Matrix Optics

by
SHAOMIN WANG
and
DAOMU ZHAO

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Foreword

In order to study Gaussian beams and optical resonators, *ABCD* matrix (ray transfer matrix) was introduced by Kogelnik and Li about 35 years ago. It is a very important implement in treating problems of propagation, transformation and self-consistent solution of laser beams inside and outside the optical resonators. A book: *Introduction to Matrix Methods in Optics* written by Gerrard and Burch and published in 1975 was a highly condensation on matrix optics for the previous ten years.

Since then, the foreword writer learning from above and starting from practice: dam deformation measured by laser and by means of Fresnel lenses in vacuum line has been considered and established. It is a misaligned optical system and it is also a diffractive optical system. *ABCD* matrix is thus augmented as 4×4 matrix. Just at that time, some novel phenomena of optical array were found in the United States, which could be treated conveniently by means of 4×4 matrix, and so, a new branch of optics is established. On the other hand, some new laser beams are found by the concept of the nature of diffraction. *Array Optics*, *A CO_2 Laser with New Beam* and *Dam Deformation Measured by Laser* have been awarded by National Natural Science (in 1990), National Technical Invention (in 1997) and National Scientific & Technical Progress (in 2000) Encouragements of China. In other words, matrix optics interpenetrates these three subjects, as examples, how to develop matrix optics and how to apply matrix optics? They are embedded in Part II of this book. In order to understand Part II and make this book as a teaching material for the undergraduate students and graduate students in Department of Physics and Department of Optics or Department of Photonics, Part I is provided as fundamentals. However, matrix algebra, geometrical optics, physical optics and laser physics in part will be needed beforehand.

It has been as a teaching material since 1981 and used in China and in Europe for 12 years. The general response is that matrix optics is not only useful but also powerful. Return back to homeland, Hangzhou University asked me to arrange that as a book and I invited one of my efficient graduate students—Mr. Daomu Zhao as a cooperator to translate all the materials into Chinese language named as *Introduction to Matrix Optics*, which was published by Hangzhou University Press in 1994 and awarded as Outstanding Teaching Material Encouragement (in 1995) by State Education Commission of China. In 1998 four universities in Hangzhou were combined as Zhejiang University, and a number of my foreign friends also asked me to translate that book back into English language again. I discussed with Mr. Daomu Zhao and Mr. Jinan Lin, the Director of Editorial Office of CHEP-Springer, the solution is that we have to rewrite a new book named *Matrix Optics* due to the quick developments of matrix optics, which include the contributions inside and outside China and also of Mr. Daomu Zhao and mine during the recent six years. However, it is impossible to embrace all of the developments in this book. And we hope sincerely that we could receive suggestions, corrections especially for the English language in order to make the Second Edition of this book better.

Having been developed and taught on matrix optics about 20 years, now it forms an international book. Besides the main cooperator—Associate Prof. Daomu Zhao who drew up this book mainly, I also wish to acknowledge my indebtedness to the other cooperators in history or at present: Prof. E.Wolf and Prof. H.Kogelnik of the United States; Prof. P.A.Belanger of Canada; Prof. H.Weber and Prof. T.Tschudi of Germany; Prof. L.Ronchi and Prof. S.Solimeno of Italy; Prof. E.Bernabeu, Prof. J.Alda and Dr. M.Porras of Spain; Prof. M.Nakatsuka and Dr.K.Tanaka of Japan; and Prof. X.Deng, Prof. D.Fan, Prof. S.Mei, Prof. F.Li, Prof. G.Zhou, Prof. Z.Lu, Prof. G.Wei, Prof. B.Lü, Prof. C.Pan, Prof. C.Xia, Prof. B.Jiang, Prof. C.Ying, Prof. Q.Lin and Prof. X.Lu; Associate Prof. X.Wang and Associate Prof. X.Jiang; Eng.Y.Wang and Eng.F.Huang.

