PEARSON

HZ BOOKS
华章教育

华章数学原版精品系列

代数

（英文版·第2版）

ALGEBRA

Michael Artin

Second Edition

（美）Michael Artin
麻省理工学院 著

机械工业出版社
China Machine Press

代数

（英文版·第2版）

Algebra

(Second Edition)

本书由著名代数学家与代数几何学家Michael Artin所著，是作者在代数领域数十年的智慧和经验的结晶。书中既介绍了矩阵运算、群、向量空间、线性算子、对称等较为基本的内容，又介绍了环、模型、域、伽罗瓦理论等较为高深的内容。本书对于提高数学理解能力，增强对代数的兴趣是非常有益处的。此外，本书的可阅读性强，书中的习题也很有针对性，能让读者很快地掌握分析和思考的方法。

作者结合这20年来的教学经历及读者的反馈，对本版进行了全面更新，更强调对称性、线性群、二次数域和格等具体主题。本版的具体更新情况如下：

- 新增球面、乘积环和因式分解的计算方法等内容，并补充给出一些结论的证明，如交错群是简单的、柯西定理、分裂定理等。
- 修订了对对应定理、SU2 表示、正交关系等内容的讨论，并把线性变换和因子分解都拆分为两章来介绍。
- 新增大量习题，并用星号标注出具有挑战性的习题。

本书在麻省理工学院、普林斯顿大学、哥伦比亚大学等著名学府得到了广泛采用，是代数学的经典教材之一。

作者简介

Michael Artin 当代领袖型代数学家与代数几何学家之一，美国麻省理工学院数学系荣誉退休教授。1990年至1992年，曾担任美国数学学会主席。由于他在交换代数与非交换代数、环论以及现代代数几何学等方面做出的贡献，2002年获得美国数学学会颁发的Leroy P. Steele终身成就奖。Artin的主要贡献包括他的逼近定理、在解决沙法列维奇--泰特猜测中的工作以及为推广“概形”而创建的“代数空间”概念。

客服热线：(010) 88378991, 88361066
购书热线：(010) 68326294, 88379649, 68995259
投稿热线：(010) 88379604
读者信箱：hzjsj@hzbook.com

华章网站 http://www.hzbook.com

网上购书：www.china-pub.com

封面设计·杨宇梅

PEARSON
www.pearson.com

上架指导：数学/代数
ISBN 978-7-111-36701-7

定价：79.00元

华章数学原版精品系列

代数

（英文版·第2版）

Algebra

(Second Edition)

（美）Michael Artin 著

麻省理工学院

机械工业出版社
China Machine Press

本书版权登记号：图字：01-2011-6871

图书在版编目（CIP）数据

代数（英文版·第2版）/（美）阿廷（Artin, M.）著. —北京：机械工业出版社，2011.12
（华章数学原版精品系列）
书名原文：Algebra, Second Edition

ISBN 978-7-111-36701-7

Ⅰ. 代…　Ⅱ. 阿…　Ⅲ. 代数－高等学校－教材－英文　Ⅳ. O15

中国版本图书馆 CIP 数据核字（2011）第 251649 号

机械工业出版社（北京市西城区百万庄大街22号　邮政编码　100037）
责任编辑：迟振春
北京京师印务有限公司印刷
2012年1月第1版第1次印刷
186mm×240mm · 34.75 印张
标准书号：ISBN 978-7-111-36701-7
定价：79.00 元

凡购本书，如有缺页、倒页、脱页，由本社发行部调换
客服热线：（010）88378991；88361066
购书热线：（010）68326294；88379649；68995259
投稿热线：（010）88379604
读者信箱：hzjsj@hzbook.com

Preface

Important though the general concepts and propositions may be with which the modern and industrious passion for axiomatizing and generalizing has presented us, in algebra perhaps more than anywhere else, nevertheless I am convinced that the special problems in all their complexity constitute the stock and core of mathematics, and that to master their difficulties requires on the whole the harder labor.

—Herman Weyl

This book began many years ago in the form of supplementary notes for my algebra classes. I wanted to discuss some concrete topics such as symmetry, linear groups, and quadratic number fields in more detail than the text provided, and to shift the emphasis in group theory from permutation groups to matrix groups. Lattices, another recurring theme, appeared spontaneously.

My hope was that the concrete material would interest the students and that it would make the abstractions more understandable – in short, that they could get farther by learning both at the same time. This worked pretty well. It took me quite a while to decide what to include, but I gradually handed out more notes and eventually began teaching from them without another text. Though this produced a book that is different from most others, the problems I encountered while fitting the parts together caused me many headaches. I can't recommend the method.

