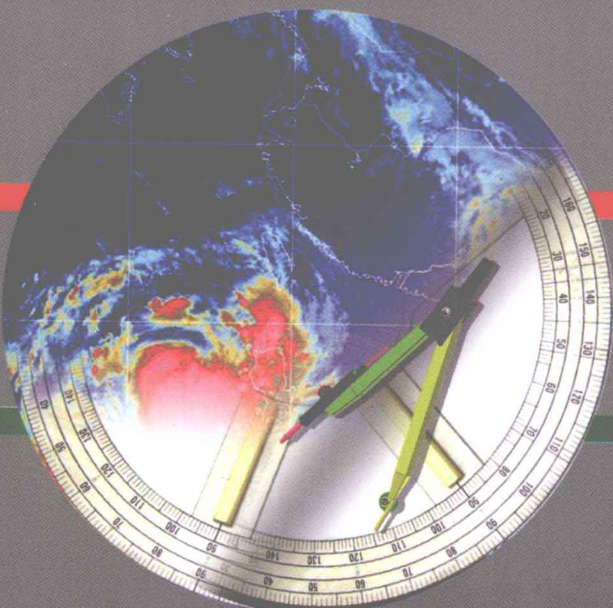
航海专业数学

Nautical Mathematics

（英文版）

主编　戴　冉　赵志垒



大连海事大学出版社

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Introduction

With China's accession to the WTO and the economic globalization, the world trade grows faster, and ships carry an estimated 90% of world trade. Therefore, water transportation plays special and vital roles which cannot be replaced by any other means. As a big shipping country with rich seafarer resources, it is of great importance to improve the overall quality of our country seafarers as to remain competitive in the labour market.

The advancement of the navigation technology brings out the higher demand for crew members' quality. They have to possess solid theoretical foundation and direct practice with theories. Through years of accumulation and practice, they have to be able to analyze and solve problems in all kinds of sailing conditions. In order to be conformable with epoch development and meet the higher requirements of maritime higher education, this book pays attention to cultivating students' ability to analyze and solve problems, not only guaranteeing necessary theoretical foundation, but also being characteristic of system and applicability. This book is a textbook of a main special basis course for students majoring in higher navigation; also, it can be referred by captain, officers, managers in shipping companies and scientific research personnel.

This book amounts to chief editors of Dai Ran and Zhao Zhilei.

This book's illustrations amount to Ding Yong and Liu Dexin.

Due to limited level, there will inevitably be lighter inadequate, so we'd like to invite your comments and suggestions. Thank you.

Editors

Contents

Chapter One Nautical Numerical Interpolation

Calculation method (numerical analysis), which is also named as scientific calculation, has the core idea that it is to obtain an approximate solution of certain continuous variables through limited steps of plus-minus and multiplication-division of four arithmetic operations. The limit theory provides the theoretical basis for approximate calculation: if an infinite sequence $\{x_n\}$ converges to a certain limit value x^*, then we can take the value x_N as the approximate solution of x^*. As long as the sequence number taken is big enough, the D-value $|x_N - x^*|$ will not exceed a certain preset positive number ε small enough. According to the Taylor expansion, a certain sufficiently smooth real function $f(x)$ can be expressed as a polynomial including infinite terms; then utilizing limit theory, we can take the first N terms of this polynomial and express them as $f(x)$ approximately, and thus we can obtain the approximation of $f(x)$ through limited steps of plus-minus and multiplication-division four arithmetic operations.

The limit theory and Taylor expansion provide us with theoretical basis for approximate calculation within greater scope, and the interpolation further provides practice guidelines for approximate calculation. In practice, for a certain properly smooth real function, we hope to obtain its function value at an arbitrary point in certain interval $[a, b]$ with less time. Provided that the interval is not very long, we can use other methods to work out some function valuees $f(x_0), f(x_1), \ldots f(x_n)$ at discrete points $x_0, x_1, \ldots, x_n$ in advance, then accordingly to construct n times or less than n times interpolation polynomial $p(x)$ can be used to conduct corresponding calculations.