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Shaomin Wang

Hangzhou

August 2000

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Part I

Fundamentals of Matrix Optics

Chapter 1
Geometrical Optics

1.1 Maxwell's equations and ray equation

The propagation of electromagnetic wave can be described completely by Maxwell's equations. The electromagnetic wave is a kind of physical reality and not a kind of probability wave, which could be described by a space continuous field. The introduction of optical particle only means the discontinuous properties of electromagnetic energy, one optical particle is related with one electromagnetic mode. Accordingly, the propagation of one optical wave corresponds to the movement of one electromagnetic mode[1-1].

Maxwell's equations[1-2] read

$$\begin{cases} \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}, \\ \nabla \times \vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t}, \\ \nabla \cdot \vec{D} = \rho, \\ \nabla \cdot \vec{B} = 0. \end{cases} \tag{1.1-1}$$

Where $\vec{E}, \vec{H}, \vec{D}, \vec{B}$ are the electric or magnetic field vectors, $\vec{J}$ is the electric current density and ρ stands for the electric charge density.

Moreover, in most optical problems only dielectric media are of interest with $\vec{J} = 0$ and $\rho = 0$. Under these assumptions Maxwell's equations reduce to

$$\begin{cases} \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}, \\ \nabla \times \vec{H} = \dfrac{\partial \vec{D}}{\partial t}, \\ \nabla \cdot \vec{D} = 0, \\ \nabla \cdot \vec{B} = 0. \end{cases} \tag{1.1-2}$$

For monochromatic fields and media with linear response we have

$$\vec{B} = \mu_0 \vec{H}, \quad \vec{D} = n^2 \varepsilon_0 \vec{E}. \tag{1.1-3}$$

Where ε_0 is the dielectric constant in vacuum, n is the refractive index. Generally, the optical media are not magnetic, the magnetic permeability in vacuum is directly used in the above equation. The problem for propagation of optical waves could be solved by using Maxwell's equations.

A connection between geometric optics and diffraction theory can be established by introducing the so-called eikonal function. To this end, let us consider an isotropic and inhomogeneous medium. The

refractive index n is assumed to be real and a slowly varying function. According to Eqs. (1.1-2) and (1.1-3), the scalar wave equation becomes

$$\nabla^2\vec{E}+\nabla(\vec{E}\cdot\frac{\nabla n^2}{n^2})-n^2\varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2}=0. \tag{1.1-4}$$

It is evident that the plane wave cannot be a solution any longer. However, if n is varying slowly, the solution will differ only slightly from the plane wave. Hence we can represent the field as a quasi plane wave, with

$$\vec{E}=\vec{E}_0(\vec{r})\exp\{i[k_0L(\vec{r})-\omega t]\}, \tag{1.1-5}$$

where $k_0=\omega\sqrt{\varepsilon_0\mu_0}=2\pi/\lambda_0$ (λ_0 is the optical wavelength in vacuum) represents the wave number in vacuum, $L(\vec{r})$ is the eikonal function. Inserting Eq.(1.1-5) into (1.1-4) yields after some extensive but simple calculations

$$\begin{aligned}&\vec{E}_0[n^2-(\nabla L)^2]+\frac{i}{k_0}[\nabla^2L\cdot\vec{E}_0+2(\nabla L\cdot\nabla)\vec{E}_0+\nabla L(\vec{E}_0\cdot\nabla\ln n)]\\&+\frac{1}{k_0^2}[\nabla^2\vec{E}_0+2\nabla(\vec{E}_0\cdot\nabla\ln n)]=0.\end{aligned} \tag{1.1-6}$$

In the limit of geometric optics, that is $\lambda\to 0$ or $k_0\to\infty$, the above equation gives

$$(\nabla L)^2=n^2. \tag{1.1-7}$$

That is the well-known eikonal equation. The equation L=constant defines surfaces of constant phase, which represents wave fronts. The unit vector $\vec{s}$ normal to the wave fronts is given by

$$\vec{s}=\frac{\nabla L}{|\nabla L|}=\frac{\nabla L}{n}. \tag{1.1-8}$$

The ray propagation through an inhomogeneous medium can be described by a vector function $\vec{r}(s)$, where s is the curvilinear coordinate measuring the length of the trajectory described. By using the relation $\mathrm{d}\vec{r}=\vec{s}\mathrm{d}s$, Eq.(1.1-8) yields

$$\frac{\mathrm{d}}{\mathrm{d}s}(n\frac{\mathrm{d}\vec{r}}{\mathrm{d}s})=\nabla n. \tag{1.1-9}$$

This equation is called the ray equation. Integrated for assigned $n(\vec{r})$ it gives the ray trajectory.

