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BERNARD
BOLZANO

THEORY OF SCIENCE

VOLUME TWO

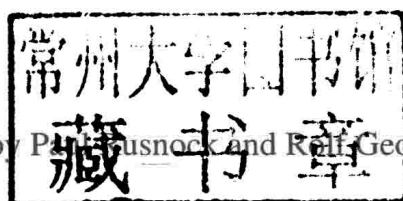
Translated by
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Bernard Bolzano

Theory of Science

Volume Two

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Bernard Bolzano

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INTRODUCTION TO VOLUME TWO

Volume two of the *Theory of Science* completes the Theory of Elements. In line with the then customary order, it contains two main parts, devoted to propositions (§§121–222) and inferences (§§223–268), respectively. The discussion of propositions is further divided into two parts, dealing with propositions in general (§§121–194) and true propositions (§§195–222). It is in the first of these subdivisions, rather than the part on inferences, where Bolzano presents his variation logic—the theory of consequence (which he calls deducibility) and related notions. The part on true propositions is devoted almost exclusively to the relation of ground to consequence, which Bolzano maintains is quite different from the relation of deducibility.

To our mind, the most noteworthy parts of Volume two are those dealing with propositional form, Bolzano's variation logic, and the theory of ground and consequence. Most of our introductory remarks will therefore deal with these topics. We close with a brief survey of Bolzano's theory of inferences.

On Propositions

Bolzano aimed not merely to set out his new theories, but also to convince his readers to accept them. To this end, he gave lengthy consideration to the views of other logicians, especially those most likely to be familiar to his readers, acknowledging agreement where it existed, and making a case for his position when it was lacking. In the case of the theory of propositions, a great many contemporary treatments of the subject, especially those written in German, closely followed Kant, though the views of Schelling and Hegel were starting to become more popular. Bolzano mainly addressed the Kantian bloc in his critical remarks, usually finding the texts of the absolute idealists either too obscure to assess or too outlandish to merit serious attention.

Though Kant's writings sparked renewed interest in logic, he also held views that strongly discouraged further research, since he maintained that this science had been complete from the time of Aristotle, neither requiring nor admitting any additions or essential improvements. This conservatism was especially apparent in Kant's remarks on logical form. He and his followers said repeatedly that logic dealt only with the *form* of thought, disregarding matter as lying outside its province, a claim that presupposed a sharp distinction between form and matter, one which presumably had been definitively

established by Aristotle. He evidently thought the question of logical form a relatively simple one, whose principles could be exhausted in a brief survey.¹

The particulars of Kant's doctrine, however, are disappointing. For example, in the Jäsche Logic, he had only this to say about the form-matter distinction with respect to concepts:

In every concept, form and matter are to be distinguished. The matter of a concept is the object, its form is generality.²

Kant's remarks about judgements were not terribly helpful either. In the case of categorical judgements, he tells us, the subject- and predicate-concepts are the matter, the copula the form; in hypothetical judgements, the antecedent and consequent categorical judgements are said to be the matter, the *consequentia* (i.e., what is expressed by 'if . . . then') the form; and similarly for disjunctive judgements.³

There are a number of problems with these characterisations. Notice, to begin with, that the copula, which belongs to the form in categorical judgements, is part of the matter in hypothetical judgements, so that the same element is claimed to be both formal and material, suggesting that the form-matter distinction is not absolute after all. Furthermore, consider the distinction between analytic and synthetic judgements in Kant's sense, e.g.:

- "Extension (b) belongs to every x to which the concept of a body (a+b) belongs" is an example of an analytic proposition.
- "Attraction (c) belongs to every x to which the concept of body (a+b) belongs" is an example of a synthetic proposition.⁴

To modern eyes, Kant's examples would seem to indicate quite clearly that this is a *formal* distinction, for the difference between "Every (a+b) is b" and "Every (a+b) is c" seems to be obviously one of form. Yet if we take Kant at his word, this cannot be, since, according to his definition, the form (i.e., the copula) is the same in all categorical judgements, regardless of whether they are analytic or synthetic.

Finally, there was the famous Kantian table of judgements, which was supposed to be the last word on what Bolzano would have called propositional form:

¹ See, for example, *Critique of Pure Reason*, B viii–ix.