In modern marine practice, different types of calculations are always needed to be done, for example, to conduct astronomical calculation by special statistical forms, to conduct freight calculation by charts, and to conduct types of business calculation by computers. The increasing popularity of computers has replaced massive and tedious manual calculations. However, because of the marine characteristics and habits, a lot of professional statistical forms are kept for mariners to refer to. This chapter is to introduce a basic calculation method—interpolation which can be utilized to conduct nautical calculation by means of nautical statistical forms.

In nautical calculations, many Functional Formulas are made use of, and the formulas are always laid out according to functional relationship, such as: $y=f(x)$. Based on the known value of x, the value of y can be obtained by referring to the relevant formula. However, you can not find all y values for each corresponding x value; that is to say, the y value you want corresponding to the known x value is not listed in the Formula and may just lie between two y values in the Formula. The method to obtain y value with values in the Formula according to the x value is termed as interpolation. Setting x as feature, the method to obtain function value y through referring to Formulas is called interpolation, and the inverse operation of interpolation is termed as inverse interpolation. According to the number of feature, the interpolation can be classified as single interpolation (one feature), double interpolation (two features) and so on. According to function character, the interpolation can be classified as proportional interpolation, interpolation by rate of change and so on.

Section One Proportional Interpolation

1 Proportional Interpolation

1.1 Proportional single interpolation (function of one variable)

Proportional single interpolation is one of the conventional interpolation methods in nautical practice.

The known function $f(x)$ is as shown in the Table 1-1-1.

Table 1-1-1 Function Table

Feature (independent variable)	Function value
x_0	y_0
x_1	y_1
…	…

For the feature x between tabulated data x_0 and x_1, with the method of proportional interpolation, try to obtain the corresponding function value.

For function $f(x)$ in interval (x_0, x_1), we can see it as a linear function to conduct interpolation. As seen in Fig.1-1-1, the triangle *abc* is similar to the triangle *aed*, so their corresponding sides are directly proportional to each other, that is:

$$\frac{y-y_0}{y_1-y_0}=\frac{x-x_0}{x_1-x_0}$$

$$y=y_0+\frac{x-x_0}{x_1-x_0}(y_1-y_0)$$

$$=y_0+\frac{y_1-y_0}{x_1-x_0}(x-x_0) \qquad (1\text{-}1\text{-}1)$$

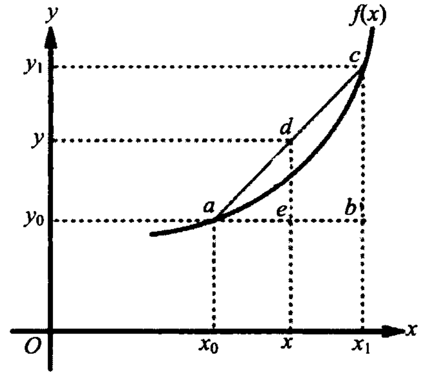

Fig. 1-1-1 Geometric Interpretation Diagram for Proportional Interpolation

Where y_1-y_0 was the tubular difference;

x_1-x_0 was the tubular interval;

$x-x_0$ was the interpolated interval.

The formula above is the calculation formula for proportional single interpolation.

The geometric interpretation of proportional single interpolation is to replace a length of a curve with a length of straight line, that is, to adopt the straight line $\overline{ac}$ between point (x_0, y_0) and point (x_1, y_1) to replace the curve $\overset{\frown}{ac}$.

As seen from Fig. 1-1-1, if $f(x)$ were a linear function the straight line $\overline{ac}$, then the y value obtained through linear interpolation would be accurate; and if $f(x)$ were a nonlinear function as the curve $\overset{\frown}{ac}$, then the y value obtained through linear interpolation would have the error of $\overline{df}$. Thus:

(1) In order to make the calculation simple and convenient, as long as the error lies within the acceptable limit, then the linear interpolation can be adopted.