For a homogeneous medium (n=constant) the ray equation reduces to

$$\frac{\mathrm{d}^2\vec{r}}{\mathrm{d}s^2}=0. \tag{1.1-10}$$

Which admits the integral

$$\vec{r}=a\vec{s}+b. \tag{1.1-11}$$

This means that a ray propagation through a homogeneous medium describes a straight line.

The identity $\nabla\times\nabla L=0$, and Eq.(1.1-8) implies the vanishing of the integral

$$\oint n\vec{s}\cdot\mathrm{d}\vec{l}=0 \tag{1.1-12}$$

for a generic closed path.

Now, let us consider two points P_1, P_2 and two different paths *I*, *II*. Then, in view of Eq.(1.1-12)

$$\int_{P_1,I}^{P_2} n\vec{s}\cdot\mathrm{d}\vec{l}=\int_{P_1,II}^{P_2} n\vec{s}\cdot\mathrm{d}\vec{l}. \tag{1.1-13}$$

If path I coincides with a ray having $\vec{s}$ parallel to $\mathrm{d}\vec{l}$, we obtain from the above equation:

$$\int_{P_1,I}^{P_2} n\mathrm{d}l = \int_{P_1,II}^{P_2} n\vec{s}\cdot\mathrm{d}\vec{l} \le \int_{P_1,II}^{P_2} n\mathrm{d}l. \tag{1.1-14}$$

The integral $\int n\mathrm{d}l$ taken along a ray between two points P_1, P_2 is smaller than along any other path connecting these two points. This property is well known as Fermat's principle. It means that the light ray between two points propagates in such a way, that the time has a minimum value

$$\int_{P_1,I}^{P_2} n\mathrm{d}l = \text{minmum}. \tag{1.1-15}$$

It can be shown that generally the integral is an extremum, e.g., there are some special cases in which the integral is a maximum. If $n(\vec{r})$ is given, Eq.(1.1-15) can be used for calculating the ray trajectories by using variation techniques.

Practically, the ray equation (1.1-9) and the eikonal equation (1.1-7) have the similarly physical meaning, they are different descriptions for ray propagation in geometrical optics. Eq.(1.1-9) is related to second-order differential equation of $\vec{r}$ and it is very difficult to obtain its analytic solution, but for most cases, the rays propagate forward closely along the optical axis z and the included angle between the propagation ray and optical axis is small. Under this case, then

$$\mathrm{d}s \approx \mathrm{d}z \tag{1.1-16}$$

and the ray equation reduces to

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(n\frac{\mathrm{d}\vec{r}}{\mathrm{d}z}\right) = \nabla n. \tag{1.1-17}$$

It is a paraxial ray equation.

1.2 Definition of ray transfer matrix

A ray is defined by a line normal to the wave front, and it is completely characterized by the position $\vec{q}$ and the momentum $\vec{p}$ of the ray. If the ray is referred by axis z, then it can be described by[1-3]

$$q_x = x, q_y = y; \tag{1.2-1}$$

and

$$\begin{cases} p_x = n(x,y,z)x'/[1+(x')^2+(y')^2]^{1/2}, \\ p_y = n(x,y,z)y'/[1+(x')^2+(y')^2]^{1/2}. \end{cases} \tag{1.2-2}$$

In which

$$x' = \mathrm{d}x/\mathrm{d}z, y' = \mathrm{d}y/\mathrm{d}z \tag{1.2-3}$$

are slope of the ray, and $n(x,y,z)$ means refractive index at the x, y and z.

Usually, the reader is interested only in paraxial approximation and in two-dimensional systems, then it is convenient that

$$r=(x, \text{ or } y), r'=(p_x/n, \text{ or } p_y/n)=(x', \text{ or } y') \tag{1.2-4}$$

as ray parameters to represent the behaviour.