² *Logik*, ed. Jäsche, §2 (Academy edition, Vol. 9, p. 91). Bolzano discusses the Kantian accounts of the form-matter distinction for concepts in §116, in Volume 1.

³ *Logik*, ed. Jäsche, §§24, 25, 27 (Academy edition, Vol. 9, pp. 105–106).

⁴ *Logik*, ed. Jäsche, §36 (Academy edition, Vol. 9, p. 111).

| <i>Quantity</i> | <i>Quality</i> | <i>Relation</i> | <i>Modality</i> |
|-----------------|----------------|-----------------|-----------------|
| Universal | Affirmative | Categorical | Problematic |
| Particular | Negative | Hypothetical | Assertoric |
| Singular | Infinite | Disjunctive | Apodeictic |

Kant's table, as noted by the Kneales in their classic history of logic, is a rather ramshackle affair, which moreover is inconsistent with the claim that *Aristotle* had completed logic:

The concern which Kant shows . . . for the purity of logic is difficult to reconcile with his own practice. . . . For it was he with his transcendentalism who began the production of the curious mixture of metaphysics and epistemology which was presented as logic by Hegel and the other Idealists of the nineteenth century. But it is more interesting to notice that he was apparently unaware of the value of any contributions made to logic after the time of Aristotle, and that the doctrine which he regarded as the complete and perfect discovery of Aristotle was in fact a peculiarly confused version of the traditional mixture of Aristotelian and Stoic elements.

[. . .]

Kant seems to attach some importance to the symmetry of his scheme, and he apparently holds (a) that every judgement can be placed in one of the three divisions under each heading, and (b) that each division under one heading can be combined with each division under each of the others. These assumptions are mistaken. We cannot, for example, have a negative hypothetical judgement. For the presence of negation in either the antecedent or the consequent of a hypothetical judgement does not make the whole negative, and the negation of the whole is not itself hypothetical. [Etc.]¹

Bolzano devotes a long appendix to a critical examination of the Kantian table along with the various justifications that had been offered for it.² It will come as little surprise that he does not consider the table well-conceived, still less definitive. More aware of the history of the science and its current state,

¹ William and Martha Kneale, *The Development of Logic* (Oxford: Clarendon Press, 1962), p. 355–356.

² §§185 ff.

he also found Kant's remarks on the completed state of logic most unfortunate:

[O]ne of KANT's literary sins was that he attempted to deprive us of a wholesome faith in the perfectibility of logic through an assertion very welcome to human indolence, namely, that *logic is a science which has been complete and closed since the time of Aristotle*. It seems to me that it would be much better to assert as a kind of practical postulate that faith in the perpetual perfectibility not only of *logic* but of *all science* should be maintained. And what is it at bottom other than pride which would lead us to claim that in all future time a certain science will not be presented in a better and more perfect way than it appears at present (namely, through our efforts)?¹

His own views are worked out in terms of a radically different notion of form. Thinking that previous logicians must have been on to something in their insistence that logic deals with forms (of concepts, propositions, inferences, etc.), he looked for a reasonable sense to attach to this expression.

[P]erhaps there is another sense in which it can be justified. This must indeed be expected, since otherwise this view could hardly have come to be so widely accepted. Let us then find this sense. Nobody can deny that the distinctions that are made in logic should be of such a kind that every special heading should encompass an entire species of propositions, i.e., not only a single, but several different propositions. From this it follows immediately that the attributes which determine the logical category into which a certain proposition belongs can only concern matters which several propositions have in common. Now if such attributes are called the common *form* of these propositions, i.e., their shape, then we can justly claim that all distinctions made in logic concern only their form, i.e., only matters which several, or even infinitely many, propositions have in common.

All the same, he continues,

I do not think . . . that these remarks suffice to determine whether a given distinction belongs in logic. For, according to this definition, there will be infinitely many distinctions which concern

¹ §9, note 2 [I.40].

only the forms of propositions; but logic is supposed to distinguish only forms that have a particular use, i.e., it is to introduce us only to those kinds of propositions that require a special scientific investigation.¹

As we noted in our introduction to Volume One, Bolzano takes ‘form’ as used in logic to refer primarily to classes of logical objects (ideas, propositions, inferences, etc.) with some features in common or, more properly, to linguistic expressions containing signs for variables which determine such classes. Corresponding to the class containing:

Socrates has wisdom, Aristotle has wisdom, etc.

for example, we might have the expression:

X has wisdom

along with a rule telling us what sorts of ideas can occupy the place indicated by ‘ X ’.