(2) As to nonlinear functions, the less the tubular interval, the less the error of the function value obtained through linear interpolation.

Case 1-1-1: a certain ship's static hydraulic natural parameters are shown as in Table 1-1-2. Given the molded draft = 7.25 m, find this ship's displacement Δ and its deadweight DW.

Table 1-1-2 Hydrostatic Data Table

Molded draft (m)	Displacement (Δ/t)	DW (t)	Tons per centimeter (TPC/t·cm^{-1})	Moment to change trim on centimetre (MTC/9.81 kN·m·cm^{-1})	Transverse metacentric height above the baseline (KM/m)	Distance from midship to center of buoyancy (x_b/m)	Distance from midship to center of floatation (x_f/m)
6.00	11 860	6 295	23.02	177.25	8.840	0.164	−0.880
6.20	12 340	6 776	23.17	179.60	8.800	0.120	−1.130
6.40	12 820	7 255	23.32	182.00	8.760	0.068	−1.400
6.60	13 280	7 715	23.46	184.50	8.738	0.015	−1.710
6.80	13 760	8 195	23.63	187.00	8.720	−0.048	−2.040
7.00	14 240	8 676	23.78	189.75	8.710	−0.114	−2.400
7.20	14 710	9 145	23.95	192.50	8.710	−0.192	−2.750
7.40	15 200	9 635	24.11	196.00	8.714	−0.280	−3.135
7.60	15 680	10 115	24.29	198.50	8.720	−0.370	−3.510
7.80	16 180	10 615	24.46	202.00	8.740	−0.488	−3.895
8.80	18 680	13 115	25.39	222.50	8.894	−1.050	−5.450

To find Δ:

$$\Delta = \Delta_0 + \frac{\Delta_1 - \Delta_0}{d_1 - d_0}(d - d_0)$$

$$= 14\,710 + \frac{15\,200 - 14\,710}{7.40 - 7.20}(7.25 - 7.20)$$

$$= 14\,832.5 \text{ t}$$

To find DW:

$$DW = 9\,145 + \frac{9\,635 - 9\,145}{7.40 - 7.20}(7.25 - 7.20)$$

$$= 9\,267.5 \text{ t}$$

Case 1-1-2: a certain ship's standard compass deviations are shown as Table 1-1-3. Given ship's compass course = 070°, find the compass deviation at this compass course.

Table 1-1-3 A Certain Ship's Standard Compass Deviations Table

Compass course CC/°	Deviation /°	Compass course CC/°	Deviation /°	Compass course CC/°	Deviation /°	Compass course CC/°	Deviation /°
000	2.8	090	−2.5	180	−1.0	270	1.9
015	2.6	105	−3.4	195	0.2	285	1.8
030	2.0	120	−3.9	210	1.2	300	1.9
045	1.2	135	−3.8	225	1.8	315	2.0
060	0.1	150	−3.1	240	1.9	330	2.3
075	−1.2	165	−2.2	255	2.0	345	2.6
090	−2.5	180	−1.0	270	1.9	360	2.8

To find the deviation:

$$\text{Deviation} = +0°.1 + \frac{(-1°.2)-(+0°.1)}{075°-060°} \times (070°-060°) = -0°.8$$

1.2 Inverse proportional interpolation

The inverse operation of proportional interpolation is named as inverse proportional interpolation. That is to determine the feature x under condition that the function value y is known. The calculation formula is shown in the following:

$$x = x_0 + \frac{x_1 - x_0}{y_1 - y_0}(y - y_0) \tag{1-1-2}$$

2 Proportional Double Interpolation (Function of Two Variables)

When there are two variables in a function, the proportional double interpolation method is used to find the approximate solution. The proportional double interpolation is the most popular interpolation method in nautical calculations. Its use will be introduced through the following case.

Case 1-1-3: Let h be the target height, α the vertical angle, given the horizontal distance D=$h$cotα (n mile), with the formula 1-1-1, and Table 1-1-4 is used, find the horizontal distance D (n mile) when h=13.4 m and α=4′.4.