There is more emphasis on special topics here than in most algebra books. They tended to expand when the sections were rewritten, because I noticed over the years that, in contrast to abstract concepts, with concrete mathematics students often prefer more to less. As a result, the topics mentioned above have become major parts of the book.

In writing the book, I tried to follow these principles:

1. The basic examples should precede the abstract definitions.
2. Technical points should be presented only if they are used elsewhere in the book.
3. All topics should be important for the average mathematician.

Although these principles may sound like motherhood and the flag, I found it useful to have them stated explicitly. They are, of course, violated here and there.

The chapters are organized in the order in which I usually teach a course, with linear algebra, group theory, and geometry making up the first semester. Rings are first introduced in Chapter 11, though that chapter is logically independent of many earlier ones. I chose this arrangement to emphasize the connections of algebra with geometry at the start, and because, overall, the material in the first chapters is the most important for people in other fields. The first half of the book doesn't emphasize arithmetic, but this is made up for in the later chapters.

About This Second Edition

The text has been rewritten extensively, incorporating suggestions by many people as well as the experience of teaching from it for 20 years. I have distributed revised sections to my class all along, and for the past two years the preliminary versions have been used as texts. As a result, I've received many valuable suggestions from the students. The overall organization of the book remains unchanged, though I did split two chapters that seemed long.

There are a few new items. None are lengthy, and they are balanced by cuts made elsewhere. Some of the new items are an early presentation of Jordan form (Chapter 4), a short section on continuity arguments (Chapter 5), a proof that the alternating groups are simple (Chapter 7), short discussions of spheres (Chapter 9), product rings (Chapter 11), computer methods for factoring polynomials and Cauchy's Theorem bounding the roots of a polynomial (Chapter 12), and a proof of the Splitting Theorem based on symmetric functions (Chapter 16). I've also added a number of nice exercises. But the book is long enough, so I've tried to resist the temptation to add material.

NOTES FOR THE TEACHER

This book is designed to allow you to choose among the topics. Don't try to cover the book, but do include some of the interesting special topics such as symmetry of plane figures, the geometry of SU_2, or the arithmetic of imaginary quadratic number fields. If you don't want to discuss such things in your course, then this is not the book for you.

There are relatively few prerequisites. Students should be familiar with calculus, the basic properties of the complex numbers, and mathematical induction. An acquaintance with proofs is obviously useful. The concepts from topology that are used in Chapter 9, Linear Groups, should not be regarded as prerequisites.

I recommend that you pay attention to concrete examples, especially throughout the early chapters. This is very important for the students who come to the course without a clear idea of what constitutes a proof.

One could spend an entire semester on the first five chapters, but since the real fun starts with symmetry in Chapter 6, that would defeat the purpose of the book. Try to get to Chapter 6 as soon as possible, so that it can be done at a leisurely pace. In spite of its immediate appeal, symmetry isn't an easy topic. It is easy to be carried away and leave the students behind.

These days most of the students in my classes are familiar with matrix operations and modular arithmetic when they arrive. I've not been discussing the first chapter on matrices in class, though I do assign problems from that chapter. Here are some suggestions for Chapter 2, Groups.

1. Treat the abstract material with a light touch. You can have another go at it in Chapters 6 and 7.

2. For examples, concentrate on matrix groups. Examples from symmetry are best deferred to Chapter 6.
3. Don't spend much time on arithmetic; its natural place in this book is in Chapters 12 and 13.
4. De-emphasize the quotient group construction.

Quotient groups present a pedagogical problem. While their construction is conceptually difficult, the quotient is readily presented as the image of a homomorphism in most elementary examples, and then it does not require an abstract definition. Modular arithmetic is about the only convincing example for which this is not the case. And since the integers modulo n form a ring, modular arithmetic isn't the ideal motivating example for quotients of groups. The first serious use of quotient groups comes when generators and relations are discussed in Chapter 7. I deferred the treatment of quotients to that point in early drafts of the book, but, fearing the outrage of the algebra community, I eventually moved it to Chapter 2. If you don't plan to discuss generators and relations for groups in your course, then you can defer an in-depth treatment of quotients to Chapter 11, Rings, where they play a central role, and where modular arithmetic becomes a prime motivating example.

In Chapter 3, Vector Spaces, I've tried to set up the computations with bases in such a way that the students won't have trouble keeping the indices straight. Since the notation is used throughout the book, it may be advisable to adopt it.

