# Quantum Transport in Semiconductor Submicron Structures

Edited by

Bernhard Kramer

NATO ASI Series

Series E: Applied Sciences - Vol. 326

# Quantum Transport in Semiconductor Submicron Structures

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**Kluwer Academic Publishers**

Dordrecht / Boston / London

Published in cooperation with NATO Scientific Affairs Division

Proceedings of the NATO Advanced Study Institute on
Quantum Transport in Semiconductor Submicron Structures
Bad Lauterberg, Germany
August 20–31, 1995

A C.I.P. Catalogue record for this book is available from the Library of Congress.

ISBN 0-7923-4190-2

Published by Kluwer Academic Publishers,
P.O. Box 17, 3300 AA Dordrecht, The Netherlands.

Kluwer Academic Publishers incorporates the publishing programmes of
D. Reidel, Martinus Nijhoff, Dr W. Junk and MTP Press.

Sold and distributed in the U.S.A. and Canada
by Kluwer Academic Publishers,
101 Philip Drive, Norwell, MA 02061, U.S.A.

In all other countries, sold and distributed
by Kluwer Academic Publishers Group,
P.O. Box 322, 3300 AH Dordrecht, The Netherlands.

*Printed on acid-free paper*



Printed in the Netherlands

This book contains the proceedings of a NATO Advanced Study Institute held within the programme of activities of the NATO Special Programme on *Nanoscale Science* as part of the activities of the NATO Science Committee.

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# CHAPTER 1

# Introduction

# QUANTUM TRANSPORT IN NANO–STRUCTURED SEMICONDUCTORS

*A Survey*

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**Abstract.** The quantum transport effects in semiconductor nano–structures discovered during the past two decades are summarized. Brief physical arguments for their explanation are provided. Possible directions of future research are outlined.

Due to their unique adjustability of charge carrier density by external means, semiconductor inversion layers have been proven to provide an outstanding laboratory for the investigation of quantum mechanical phenomena in condensed matter. During the past two decades, a great variety of hitherto unforeseen quantization and coherence effects in their electrical transport properties have been discovered. The most prominent example is the quantum Hall effect. The finding of the quantization of the Hall conductivity of MOSFETs in integer multiples of $e^2/h$ at low temperatures and sufficiently strong magnetic fields initiated an "industry" of experimental and theoretical research. The *Integer Quantum Hall* effect established a completely new tool for the investigation of localization phenomena. The subsequent discovery of the *Fractional Quantum Hall* effect gave rise to totally unexpected developments concerning the effects of the Coulomb interaction. Novel phases of the interacting two dimensional electronic system, like the "incompressible electron fluid", were found. New routes to well known concepts like the Wigner crystal suddenly became experimentally accessible.

With refined preparation techniques, it became possible to prepare inversion layers that are laterally structured. Quasi–one dimensional inversion layers exhibit unique quantization and fluctuation phenomena. Systems of



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two–dimensional point contacts were designed to form islands of electrons, quantum dots, which showed characteristic oscillatory transport behavior — signature of the Coulomb repulsion between the electrons. Arrays of quantum dots were discovered to allow for the systematic experimental study of signatures of chaos in quantum systems. The long–standing theoretical prediction of persistent currents in normally conducting metallic systems was experimentally verified by using a structured inversion layer imbedded in a AlGaAs/GaAs–heterostructure. Even nowadays the field is still rapidly evolving. No saturation of the activities is yet in sight. Practically every year a new effect is reported in the literature.

In the following, a brief survey of the quantum transport effects in nano–structured *semiconductors* which were discovered during the past two decades is given [1]. Emphasis will be on those aspects which are not discussed in the articles. Topics which are explained in detail in the later chapters will only be briefly addressed.

## 1. The Mesoscopic Regime

### 1.1. FROM DIFFUSIVE TO QUANTUM TRANSPORT

The classical charge transport in metals is described by the Drude theory [2]. The basic result is that the DC–conductivity of a metal is

$$\sigma = \frac{ne^2\tau}{m}, \tag{1}$$

with the density of the electrons (charge $-e$) $n$, the effective mass $m$ and the *mean free time* $\tau$. The latter incorporates all of the scattering processes the electrons suffer from static impurities, vacancies and dislocations, and also from other elementary processes like electron–phonon and electron–electron scattering. The basic assumption behind the Drude theory is that scatterings are *incoherent*: the electrons, "after having suffered a collision, do not remember that they existed before". Subject to the influence of the electric field, they move *diffusively* through the lattice of the metal ions. One of the consequences of this is *Matthiesen's rule*, stating that the contributions of different scattering processes are independent and additive, i. e. the total scattering rate is given by the sum of the corresponding rates.