Table 1-1-4 Horizontal Distance Table (n mile)

α/′ \ D/n mile \ h/m	10	13.4	20
4	4.6	6.2	9.3
4.4		5.7	
5	3.7	5.0	7.4

To find D:

Step 1: to find D_1 when α=4′, h=13.4 m.

$$D_1 = D_0 + \frac{D_1 - D_0}{h_1 - h_0}(h - h_0) = 4.6 + \frac{9.3-4.6}{20-10}(13.4-10) \approx 6.2 \text{ n mile}$$

Step 2: to find D_2 when a=5′, h=13.4 m.

$$D_2 = D_0 + \frac{D_1 - D_0}{h_1 - h_0}(h - h_0) = 3.7 + \frac{7.4-3.7}{20-10}(13.4-10) \approx 5.0 \text{ n mile}$$

Step 3: to find D between D_1 and D_2 when a=4′.4.

$$D = D_1 + \frac{D_1 - D_0}{\alpha_1 - \alpha_0}(\alpha - \alpha_0) = 6.2 + \frac{5.0 - 6.2}{5 - 4}(4.4 - 4) \approx 5.7 \text{ n mile}$$

The conclusion from the above is that the proportional double interpolation would be conducted through three steps of proportional single interpolations. Due to the fact that Table 1-1-3 was calculated according to nonlinear function $D=h\cot a$, some errors would be produced when we adopt proportional interpolation. That is: in step 1 and step 2, since that α was set as a constant, then function $D=h\cot a$ turned into a linear function, thus D_1 and D_2 are accurate, having no error (without considering rounded error); in step 3, that a was set as a variable, then function $D=h\cot a$ turned into a nonlinear function, thus D would have some error due to proportional interpolation.

In nautical practice, we often adopt a more simple proportional double interpolation method where the precision is lower than the above method. However, its calculation process is simpler, and its use also meets nautical requirements (please refer to the formula 1-2-3). If there were three features in a function, then we can conduct interpolation based on the principle of the proportional double interpolation method. For example, when you look up the low altitude solar azimuth by *Sun's Azimuth Table*, you have to conduct proportional triple interpolation which will be explained further on.

Proportional interpolation is used to find the function value y corresponding to a feature which is between tabular data x_0 and x_1. However, in nautical practice, the reference feature x may be beyond the tabular features. For example, this situation may occur when you look up the tide height differences in standard port and secondary port by *Tide Table*, and you can still conduct calculations based on the proportional interpolation's principles. Because the reference feature is beyond the tabular features, the calculation method is referred to as proportional extrapolation.

Section Two Interpolation by Rate of Change

From section one we've known, for nonlinear functions, the proportional interpolation will cause an error. So, in order to reduce the error, the interpolation by rate of change is used.

1 Single Interpolation by Rate of Change(Function of One Variable)

As we know, the calculation formula of proportional interpolation is:

$$y = y_0 + \frac{y_1 - y_0}{x_1 - x_0}(x - x_0)$$

Where $\frac{y_1 - y_0}{x_1 - x_0}$ lies in the interval (x_0, x_1), which indicates the function's average rate of change.

The method of proportional interpolation, conducts interpolation according to the average rate of change, thus, it can be also called interpolation of average rate of change. When a function's change rates are almost the same, the method of proportional interpolation can be made good use of. However, some functions' change rates are markedly different; the proportional interpolation will cause a major error. For tables worked out by these functions, the proportional interpolation cannot be applied to but instead interpolation by rate of change. According to the proportional interpolation's calculation formula, the calculation formula of interpolation by rate of change is:

$$y = y_0 + \frac{dy}{dx}(x - x_0) \tag{1-2-1}$$

For function tables which need to conduct interpolation by rate of change, the corresponding change rates are always given in the tables.

Case 1-2-1: Table 1-2-1 was worked out by using the function $y=x^2$. Find the y value when $x=2.3$.