The matrix exponential that is defined in Chapter 5 is used in the description of one-parameter groups in Chapter 10, so if you plan to include one-parameter groups, you will need to discuss the matrix exponential at some point. But you must resist the temptation to give differential equations their due. You will be forgiven because you are teaching algebra.

Except for its first two sections, Chapter 7, again on groups, contains optional material. A section on the Todd-Coxeter algorithm is included to justify the discussion of generators and relations, which is pretty useless without it. It is fun, too.

There is nothing unusual in Chapter 8, on bilinear forms. I haven't overcome the main pedagogical problem with this topic – that there are too many variations on the same theme, but have tried to keep the discussion short by concentrating on the real and complex cases.

In the chapter on linear groups, Chapter 9, plan to spend time on the geometry of SU_2. My students complained about that chapter every year until I expanded the section on SU_2, after which they began asking for supplementary reading, wanting to learn more. Many of our students aren't familiar with the concepts from topology when they take the course, but I've found that the problems caused by the students' lack of familiarity can be managed. Indeed, this is a good place for them to get an idea of a manifold.

I resisted including group representations, Chapter 10, for a number of years, on the grounds that it is too hard. But students often requested it, and I kept asking myself: If the chemists can teach it, why can't we? Eventually the internal logic of the book won out and group representations went in. As a dividend, hermitian forms got an application.

You may find the discussion of quadratic number fields in Chapter 13 too long for a general algebra course. With this possibility in mind, I've arranged the material so that the end of Section 13.4, on ideal factorization, is a natural stopping point.

It seemed to me that one should mention the most important examples of fields in a beginning algebra course, so I put a discussion of function fields into Chapter 15. There is always the question of whether or not Galois theory should be presented in an undergraduate course, but as a culmination of the discussion of symmetry, it belongs here.

Some of the harder exercises are marked with an asterisk.

Though I've taught algebra for years, various aspects of this book remain experimental, and I would be very grateful for critical comments and suggestions from the people who use it.

ACKNOWLEDGMENTS

Mainly, I want to thank the students who have been in my classes over the years for making them so exciting. Many of you will recognize your own contributions, and I hope that you will forgive me for not naming you individually.

Acknowledgments for the First Edition

Several people used my notes and made valuable suggestions – Jay Goldman, Steve Kleiman, Richard Schafer, and Joe Silverman among them. Harold Stark helped me with the number theory, and Gil Strang with the linear algebra. Also, the following people read the manuscript and commented on it: Ellen Kirkman, Al Levine, Barbara Peskin, and John Tate. I want to thank Barbara Peskin especially for reading the whole thing twice during the final year.

The figures which needed mathematical precision were made on the computer by George Fann and Bill Schelter. I could not have done them by myself. Many thanks also to Marge Zabierek, who retyped the manuscript annually for about eight years before it was put onto the computer where I could do the revisions myself, and to Mary Roybal for her careful and expert job of editing the manuscript.

I haven't consulted other books very much while writing this one, but the classics by Birkhoff and MacLane and by van der Waerden from which I learned the subject influenced me a great deal, as did Herstein's book, which I used as a text for many years. I also found some good exercises in the books by Noble and by Paley and Weichsel.

Acknowledgments for the Second Edition

Many people have commented on the first edition – a few are mentioned in the text. I'm afraid that I will have forgotten to mention most of you.

I want to thank these people especially: Annette A' Campo and Paolo Maroscia made careful translations of the first edition, and gave me many corrections. Nathaniel Kuhn and James Lepowsky made valuable suggestions. Annette and Nat finally got through my thick skull how one should prove the orthogonality relations.

I thank the people who reviewed the manuscript for their suggestions. They include Alberto Corso, Thomas C. Craven, Sergi Elizade, Luis Finotti, Peter A. Linnell, Brad Shelton, Hema Srinivasan, and Nik Weaver. Toward the end of the process, Roger Lipsett read and commented on the entire revised manuscript. Brett Coonley helped with the many technical problems that arose when the manuscript was put into TeX.

Many thanks, also, to Caroline Celano at Pearson for her careful and thorough editing of the manuscript and to Patty Donovan at Laserwords, who always responded graciously to my requests for yet another emendation, though her patience must have been tried at times.

And I talk to Gil Strang and Harold Stark often, about everything.