At sufficiently low temperatures this assumption breaks down. The quantum mechanical nature of the electrons comes into play. Incoherent processes that destroy the "phase memory" of the electrons, as electron–phonon scattering, are more or less frozen out. What remains is scattering at the impurities which is *not* incoherent. The quantum mechanical state of an electron depends on the configuration of *all* of the imperfections. This important fact, which is the backbone of the physics of almost all of the mesoscopic transport phenomena, became obvious only about twenty years ago when at temperatures close to absolute zero the weak localization correction to metallic conduction in thin metallic films — quasi–two dimensional metallic systems — was discovered [3].

The thickness of the inversion layers in semiconductor hetero–structures is of the order of 5nm. Therefore, they can be considered as almost ideally two dimensional. They are perfect laboratories for the investigation of quantum coherent transport phenomena because it is possible to change the electronic properties by doping, and the electron density by applying an external gate voltage, in contrast to metallic systems. In addition, the lateral structure of the inversion layers can be systematically influenced by voltages at external gates. This enables us to construct single point contacts and also small islands of confined electrons — "artificial atoms" — which show transport quantization properties that are not at all predicted by the semi–classical theory, and they are *externally tunable* [4]. The arsenal of tools for the systematic investigation of quantum transport effects in structured semiconductors is completed by externally applied magnetic fields. This causes a number of additional, most surprising effects which are also not foreseen when using the semi–classical theory of electron transport.

## 1.2. MESOSCOPIC LENGTHS SCALES

There are several lengths scales which can be used to characterize the mesoscopic transport regime. The presence of imperfections in a metallic system gives rise to the *elastic mean free path*

$$\ell = v_F \tau \qquad (2)$$

with the Fermi velocity $v_F$. It is the only limiting length for transport at $T = 0$ and is independent of the temperature. The mean free time $\tau$ has to be determined by quantum mechanical theory. If the perturbation introduced by the impurities is only weak one can use perturbation theory. In lowest order, $\tau^{-1} \propto \overline{V^2}$ where $V$ is the random impurity potential. It is very important to note here that *the elastic mean free path has nothing to do with the destruction of phase coherence.* In principle, the underlying impurity scattering can be exactly taken into account by diagonalizing the Hamiltonian of the electron in the presence of the impurity potential. In metallic systems, $\ell$ is usually of the order of nanometers. In very pure semiconductor hetero–structures the mean free path can be much longer than $10\mu$m, several orders of magnitude larger than the interatomic distance!

At finite temperatures, there are basically two additional limiting influences on the transport. First of all, the conductivity is an average over the states within an interval $\Delta E \propto kT$ near the Fermi level, as one can easily

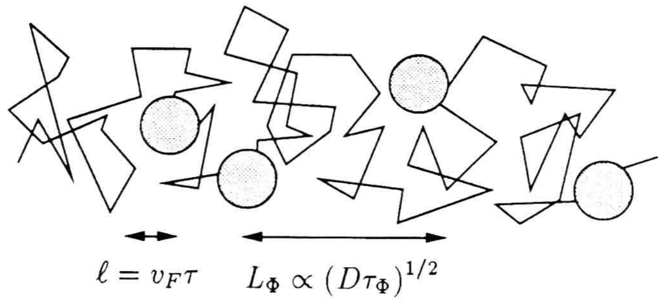


*Figure 1.* Diffusive motion of a particle in an impure metal at a temperature close to absolute zero under the influence of impurity scattering (mean free path $\ell = v_F\tau$), and rare phase randomizing scattering processes (shaded circles, phase coherence length $L_\varphi$).

