

Principles of **Plasma Physics** for Engineers and Scientists



Umran S. Inan and Marek Gołkowski

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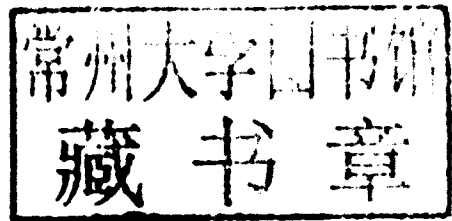
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Principles of Plasma Physics for Engineers and Scientists

This unified introduction provides the tools and techniques needed to analyze plasmas, and connects plasma phenomena to other fields of study. Combining mathematical rigor with qualitative explanations, and linking theory to practice with example problems, this is a perfect textbook for senior undergraduate and graduate students taking a one-semester introductory course in plasma physics.

For the first time, material is presented in the context of unifying principles, illustrated using organizational charts, and structured in a successive progression from single-particle motion to kinetic theory and average values, through to the collective phenomena of waves in plasma. This provides students with a stronger understanding of the topics covered, their interconnections, and when different types of plasma models are applicable. Furthermore, mathematical derivations are rigorous yet concise, so physical understanding is not lost in lengthy mathematical treatments. Worked examples illustrate practical applications of theory, and students can test their new knowledge with 90 end-of-chapter problems.

Umran Inan is a Professor of Electrical Engineering at Stanford University, where he has led pioneering research on very low frequency studies of the ionosphere and radiation belts, space plasma physics, and electromagnetics for over 30 years. He also currently serves as President of Koç University in Istanbul, Turkey. As a committed teacher, he has supervised the Ph.D. dissertations of 42 students and has authored two previous books that have become standard textbooks for electromagnetics courses, as well as receiving numerous awards including the Tau Beta Pi Excellence in Undergraduate Teaching Award and the Outstanding Service Award from the Electrical Engineering Department for excellence in teaching. He is a Fellow of the Institute for Electrical and Electronics Engineers (IEEE), the American Geophysical Union (AGU), and the American Physical Society (APS), and is the recipient of the 2008 Appleton Prize from the International Union of Radio Science and the Royal Society, the 2007 Allan Cox Medal of Stanford for Faculty Excellence in fostering undergraduate research, and the 2010 Special Science Award given by the Scientific and Technological Research Council of Turkey.

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**To my parents, my beautiful wife Elif, my dear children
Ayşe and Ali, and my very special granddaughter Ayla.**
USI

**To my father, who taught me to appreciate physics,
my mother, who taught me to appreciate writing, and
my wife, who gave me the support and motivation to
finish this project.**
MG

I have been teaching Introductory Plasma Physics to senior undergraduates and beginning graduate students for many years, and I find the level of the presentation of material, the order that the topics are presented, and the overall length of the book to be an excellent match for my needs in a textbook.

David Hammer, Cornell University

The authors have done an excellent job in introducing the vast scope of plasma physics for basic plasma physics courses. The schematic illustrations and flow charts used are especially helpful in understanding the complexities involved in the hierarchical nature of plasmas. Mathematics is kept at just the right level for the intended readers and the descriptions of the physical processes are clear. Although this book is targeted to advanced undergraduate or beginning graduate students, it will be a good addition to the personal library of every plasma physicist.

Gurudas Ganguli, Naval Research Laboratory

This new book provides an excellent summary of the basic processes occurring in plasmas together with a comprehensive introduction to the mathematical formulation of fluid (MHD) and kinetic theory. It provides an excellent introduction to the subject suitable for senior undergraduate students or entry-level graduate students.