Finally, I want to thank the many MIT undergraduates who read and commented on the revised text and corrected errors. The readers include Nerses Aramyan, Reuben Aronson, Mark Chen, Jeremiah Edwards, Giuliano Giacaglia, Li-Mei Lim, Ana Malagon, Maria Monks, and Charmaine Sia. I came to rely heavily on them, especially on Nerses, Li-Mei, and Charmaine.

"One, two, three, five, four..."
"No Daddy, it's one, two, three, four, five."
"Well if I want to say one, two, three, five, four, why can't I?"
"That's not how it goes."

—Carolyn Artin

教师服务登记表

尊敬的老师：

您好！感谢您购买我们出版的______________________________教材。

机械工业出版社华章公司为了进一步加强与高校教师的联系与沟通，更好地为高校教师服务，特制此表，请您填妥后发回给我们，我们将定期向您寄送华章公司最新的图书出版信息！感谢合作！

个人资料（请用正楷完整填写）

<table>
<tr><td>教师姓名</td><td></td><td>☐先生
☐女士</td><td>出生年月</td><td></td><td>职务</td><td></td><td colspan="2">职称：☐教授 ☐副教授
☐讲师 ☐助教 ☐其他</td></tr>
<tr><td>学校</td><td colspan="2"></td><td>学院</td><td colspan="3"></td><td>系别</td><td></td></tr>
<tr><td rowspan="2">联系
电话</td><td colspan="3" rowspan="2">办公：
宅电：
移动：</td><td>联系地址
及邮编</td><td colspan="4"></td></tr>
<tr><td>E-mail</td><td colspan="4"></td></tr>
<tr><td>学历</td><td></td><td>毕业院校</td><td colspan="2"></td><td colspan="2">国外进修及讲学经历</td><td colspan="2"></td></tr>
<tr><td>研究领域</td><td colspan="8"></td></tr>
</table>

<table>
<tr><td>主讲课程</td><td>现用教材名</td><td>作者及
出版社</td><td>共同授
课教师</td><td>教材满意度</td></tr>
<tr><td>课程：
☐专 ☐本 ☐研
人数： 学期：☐春☐秋</td><td></td><td></td><td></td><td>☐满意 ☐一般
☐不满意 ☐希望更换</td></tr>
<tr><td>课程：
☐专 ☐本 ☐研
人数： 学期：☐春☐秋</td><td></td><td></td><td></td><td>☐满意 ☐一般
☐不满意 ☐希望更换</td></tr>
<tr><td colspan="5">样书申请</td></tr>
<tr><td colspan="2">已出版著作</td><td colspan="3">已出版译作</td></tr>
<tr><td colspan="2">是否愿意从事翻译/著作工作 ☐是 ☐否</td><td colspan="3">方向</td></tr>
<tr><td colspan="5">意见和建议</td></tr>
</table>

填妥后请选择以下任何一种方式将此表返回：（如方便请赐名片）

地 址：北京市西城区百万庄南街1号 华章公司营销中心 邮编：100037

电 话：(010) 68353079 88378995 传真：(010)68995260

E-mail:hzedu@hzbook.com markerting@hzbook.com 图书详情可登录http://www.hzbook.com网站查询

Contents

CHAPTER 1

Matrices

Erstlich wird alles dasjenige eine Grösse genennt,
welches einer Vermehrung oder einer Verminderung fähig ist,
oder wozu sich noch etwas hinzusetzen oder davon wegnehmen läßt.

—Leonhard Euler[1]

Matrices play a central role in this book. They form an important part of the theory, and many concrete examples are based on them. Therefore it is essential to develop facility in matrix manipulation. Since matrices pervade mathematics, the techniques you will need are sure to be useful elsewhere.

1.1 THE BASIC OPERATIONS

Let m and n be positive integers. An $m \times n$ *matrix* is a collection of mn numbers arranged in a rectangular array

$$\text{(1.1.1)} \qquad m \text{ rows} \overset{n \text{ columns}}{\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}}$$

For example, $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 5 \end{bmatrix}$ is a 2×3 matrix (two rows and three columns). We usually introduce a symbol such as A to denote a matrix.

The numbers in a matrix are the *matrix entries*. They may be denoted by a_{ij}, where i and j are indices (integers) with $1 \le i \le m$ and $1 \le j \le n$, the index i is the *row index*, and j is the *column index*. So a_{ij} is the entry that appears in the ith row and jth column of the matrix:

$$i \overset{j}{\begin{bmatrix} & \vdots & \\ \cdots & a_{ij} & \cdots \\ & \vdots & \end{bmatrix}}$$

[1] This is the opening sentence of Euler's book *Algebra*, which was published in St. Petersburg in 1770.