see by using the Kubo formula. Since the phases of different eigenstates are completely independent, we expect a decay of the average correlation function on a time scale

$$\tau_T(T) \propto (kT)^{-1}. \tag{3}$$

This is usually interpreted as a *temperature-induced phase coherence time.* On the other hand, interactions with other elementary excitations as phonons or the Boson-like pair excitations of the electrons, lead to mixing of the one-electron states. These scatterings are in general inelastic and therefore lead to *phase incoherence* with a temperature dependent characteristic time $\tau_i(T)$ which is the mean free time between *inelastic* scattering events. If one assumes that at low temperatures phase randomizing processes are sufficiently rare in comparison with the mean free time due to the impurities one can determine a *phase coherence length* by assuming diffusive transport — due to the impurity scattering — between two phase destroying scattering events (Fig. 1),

$$L_\varphi = \sqrt{D\tau_\varphi(T)}. \tag{4}$$

The phase coherence time $\tau_\varphi$ is the mean free time between two successive phase randomizing events. The diffusion constant $D$ contains only the impurity scattering which does not destroy quantum coherence. It is related to the residual conductivity via the Einstein relation

$$\sigma = e^2\rho D \tag{5}$$

where $\rho$ is the state density at the Fermi energy. In general, the relation between $\tau_\varphi$ and $L_\varphi$ is more complicated. For instance, in the hopping region, where $D = 0$, $L_\varphi$ is given be the mean hopping distance.

The temperature dependence of $L_\varphi$ is determined by the nature of the contributing scattering processes and is presently a subject of the research world–wide. The understanding is far from complete. Generally, one assumes for the phase coherence time in a metal ($D \neq 0$)

$$\tau_\varphi(T) \propto T^{-p} \tag{6}$$

with $1 \leq p \leq 5$ depending on the nature of the scattering, the temperature and other parameters. At low temperature, the smallest of the phase coherence times limits the transport. If $p > 1$, the phase coherence at very low temperatures is eventually given by $\tau_T \propto (kT)^{-1}$ such that $L_\varphi \propto T^{-1/2}$.

It is now easy to provide a *criterium for mesoscopic transport*: the temperature has to be so low that $L_\varphi(T) > L$, the geometrical diameter of the sample. Typically, in metals $L_\varphi(1K) = O(1\mu\text{m})$. In semiconductor systems, especially when a magnetic field is applied, $L_\varphi$ can be considerably larger. We can also specifiy now what we mean by a $d$–dimensional mesoscopic system: if the thickness of the system in, say, the $z$–direction is smaller than $L_\varphi$ we have a two dimensional system. When in addition $L_\varphi$ is larger than the extensions in the $x$ and $y$–directions, the dimensionality will be further reduced to $d = 1$ and $d = 0$, respectively.

## 2. Mesoscopic Transport Phenomena

### 2.1. THE INTEGER QUANTUM HALL EFFECT

The Quantum Hall Effect was discovered in 1980 by Klaus von Klitzing when he investigated the magneto–transport properties of the inversion layer in a Silicon MOSFET at low temperature ($T \approx 1\text{K}$) and at high magnetic field ($B \approx 20\text{T}$) [5]. He found that when the (negative) voltage at the gate of the transistor was increased, the Hall voltage did not decrease monotonically. Such a decrease is indeed expected according to the classical theory of the Hall effect, when assuming that the charge density in the inversion layer decreases monotonically with increasing gate voltage. Instead, the Hall voltage was found to remain constant in certain regions. Here, the voltage parallel to the source–drain current turned out to be unmeasurably small. The corresponding values of the Hall resistance $R_H$ were precisely given by integer fractions of $R_K = h/e^2$,

$$R_H = \frac{1}{i} R_K \qquad (i = 1, 2, 3, \cdots). \tag{7}$$

The Hall conductance $\Gamma_H \equiv 1/R_H$ is then quantized in units of $e^2/h$, the Sommerfeld constant.

While the relative accuracy of the quantization in first experiment was only of the order of e few $10^{-6}$, later experiments, done at lower temperatures, $T \approx 50\text{mK}$, and different samples, AlGaAs/GaAs hetero–structures, showed a dramatic increase in precision. Nowadays, the reproducibility of the plateaus is better than $10^{-8}$ such that the Quantum Hall Effect is used as a standard for the electrical resistance.