Richard M. Thorne, University of California at Los Angeles

Preface

This book is intended to provide a general introduction to plasma phenomena at a level appropriate for advanced undergraduate students or beginning graduate students. The reader is expected to have had exposure to basic electromagnetic principles including Maxwell's equations and the propagation of plane waves in free space. Despite its importance in both science and engineering the body of literature on plasma physics is often not easily accessible to the non-specialist, let alone the beginner. The diversity of topics and applications in plasma physics has created a field that is fragmented by topic-specific assumptions and rarely presented in a unified manner with clarity. In this book we strive to provide a foundation for understanding a wide range of plasma phenomena and applications. The text organization is a successive progression through interconnected physical models, allowing diverse topics to be presented in the context of unifying principles. The presentation of material is intended to be compact yet thorough, giving the reader the necessary tools for further specialized study. We have sought a balance between mathematical rigor championed by theorists and practical considerations important to experimenters and engineers. Considerable effort has been made to provide explanations that yield physical insight and illustrations of concepts through relevant examples from science and technology.

The material presented in this book was initially put together as class notes for the EE356 Elementary Plasma Physics course, newly introduced and taught by one of us (USI) at Stanford University in the spring quarter of 1998. The course was then taught regularly every other year, for graduate students from the departments of Electrical Engineering, Materials Science, Mechanical Engineering, Applied Physics, and Physics. Over the years, several

PhD students, including Nikolai Lehtinen, Georgios Veronis, Jacob Bortnik, Michael Chevalier, Timothy Chevalier, and Prajwal Kulkarni, contributed to the course in their work as teaching assistants. The course was co-taught by Prajwal Kulkarni and one of us (MG) in the Spring of 2008, and by Brant Carlson in the Spring of 2010. We offer our thanks to each of these colleagues for their enthusiastic help and contributions, as well as to the many students enrolled in the course who helped improve its content with their contributions.

More generally, we owe considerable gratitude to all the other researchers and students of the Very Low Frequency Group at Stanford University who have been a source of valuable feedback and expertise, and to our administrative assistants, Shaolan Min and Helen Wentong Niu, for their contributions. We would like specifically to acknowledge Dr. Prajwal Kulkarni, for his pedagogical insights that have helped shape this text, and Dr. Brant Carlson, for valuable help in editing the manuscript.

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Introduction

This text concerns the basic elementary physics of *plasmas*, which are a special class of gases made up of a large number of electrons and ionized atoms and molecules, in addition to neutral atoms and molecules as are present in a normal (non-ionized) gas. The most important distinction between a plasma and a normal gas is the fact that mutual Coulomb interactions between charged particles are important in the dynamics of a plasma and cannot be disregarded. When a neutral gas is raised to a sufficiently high temperature, or when it is subjected to electric fields of sufficient intensity, the atoms and molecules of the gas may become ionized, electrons being stripped off by collisions as a result of the heightened thermal agitation of the particles. Ionization in gases can also be produced as a result of illumination with ultraviolet light or X-rays, by bombarding the substance with energetic electrons and ions, or in other ways. When a gas is ionized, even to a rather small degree, its dynamical behavior is typically dominated by the electromagnetic forces acting on the free ions and electrons, and it begins to conduct electricity. The charged particles in such an ionized gas interact with electromagnetic fields, and the organized motions of these charge carriers (e.g., electric currents, fluctuations in charge density) can in turn produce electromagnetic fields. The ability of an ionized gas to sustain electric current is particularly important in the presence of a magnetic field. The presence of mobile charged particles in a magnetic field yields a Lorentz force $q\mathbf{v} \times \mathbf{B}$. When applied to a collection of particles this force leads to an electromagnetic body force $\mathbf{J} \times \mathbf{B}$ which can dominate the gas dynamics. As a result, the most novel and spectacular behavior of plasmas is exhibited in the context of their interaction with a magnetic field.

During the 1920s, I. Langmuir and colleagues first showed that characteristic electrical oscillations of very high frequency can exist

in an ionized gas that is neutral or quasi-neutral, and introduced the terms *plasma* and *plasma oscillations*,¹ in recognition of the fact that these oscillations resembled those of jelly-like substances [1, 2]. When subjected to a static electric field, the charge carriers in an ionized gas rapidly redistribute themselves in such a way that most of the gas is shielded from the field, in a manner quite similar to the redistribution of charge which occurs within a metallic conductor placed in an electric field, resulting in zero electric field everywhere inside. Langmuir gave the name “plasma” specifically to the relatively field-free regions of the ionized gas, which are not influenced by the boundaries. Near the boundaries, typically metallic surfaces held at prescribed potentials, strong space-charge fields exist in a transition region Langmuir termed the *plasma sheath*. The sheath region has properties that differ from the plasma, since the motions of charged particles within the sheath are predominantly influenced by the potential of the boundary. The particles in the sheath form an electrical screen between the plasma and the boundary. We will find later that the screening distance is a function of the density of charged particles and of their temperature.