If the paraxial ray is passing through a linear optical system as shown in Fig.1-1, it is positive that there are linear relations between the outgoing and the incoming parameters as follows

$$\begin{cases} r_2 = ar_1 + br_1', \\ r_2' = cr_1 + dr_1'. \end{cases} \tag{1.2-5}$$

Where a, b, c and d denote linear coefficients.

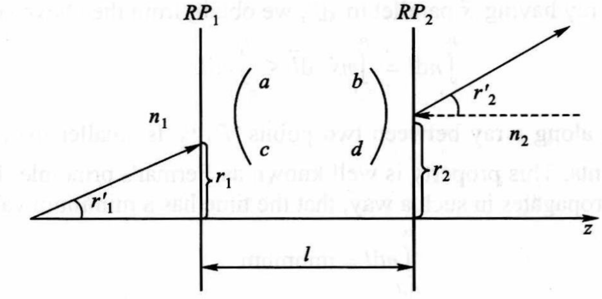

Fig.1-1 Paraxial ray passing through a linear optical system.

Make them in a matrix form, then we get

$$\begin{pmatrix} r_2 \\ r_2' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} r_1 \\ r_1' \end{pmatrix}, \text{ or } \vec{r}_2 = \underrightarrow{M}\vec{r}_1. \tag{1.2-6}$$

Which is called ray transfer matrix or *ABCD* matrix[1-4,1-5], with

$$\det(\underrightarrow{M}) = ad - bc = n_1 / n_2. \tag{1.2-7}$$

Notice that the determinant of the *ABCD* matrix for any linear optical element or system is equal to the ratio between n_1 and n_2. If only $n_1 = n_2$, the determinant will be reduced to unit and the matrix will become symmetric reciprocal. The determinant is useful to check the calculations and has clear physical meaning.

It is convenient to apply matrix multiplication when several of the optical elements are linked up, which is shown in Fig.1-2, but note the multiplication order.

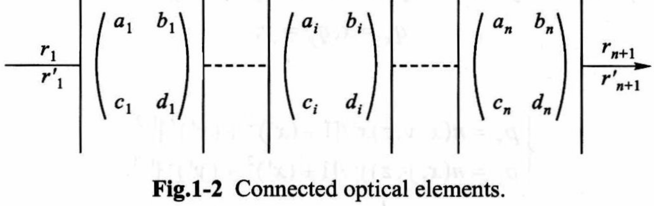

Fig.1-2 Connected optical elements.

According to Fig.1-2, we have

$$\begin{pmatrix} r_{n+1} \\ r_{n+1}' \end{pmatrix} = \begin{pmatrix} a_n & b_n \\ c_n & d_n \end{pmatrix} \cdots \begin{pmatrix} a_i & b_i \\ c_i & d_i \end{pmatrix} \cdots \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \begin{pmatrix} r_1 \\ r_1' \end{pmatrix} \tag{1.2-8}$$

$$\det(\underrightarrow{M}) = \det(\underrightarrow{M_n}) \cdots \det(\underrightarrow{M_i}) \cdots \det(\underrightarrow{M_1}). \tag{1.2-9}$$

When the *ABCD* matrix is applied to the reflection systems, the signs of ray parameters are defined in Fig.1-3. The sign of *r* is unchanged but the sign of *r'* is changed. It is equivalent to multiply a sign matrix *S* as follows

$$S = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{1.2-10}$$

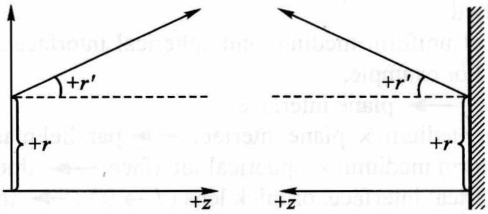

Fig.1-3 Definition of signs for ray parameters.

1.3 Derived methods for matrix elements

There are four basic methods to derive the concrete formations of ray or beam transfer matrix.