With this extremely general notion of form, it is clear that every proposition, no matter how simple, belongs to several different forms. The proposition “Socrates has wisdom”, for example, would belong not only to the form ‘ X has wisdom’, but also to ‘Socrates has y ’, ‘ X has y ’, and others still, given the complexity of the ideas “Socrates” and “wisdom”. One cannot therefore speak of *the* form of a proposition, if this is taken to indicate uniqueness. Instead, what counts as form, what as matter in a given logical object may change from one context to another. Consider, for example, a proposition such as:

If $3 < 4$ then $4 \not< 3$

We might consider it to belong to the following form:

If aRb , then $\neg bRa$.

But, depending upon the direction of our inquiries, we might just as easily consider it to belong to the form:

If $a < b$ then $b \not< a$.

Accordingly, the concept “ $<$ ” could be treated as belonging either to the matter or to the form of the proposition. The choice is ours according to Bolzano, for the distinction between form and matter is one of our making. In brief:

¹§186 [II.251–252].

In my opinion, what belongs to the *form* of a logical object are those of its attributes which determine the kind of object the logician considers it to be. Everything else which belongs to it only accidentally belongs to its *matter*.¹

It is sometimes thought that, once one reaches the *logical* concepts, one encounters pure form. Bolzano is opposed to such a view.² For him, recall, a concept is a kind of idea, and an idea is simply any part of a proposition that is not itself a complete proposition.³ Under this definition, even words such as ‘not’, ‘and’, ‘if . . . then’ designate ideas. This does not mean that they refer to objects—Bolzano understood that meaning could not be reduced to a sign standing for one or more objects—nonetheless, they designate elements which contribute to the determination of truth conditions. As such, they have as much right to be considered material elements of propositions as any other parts. Moreover, he thought that the boundaries between sciences were fixed in large part by pragmatic considerations. What counts as logical, what extra-logical is not determined, as the Kantians supposed, by a fundamental distinction rooted in the structure of the mind. Rather, this is something that is worked out based on human needs. If logicians, for example, decide that the theory of order (or set-membership, or what have you) may usefully be treated in logic and the consensus of scientific opinion agrees with this decision, then the concept of order becomes *ipso facto* a logical notion (which is not to say that it cannot also be a mathematical concept at the same time). Thus, although he had a perfectly serviceable general conception of form, *logical* form remained an open question for Bolzano: he did not claim to have a definitive account himself, and thought it unlikely that anyone else would be in a position to provide one anytime soon, if ever.

Volume two also contains proposed analyses of various kinds of propositions. Since propositions occur in sciences primarily in written form, these analyses are essentially linguistic in character. That is, Bolzano considers common sentence forms and proposes that we use others, supposed better to reflect the structure of the propositions expressed, in their stead. Although he does not go into much detail concerning the constraints of such analysis, several of his remarks make it clear that the theory of inferences is to play an important role.⁴ Our inferential practice is usually if not invariably guided by linguistic forms, and these may lead us astray, producing correct results

¹ §254 [II.516].

² See §12, no. 2 in Volume 1.

³ §48, in Volume 1.

⁴ See, e.g., §366, in Volume 3. Introspection is also involved.

in some cases but incorrect ones in others. That is, while two sentences may have the same truth-conditions, one may be more likely than the other to lead us into mistaken inferences. A perfect expression of a proposition, one that shows all of its simple parts along with the manner of their combination, will, he thinks, not give rise to any such mistaken inferences. But even with imperfect expressions, some will be better, some worse in this respect and thus more or less likely to give rise to errors. Bolzano will speak in such cases of expressions that are more or less *distinct*.