The plasma medium is often referred to as the fourth state of matter, since it has properties profoundly different from those of the gaseous, liquid, and solid states. All states of matter represent different degrees of organization, corresponding to certain values of binding energy. In the solid state, the important quantity is the binding energy of molecules in a crystal. If the average kinetic energy of a molecule exceeds the binding energy (typically a fraction of an electron volt), the crystal structure breaks up, either into a liquid or directly into a gas (e.g., iodine). Similarly, a certain minimum kinetic energy is required in order to break the bonds of the van der Waals forces in order for a liquid to change into a gas. In order for matter to make the transition to its fourth state and exist as a plasma, the kinetic energy per plasma particle must exceed the ionizing potential of atoms (typically a few electron volts). Thus, the state of matter is basically determined by the average kinetic energy per particle. Using water as a convenient example, we note that at low temperatures the bond between the H₂O molecules holds them tightly together against the low energy of molecular motion, so that the matter is in the solid state (ice). At room temperature, the

¹ The word “plasma” first appeared as a scientific term in 1839 when the Czech biologist J. Purkyně coined the term “protoplasma” to describe the jelly-like medium containing a large number of floating particles which make up the nuclei of the cells. The word “plasma” thus means a mold or form, and is also used for the liquid part of blood in which corpuscles are suspended.

increased molecular energy permits more widespread movements and currents of molecular motion, and we have the liquid state (water). Since the particle motions are random, not all particles have the same energy, with the more energetic ones escaping from the liquid surface to form a vapor above it. As the temperature of the water is further increased, a larger fraction of molecules escapes, until the whole substance is in the gaseous phase (steam). If steam is subjected to further thermal heating, illumination by UV or X-rays, or bombardment by energetic particles, it becomes ionized (plasma).

Although by far most of the Universe is ionized and is therefore in a plasma state, on our planet plasmas have to be generated by special processes and under special conditions. While we live in a bubble of essentially non-ionized gas in the midst of an otherwise ionized environment, examples of partially ionized gases or plasmas, including fire, lightning, and the aurora borealis have long been part of our natural environment. It is in this connection that early natural philosophers held that the material Universe is built of four “roots,” earth, water, air, and fire, curiously resembling our modern terminology of solid, liquid, gas, and plasma states of matter. A transient plasma exists in the Earth’s atmosphere every time a lightning stroke occurs, but is clearly not very much at home and is short-lived. Early work on electrical discharges included generation of electric sparks by rubbing a large rotating sphere of sulphur against a cloth [3], production of sparks by harnessing atmospheric electricity in rather hazardous experiments [4], and studies of dust patterns left by a spark discharge passing through the surface of an insulator [5]. However, it was only when electrical and vacuum techniques were developed to the point where long-lived and relatively stable electrical discharges were available that the physics of ionized gases emerged as a field of study. In 1879, W. Crookes published the results of his investigations of discharges at low pressure and remarked: “The phenomena in these exhausted tubes reveal to physical science a new world, a world where matter may exist in a fourth state . . .” [6]. A rich period of discoveries followed, leading to Langmuir’s coining of the word “plasma” in 1929, and continuing into the present as a most fascinating branch of physics.

Although a plasma is often considered to be the fourth state of matter, it has many properties in common with the gaseous state. At the same time, the plasma is an ionized gas in which the long range of Coulomb forces gives rise to collective interaction effects, resembling a fluid with a density higher than that of a gas. In its most general sense, a plasma is any state of matter which contains enough free, charged particles for its dynamical behavior to be dominated by electromagnetic forces. Plasma physics therefore