A. According to the definition

According to the definition of *ABCD* matrix (1.2-5), we have

$$\begin{aligned} a &= \frac{r_2}{r_1}\bigg|_{r_1'=0}, \quad b = \frac{r_2}{r_1'}\bigg|_{r_1=0}, \\ c &= \frac{r_2'}{r_1}\bigg|_{r_1'=0}, \quad d = \frac{r_2'}{r_1'}\bigg|_{r_1=0}. \end{aligned} \tag{1.3-1}$$

For instance, the ray transfer matrix formations of uniform medium and spherical interface can be derived by Eq.(1.3-1) as shown in Fig.1-4 and Fig.1-5, where R is the curvature radius of spherical interface.

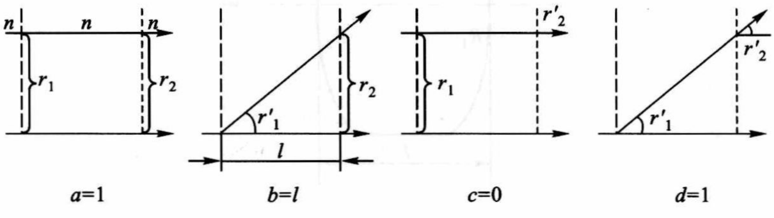

Fig.1-4 Ray transfer matrix for uniform medium.

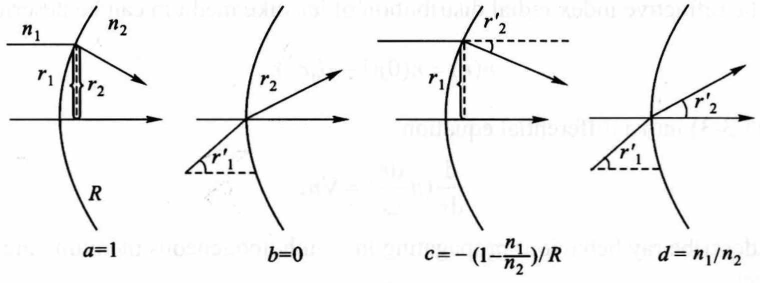

Fig.1-5 Ray transfer matrix for spherical interface.

B. Composed and simplified

The ray transfer matrices of uniform medium and spherical interface are fundamental formations, which can form some others. For example,

Spherical interface ($R \to \infty$) ➤ plane interface,
Plane interface × uniform medium × plane interface ➤ parallel plate,
Spherical interface × uniform medium × spherical interface ➤ thick lens,
Spherical interface × spherical interface, or thick lens ($l \to 0$) ➤ thin lens,
Thin lens × uniform medium × thin lens ➤ telescope,
Sign matrix S × spherical interface ($n_2 \to -n_1$) ➤ spherical reflector,
Sign matrix S × plane interface ($n_2 \to -n_1$), or spherical reflector ($R \to \infty$) ➤ flat reflector; etc.

A thick lens is shown in Fig.1-6, we can get its ray transfer matrix by composed method. A ray passing through the thick lens can be regarded as passing sequentially through a spherical interface with curvature radius R_1, a uniform medium with length l and a spherical interface with curvature radius R_2, then

$$\underset{\rightarrow}{M} = \begin{pmatrix} 1 & 0 \\ (1-n)/R_2 & n \end{pmatrix} \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -(1-1/n)/R_1 & 1/n \end{pmatrix}$$
$$= \begin{pmatrix} 1-\dfrac{(1-1/n)l}{R_1} & \dfrac{l}{n} \\ -[(n-1)(\dfrac{1}{R_1}+\dfrac{1}{R_2})-\dfrac{(n-1)^2 l}{nR_1R_2}] & 1-\dfrac{(1-1/n)l}{R_2} \end{pmatrix}. \tag{1.3-2}$$

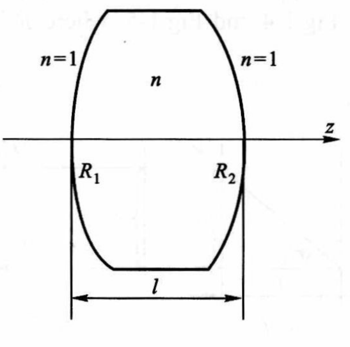

Fig.1-6 A thick lens.