Consider, for example, a sentence such as ‘It is snowing.’ If this linguistic form were taken to reflect the structure of the proposition it expresses, then the word ‘it’ would designate the subject-idea. Now traditional logic as well as common usage agree that in true propositions the subject-idea represents one or more objects. Accordingly, ‘it’ as it occurs in a true utterance of the sentence ‘It is snowing’ would represent some object or other, and from the truth of ‘It’s snowing’ we could infer the truth of ‘It exists,’ and embark upon a Quixotic search for the referent of ‘it’. If, by contrast, we paraphrase as follows: “The idea of a snowfall here and now has an object”, the proposition does concern an object (namely, an idea) and this pseudo-problem disappears, suggesting that this paraphrase is an improvement upon the original at least in this respect.¹

Given the generality of his conception of form, modern readers may be disappointed in some of the details of Bolzano’s analyses, which, at first blush, may seem merely to reflect logical tradition. To begin with, he argues, based largely on a linguistic analysis, that all propositions stand under the subject-predicate form “A has *b*”, where “has” is without distinguished tense or number.² In the case of true propositions, at least, the idea “A” represents one or more objects, and the idea “*b*” one or more attributes.³ Since he was by no means unaware of the considerable variety of different sentence forms, this thesis committed him to an ambitious program of paraphrase, recasting sentences of every sort (even some that are already in subject-predicate form) into his preferred idiom. Bolzano thought this exercise important not merely for the sake of defending his position, but also for becoming clear about the truth-conditions of sentences.

In some cases, at least, we can agree that he has a point. In many presentations of traditional logic, for example, it was said that the subject of a proposition of the form “Some A are B” was “Some A”, and also that a

¹Cf. §172.

²§§126–127.

³§196, nos. 2 and 3.

proposition was true if the objects represented by the predicate included the objects represented by the subject. But, as Bolzano observes, the idea “Some men”, for example, certainly represents, say, Caligula and Nero. If “Some men” were indeed the subject of the proposition “Some men are virtuous”, he observes, we would then seem to be obliged to say that this proposition was about Caligula and Nero, among others, and the truth of the proposition would entail the virtuousness of these men.¹ Instead, Bolzano proposes the paraphrase: “The idea of an *A* which is *B* has objectuality.” On his rendering, the proposition is not about men at all, at least not in the first instance, but rather about the *idea* “virtuous man”, and the predicate, *objectuality*, is one that applies specifically to ideas, representing the attribute of having objects. We can immediately see the kinship of this paraphrase with the modern rendering ‘ $\exists x(Ax \wedge Bx)$ ’, and it seems reasonable to regard Bolzano’s concept “objectuality” as a near-equivalent to the modern existential quantifier.

Similar remarks apply to Bolzano’s treatment of existential propositions, of which he distinguishes two quite different sorts (§§137, 142). On the one hand, we have claims such as ‘*There are* prime numbers of the form $2^p - 1$.’ In such cases, Bolzano thinks that what is really being said is that a certain idea (here, “prime number of the form $2^p - 1$ ”) has at least one object, or, more precisely, “The idea of a prime number of the form $2^p - 1$ has objectuality.” On the other hand, we also find claims where ‘existence’ appears to designate an attribute that some objects have and others lack. One might say, for example, that while mathematical objects such as numbers do not really exist, giraffes do. In the second sort of claim, Bolzano maintains, there is no doubt that objects are spoken of, and existence treated as an attribute. He thus rejects Kant’s well-known claim that existence is not a property, and with it an often repeated criticism of the ontological argument for the existence of God.²

Bolzano’s analysis of existential propositions allows him to dispense easily with the ancient riddle of non-being, according to which one cannot deny

¹ §173.

² This is not to say that Bolzano thought the ontological argument valid. In an unpublished manuscript entitled “Improvements and additions to the Logic (i.e., the *Theory of Science*)” (*Bernard Bolzano-Gesamtausgabe*, Series 2A, Vol. 12/2, p. 103), he diagnoses the error of that argument as follows: “*Judgements of existence*: The flaw of the ontological proof of the existence of God did not lie in the fact that existence is not an attribute, as Kant had claimed. Rather, the flaw is that, while existence is presupposed of every being [*Wesen*], it must first be proved that the concept of the being is an objectual one.” Cf. F. Příhonský, *Neuer Anti-Kant; oder, Prüfung der Kritik der reinen Vernunft nach den in Bolzano’s Wissenschaftslehre niedergelegten Begriffen* (Bautzen, 1850), p. 147.