A number of fundamental questions emerged as a result of the discovery of this first of the quantization effects in electrical transport. One of the conclusions of the weak localization theory of transport was that two dimensional disordered quantum systems at zero temperature cannot conduct the electrical current due to strong enhancement of quantum backscattering. All of the quantum states are localized. Under these conditions, it was hard to believe that such a precise, material-independent quantization effect could exist. The only way out was the assumption that the strong magnetic field delocalized at least a few of the states [6].

This hypothesis could be confirmed by later numerical calculations [7]. The results showed that indeed all of the states in two dimensional disordered systems in a strong magnetic field are localized. However, the localization length was found to diverge in the centers of the Landau bands, $E = 0$, with a power law

$$\lambda_0 |E|^{-\nu}, \qquad (\lambda_0 = \text{ constant}). \tag{8}$$

The critical exponent was quantitatively determined, $\nu = 2.34 \pm 0.04$, and shown to be universal, i. e. independent of the nature of the randomness, and the Landau band index. By using this divergent behavior and assuming that the largest possible localization length in the system was the temperature dependent phase coherence length it turned out to be possible to determine, for instance, the temperature dependence of the widths of the Hall plateaus. The results were consistent with the experimental findings. Further experiments done on samples with different geometrical sizes yielded even a value for the exponent that was consistent with the above result [8].

Basically, the existence of the singularities of the localization length in the centers of the Landau bands my be qualitatively understood by considering the percolation limit: for an extremely high magnetic field the magnetic length $\ell_B \equiv (h/eB)^{1/2}$ is small compared with the spatial correlation length of the random potential. Then one can show that only the Landau states centered at the positions corresponding to the randomly percolating equi-potential lines defined by $V(r_{\rm E}) = E$ contribute to the eigenstates at energy $E$ in the presence of disorder [11, 12]. The localization problem is reduced to a percolation problem: the "landscape" of the random potential is filled with water up to a given level — the energy of the state. The shore lines correspond to the equi-potential lines. For low water level, there are only isolated lakes. All shore lines are closed. The states are localized. Correspondingly, for high water levels, there are isolated mountains in a sea of water. Again all of the shore lines are closed, and the corresponding states localized. It is intuitively clear that there must be exactly one water level at which one can reach two different edges of the system by travelling along the shore lines. This corresponds to the percolation threshold, and represents the energy where the localization length diverges.

In this way, the integer Quantum Hall Effect was identified as a *degenerate metal–insulator transition.* Although there is no predictive theory up to now which explains why the plateaus in the Hall resistance are practically *exactly* given by integer fractions of the von Klitzing constant $R_K$, a large number of new quantum properties were discovered when attempting to find such a theory. A most important discovery of the past years was that the states at the critical point have multifractal properties [9, 10].

A further important discovery was that in two dimensional systems in a strong magnetic field quantum coherent edge states play an important role for the understanding of magneto–transport [13, 14]. In the above picture of the landscape filled with water they can be vizualized by considering a landscape with boundaries represented by infintely high walls. Then one of the shore lines goes around the whole system. In the semi–classical picture of magneto–transport edge states correspond to the so–called "skipping orbits" which are essentially cyclotron orbits travelling along the edges.

Edge states can have coherence lengths even of several hundred micrometers due to the absence of backscattering induced by the magnetic field. They might play an important role for the explanation of the precision of the Quantum Hall Effect.

## 2.2. FRACTIONAL QUANTUM HALL EFFECT

The integer Quantum Hall Effect initiated numerous experimental and theoretical investigations of the two dimensional electron systems in semiconductor hetero–structures. A very important discovery only a few years later [15] was the fractional Quantum Hall Effect. In highly pure AlGaAs/GaAs samples with electron mobilities higher than, say 100000 $\mathrm{Vcm/s^2}$, the Hall conductance was found to be quantized at certain *rational* multiples of $e^2/h$,

$$\frac{1}{R_H} = \frac{p}{q}\frac{e^2}{h} \qquad (p, q \text{ integers}). \tag{9}$$

First attempts to explain the additional plateaus which appeared at the rational filling factors $\nu \equiv nh/eB = p/q$ within the one–electron approximation failed. Very rapidly, it became clear that the Fractional Quantum Hall Effect was a direct manifestation of the electron–electron interaction in the two dimensional system subject to the strong magnetic field. There have been several attempts to construct the many particle states for this system [16, 17]. Numerical diagonalizations of several interacting particles