Table 1-2-1 Function Table ($y=x^2$)

x	$y=x^2$	dy/dx
2	4	4
3	9	6
4	16	8

To find y:

(1) Use method of proportional interpolation

$$y = 4 + \frac{9-4}{3-2}(2.3-2) \approx 5.5$$

(2) Use method of interpolation by rate of change based on $x=2$

$$y = 4 + 4(2.3 - 2) \approx 5.2$$

(3) Use method of interpolation by rate of change based on x=3

$$y = 9 + 6(2.3 - 3) \approx 4.8$$

(4) Calculate directly by $y=x^2$

$$y = (2.3)^2 = 5.29$$

From the above we can see:

(a) The error of y value obtained from proportional interpolation is large.

(b) The error of y value obtained from interpolation by rate of change based on x=2 is smaller.

Conclusion: when using the method of interpolation by rate of change, in order to reduce the error, we should make use of the tabular data which is the closest to the reference feature to conduct interpolation. For example, in Case 1-2-1, the reference feature x=2.3, the closest tabular feature x_0=2, with the corresponding function value y_0=4.

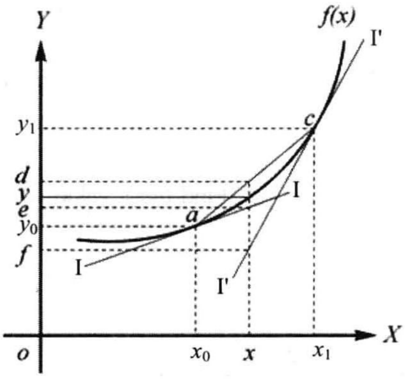

Fig. 1-2-1 Geometric Interpretation of Interpolation by Rate of Change

The geometric interpretation of interpolation by rate of change is that: to take the tangent line at the point '*a*' defined by tabular feature (x_0, y_0) to replace the curve, as is shown in Fig. 1-2-1, the exact value is $\overline{oy}$. If we adopt proportional interpolation, the error is $\overline{dy}$; if we adopt interpolation by rate of change based on x_0, the error is $\overline{ey}$; if we adopt interpolation by rate of change based on x_1, the error is $\overline{fy}$. Apparently, $\overline{ey} < \overline{dy} < \overline{fy}$.

2 Double Interpolation by Rate of Change

For a function of two variables and whose change rates are nonuniform, the double interpolation by rate of change is used.

Let a function be:

$$z = f(x, y)$$

At the point $z_0 = f(x_0, y_0)$, Taylor series of expansion is:

$$z = z_0 + \frac{\partial z}{\partial x}(x - x_0) + \frac{\partial z}{\partial y}(y - y_0) \tag{1-2-2}$$

In some nautical special statistical formulas, there are change rates $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, and they can both be used to conduct double interpolation by rate of change.

Here, another aim in using the double interpolation by rate of change is to derive a simple method for approximate calculation of proportional double interpolation. For the formula (1-2-2), if we rewrite its change rate into average change rate, then, from the calculation formula of interpolation by rate of change, we arrive at the approximate calculation formula of proportional interpolation (Table 1-2-2 is the function table of $z = f(x, y)$):

$$z = z_{00} + \frac{z_{10} - z_{00}}{x_1 - x_0}(x - x_0) + \frac{z_{01} - z_{00}}{y_1 - y_0}(y - y_0) \tag{1-2-3}$$

Table 1-2-2 Function Table (z=f(x,y))

x / z / y	x_0	x_1
y_0	z_{00}	z_{10}
y_1	z_{01}	z_{11}

If we adopt formula (1-2-3), it would only take two steps of proportional interpolation to accomplish the proportional double interpolation. In formula (1-2-3), the second term is to conduct proportional interpolation for the feature x; and the third term is to conduct proportional interpolation for the feature y; so this method is also called proportional interpolation of two features.

Case 1-2-2: with the formula (1-2-3), find the solution to case 1-1-3. (given h=13.4 m, α=4′.4, find the horizontal distance D (n mile)).