the existence of something without falling into contradictions. For, it was argued, to deny the existence of something, one must speak of that thing. But to speak meaningfully of a thing, the thing must be. Thus any denial of the existence of a thing must be either false or meaningless. Bolzano agreed that in a true proposition the subject-idea must have an object. But in the case of denials of existence, this object is not what the above argument supposes:

The matter is similar with those truths which seem to be concerned with so-called *imaginary objects*, such as “There is no round square.” We already know from §137 that propositions of this kind actually have the following sense: “The idea of a round square lacks objectuality.” Hence its subject is again an objectual idea, for only the idea “round square” is objectless; the idea of this idea (which is the subject of the proposition) is an objectual idea. Its object is that first idea.¹

Although the paraphrases Bolzano proposes are often inelegant, they do indicate that the subject-predicate form is far more plastic than has been generally recognised (one sometimes hears it said that the subject-predicate form can at most capture the logical forms of monadic first-order predicate calculi). Relations, for example, are easily accommodated within Bolzano’s subject-predicate form, a statement that a relation holds between given objects being interpreted as a statement about a *collection*. A sentence such as ‘ $2 < 3$ ’, for example, can be construed as a claim about an ordered pair, i.e., $\langle 2, 3 \rangle$, in much the same way that one encounters in contemporary logic.²

Statements of equality provide another interesting example.³ Bolzano considers, and then rejects, the view that ‘is equal to’ is the copula of the proposition expressed by a sentence of the form ‘ A is equal to B .’ This traditional interpretation of such statements suffers from two flaws, he observes: first, it supposes that such claims recognise a significant difference between A and B , in that one of them is distinguished as the subject, the other as the predicate; second, it presupposes that equality is a relation that only obtains between pairs of objects. Both of these assumptions, he argues, are false. Equality being by its nature a symmetric relation, there is no difference in meaning between the sentences ‘ A is equal to B ’ and ‘ B is equal to A .’ Furthermore, he maintains quite reasonably that statements of equality often cover infinitely many objects, e.g., when one says that all of the fractions in

¹§196 [II.329].

²Cf. §135.

³§135.

the set $\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \dots, \frac{n}{3n}, \dots\}$ are equal. He accordingly interprets statements of equality as statements about unordered collections (or sets), and thus brings them under his general subject-predicate form.

Bolzano recognises two different kinds of negation (§136). In one kind, negation attaches to the predicate, as in the proposition “Caius is unwise” or “Caius has lack-of-wisdom”. In the second kind, the scope of the negation is the entire proposition, i.e., “Not (Caius has wisdom)” or, more distinctly in Bolzano’s estimation, “The proposition ‘Caius has wisdom’ has lack-of-truth.” The difference between the two kinds of negation becomes especially evident in the case of propositions with objectless subject-ideas, e.g.:

Erewhon is inhabited.

With such propositions, the predicate negation (“Erewhon is uninhabited.”) is also false, as it too has an objectless subject-idea, while the propositional negation:

The proposition “Erewhon is inhabited” has falsity.

is true, the subject idea having as its only object the original proposition.

An especially important distinction, between purely conceptual and intuitional propositions, is introduced in §133. Recall that Bolzano defined an intuition as a simple idea with exactly one object.¹ Pure concepts were defined as ideas that are not intuitions and contain no intuitions as parts. Similarly, a proposition is called *purely conceptual* if none of its parts are intuitions. The remaining propositions are called intuitional. The distinction between conceptual and intuitional propositions, Bolzano observes, nearly but not exactly corresponds to the traditional distinction between propositions whose truth or falsity may be determined *a priori* and those whose truth or falsity may only be determined *a posteriori*.

Recalling Bolzano’s insistence that in the Theory of Elements we should only consider propositions and ideas in and of themselves, regardless of whether they have been thought or expressed, we can understand why he thought it inappropriate to classify propositions in themselves as either *a priori* or *a posteriori*. All the same, his distinction is, in part at least, an attempt to explain why only certain propositions may be known *a priori*. The objects of all humanly attainable intuitions, recall, are supposed to be presently occurring changes in the mind of the being who forms the intuitions. These objects are contingently existing particulars, and it would seem that even to form a simple idea of such an object is already to have a primitive kind of experience, or

¹ §72, in Volume 1.