, Nanyang Technological University</i>	185
Chapter 7	Teaching Calculation of Time Intervals: Comparing Mathematics Competence of Students in Macau, Hong Kong and the Netherlands <i>LI Titus Siu Pang, Vrije Universiteit Amsterdam</i>	211
Chapter 8	Teaching Number Sense via Interactive Multimedia in a Primary School in Taiwan <i>YANG Der-Ching, National Chiayi University</i> <i>CHEN Pei-Chieh, Chiayi Dongshih Elementary School</i> <i>TSAI YI FANG, Yunlin Jhennan Elementary School</i> <i>HSIEH Tien-Yu, Chiayi Dongshih Elementary School</i>	243
Chapter 9	Teaching Geometrical Theorems in Grade 8 using the “ <i>Shen Tou</i> ” Method: A Case Study in Shanghai <i>DING Liping, Sør-Trøndelag University College</i> <i>JONES Keith, The University of Southampton</i> <i>ZHANG Dianzhou, East China Normal University</i>	279
Chapter 10	Implementation of Objectives Based on the Curriculum Standards: A Case of Teaching Using Letter to Represent Number at a Chinese Primary School in Chinese Mainland <i>HUANG Xingfeng, Changshu Institute of Technology</i> <i>YANG Jinglei, Changshu Institute of Technology</i> <i>LI Shiqi, East China Normal University</i>	313
Chapter 11	Chinese Project-based Classroom Practices: Promoting Students’ Engagement in Mathematical Activities <i>XU Binyan, East China Normal University</i> <i>ZHU Guangtian, East China Normal University</i>	337
Chapter 12	A Large-Scale Video Survey on Taiwanese Fourth-Grade Classrooms of Mathematical Teaching Behaviors <i>LEE Yuan-Shun, University of Taipei</i> <i>LIN Fou-Lai, National Taiwan Normal University</i>	373
Chapter 13	Features of Exemplary Lessons under the Curriculum Reform in Chinese Mainland: A Study of Thirteen Elementary Mathematics Lessons <i>MA Yunpeng, Northeast Normal University</i> <i>ZHAO Dongchen, Harbin Normal University</i>	408