C. To solve ray differential equation

For instance, the refractive index radial distribution of lenslike medium can be described by

$$n(r) = n(0)(1-\beta_0 r^2). \tag{1.3-3}$$

Substituting Eq.(1.3-3) into a differential equation

$$\frac{\mathrm{d}}{\mathrm{d}s}(n\frac{\mathrm{d}r}{\mathrm{d}s}) = \nabla n, \tag{1.3-4}$$

which is used to describe ray behaviour propagating in an inhomogeneous medium; and taking paraxial approximation, get

$$\frac{d^2r}{dz^2}+2\beta_0 r=0. \tag{1.3-5}$$

The solutions of Eq.(1.3-5) are

$$\begin{cases} r_2=\cos(l\sqrt{2\beta_0})r_1+\dfrac{1}{\sqrt{2\beta_0}}\sin(l\sqrt{2\beta_0})r_1', \\ r_2'=-\sqrt{2\beta_0}\sin(l\sqrt{2\beta_0})r_1+\cos(l\sqrt{2\beta_0})r_1', \end{cases} \tag{1.3-6}$$

for $\beta_0>0$; and

$$\begin{cases} r_2=\mathrm{ch}(l\sqrt{2\beta_0})r_1+\dfrac{1}{\sqrt{2\beta_0}}\mathrm{sh}(l\sqrt{2\beta_0})r_1', \\ r_2'=-\sqrt{2\beta_0}\mathrm{sh}(l\sqrt{2\beta_0})r_1+\mathrm{ch}(l\sqrt{2\beta_0})r_1', \end{cases} \tag{1.3-7}$$

for $\beta_0<0$. Make them in the matrix forms, then we get

$$\begin{pmatrix} r_2 \\ r_2' \end{pmatrix}=\begin{pmatrix} \cos(l\sqrt{2\beta_0}) & \dfrac{1}{\sqrt{2\beta_0}}\sin(l\sqrt{2\beta_0}) \\ -\sqrt{2\beta_0}\sin(l\sqrt{2\beta_0}) & \cos(l\sqrt{2\beta_0}) \end{pmatrix}\begin{pmatrix} r_1 \\ r_1' \end{pmatrix}, \text{ for } \beta_0>0 \tag{1.3-8}$$

and

$$\begin{pmatrix} r_2 \\ r_2' \end{pmatrix}=\begin{pmatrix} \mathrm{ch}(l\sqrt{2\beta_0}) & \dfrac{1}{\sqrt{2\beta_0}}\mathrm{sh}(l\sqrt{2\beta_0}) \\ -\sqrt{2\beta_0}\mathrm{sh}(l\sqrt{2\beta_0}) & \mathrm{ch}(l\sqrt{2\beta_0}) \end{pmatrix}\begin{pmatrix} r_1 \\ r_1' \end{pmatrix}, \text{ for } \beta_0<0. \tag{1.3-9}$$

As we know that, from Eq.(1.3-8) the lenslike medium is equivalent to a positive lens; from Eq.(1.3-9) the lenslike medium is equivalent to a negative lens. But the effects of the lenslike medium and the general lens on radial phase difference are different, the former is realized by changing the radial distribution of refractive index, the latter is realized by changing the spherical interface curvatures and the thickness of the lens.

D. The equivalent transfer method

The equivalent transfer method will be introduced in Chapter 3.

The transfer matrix for some common optical elements and media could be obtained by using above four derived methods, the concrete results are illustrated in Appendix 1. In which, the ray transfer flow graphs will be discussed in Chapter 5.

1.4 Reference plane moving technique

The ray transfer matrix elements in Appendix 1 depend on selected reference planes besides the intrinsic properties of the optical elements or systems.

Therefore, one can apply reference plane moving technique to get different matrix formalisms or to find required matrix formalisms as shown in Fig.1-7. The new ray transfer matrix can be decided by

$$\begin{aligned}\begin{pmatrix} a^M & b^M \\ c^M & d^M \end{pmatrix}&=\begin{pmatrix} 1 & b_2^+ \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & b_1^+ \\ 0 & 1 \end{pmatrix} \\ &=\begin{pmatrix} a+cb_2^+ & b+ab_1^++db_2^++cb_1^+b_2^+ \\ c & d+cb_1^+ \end{pmatrix}. \end{aligned} \tag{1.4-1}$$