Table 1-2-3 Table of Horizontal Distance D (n mile)

a'' \ D/n mile \ h/m	10	20
3	6.2	12.3
4	4.6	9.3
5	3.7	7.4

To find D:

$$z = z_{00} + \frac{z_{10} - z_{00}}{x_1 - x_0}(x - x_0) + \frac{z_{01} - z_{00}}{y_1 - y_0}(y - y_0)$$

$$= 4.6 + \frac{9.3 - 4.6}{20 - 10}(13.4 - 10) + \frac{3.7 - 4.6}{5 - 4}(4.4 - 4)$$

$$\approx 5.8 \text{ n mile}$$

The calculation result above is basically identical with the result (D=5.7) calculated through proportional double interpolation method, and, it's relatively simple. Thus, in nautical practice, this method is often used to conduct proportional double interpolation.

In conclusion, when looking up a function table worked out through a nonlinear function, whether you use proportional interpolation or use interpolation by rate of change, some errors would be caused. In nautical practice, we often use the method of proportional interpolation. If a function table lists change rates, they should be adopted to conduct interpolation by rate of change.

Questions and Revision Exercises

1. What is the theoretical basis to conduct approximate calculation with interpolation?
2. What is the implication of interpolation in nautical calculation?
3. In nautical calculation, what is the classification method to classify the interpolation? Also, list the categories.
4. How is the error caused when the proportional interpolation method is adopted?
5. List the formulas of the proportional interpolation and the interpolation by rate of change, and explain the differences in geometric properties between interpolation by rate of change and proportional interpolation.

6. For linear proportional interpolation, the less of tabular interval, the less the interpolated error. Why?

The less of interpolated interval, the less the interpolated error. Why?

7. What should we pay attention to when using interpolation by rate of change? Why?

8. In the table P1-1 below, given a=25.3, b=29°35′, find c.

Table P1-1 Function Table (c=f(a, b))

b \ a (c)	24	25	26
28°	46.5	47.4	48.2
29°	45.8	46.6	47.3
30°	45.0	45.9	46.7

9. Table P1-2, shows the correction of moon's altitude,

Table P1-2

Altitude \ Horizontal parallax	54′	55′	56′
53°	46.5	47.4	48.2
54°	45.8	46.6	47.3
55°	45.0	45.9	46.7

(1) Given altitude=53°20′, horizontal parallax=54′.6, find the correction of the moon's altitude.

(2) Given altitude=54°40′, horizontal parallax=55′.3, find the correction of the moon's altitude.

Chapter Two Spherical Trigonometry

Spherical trigonometry is a part of spherical geometry, mainly used to find, explain and deal with the relationship in sides and angles of polygons (especially for triangles) on spherical surfaces. A spherical triangle is similar to a plane triangle in many ways. Many formulas related to spherical triangles are derived from the knowledge of plane triangles. However, there are still many differences between them, the significant difference is that the sum of inner angles of a spherical triangle is not 180°. In nautical practice, celestial navigation and the great circle route are both based on the theory of spherical trigonometry, mainly studying the spherical triangle's properties and using its solving methods, to provide foundations of mathematics for specialized courses in navigation.

Section One Spherical Geometry

1 Sphere and Spherical Surface

A sphere is a figure such that all points of its surface are equal distant from a certain point within it, called the centre of sphere, and the surface is called the spherical surface. Any straight line drawn from the centre of the sphere to the surface is called a radius, and a straight line drawn through the centre and terminating both ways at the surface is called a diameter. For any sphere, all its radiuses are equal, so are all of its diameters. Similarly, two spheres with equal radiuses or equal diameters are congruent.

2 Circles on Spherical Surfaces

The line on the spherical surface crossed by any plane is a circle.

As shown in Fig.2-1-1, the spherical surface is crossed by plane π, and A is an arbitrary point on the sphere. Drop a vertical line $\overline{OO'}$ from centre of the sphere O to cross section π, with the connection lines $\overline{OA}$ and $\overline{O'A}$, forming a right angled triangle OAO', $\angle OO'A = 90^\circ$, then: