中央研究院

# 民族學研究所集刊

第二十三期

目　　錄




中華民國五十六年春季

臺灣・南港

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# NUMERICAL KINSHIP NOTATION SYSTEM

## Mathematical Model of Genealogical Space

JOHN H. T. HARVEY and PIN-HSIUNG LIU

The idea on which this paper is based is the fruit of my discussions with Mr. Harvey at Harvard during summer 1965. By the end of October we presented the first draft of this paper to Professors Maybury-Lewis and White for comment. In the meantime a new category diagram was adopted, differing from the one used in the first draft, which enabled us to reach new results and to compose a second draft by the end of December. In addition to the numerical system for kinship categories, we already started to establish the basis for the numerical system of kinship types and its operational rules.

As the system opens a new field for kinship study, we also planned to set up a transformational analysis of kinship terminology, as well as a formal analysis of kin group, especially for the section system, to show the applicability of the group theory and other mathematical methods to this study. This would lead it to genuine kinship algebra (which we would like to name kinology) the usability of which has been in doubt up to now.

Due to certain deficiencies in the second draft we started to work out a third draft for clarification and addition. But through my sudden depature from Cambridge, this could not be realized and our cooperation had to be postponed. Considering, however, the possible usefulness of the study for my colleagues and the fact that the working out of the final draft will require a considerable length of time, I herewith publish the major part of the second draft in its present form.

I take the opportunity to express my gratitude to the Harvard-Yenching Institute for having received me as a visiting scholar during the years 1964-1966.

P. H. L.

### KINSHIP CATEGORIES

We define a *kinship category* as a set consisting of all egocentric relationships expressible by a given sequence of sex-generalized lineal links, i. e., in terms of parent and child.

The eight primary relationships of the traditional languaged-based kinship notation are not uncommonly supplemented or replaced by four without specification of sex. A notation proposed by Romney (Romney and D'Andrade 1964), which analyzes these twelve terms into relational and sexual components, provides a sex variable "a" in addition to sex constants "m" and "f" to the same effect, and expresses what are essentially kinship categories by sequences of the relation markers "+", parent link, "−", child link, "0", sibling link, and "=", marriage bond.

The further step of reducing to two primitive relationships is implicit in a notation suggested by Radcliffe-Brown (Radcliffe-Brown 1930), where the up-down angle for a collateral link and the down-up angle for an affinal link transparently are ordered pairs of the up-slant for an ascendent link and the down-slant for a descendent link with the intervening terms unstated.

It is important to note that, unlike generalization of sex, decomposition of non-lineal links into lineal *transitions* involves no loss of information, except where "parent's child" may have been utilized to distinguish half-sibling from full "sibling". This is because it merely exploits the redundancy in the traditional system that such transitions must be mediated by sequences of non-lineal links, minimally a collateral link between ascent and descent and an affinal link between descent and ascent, the sequences alternating unless step-relations are intended.

It is obvious that a collateral link may be replaced by ascent to and descent from *lowest common ascendents LCA*, the understood parents, and an affinal link by descent to and ascent from *highest common descendents HCD*, the actual or potential offspring. If the latter is considered an artifice, we might point out the genealogical irrelevance of marriage without issue, and the sociological implications of the linguistic phenomenon of teknonymy.

A kinship category, then, is equivalent to a kinship term compounded from the primary relationships "Parent", "Child", "Sibling", and "Spouse", with the exception that the seldom exercised option of indicating half-siblinghood is not open.

The advantage of reduction to a pair of primitive relations, particularly to a reciprocal pair with suggestive mathematical analogs, is that it opens up the possibility of applying powerful techniques to operations on complex terms.

## NUMERICAL KINSHIP CATEGORIES

We pursue the mathematical analogy by giving each kinship category a numerical designation, or *numerical kinship category C.*

If we map the sequence of parent and child links defining a kinship category into a corresponding sequence of +1's and −1's, respectively, it is apparent that we may consolidate by summing subsequences of like sign, so that a *digit d* stands for *d* consecutive generations in a given direction.

Since signs now necessarily alternate, we may adopt an *alternate sign place system* which marks sign by position only, just as ordinary numerical place systems mark successive powers of the base by position only. We assign the first *place* and all subsequent odd places to positive values, or generations of ascent, the second place and all subsequent even places to negative values, or generations of descent. A negative value for an initial digit is indicated by a preposed zero, or empty first place. Similarly, for reasons that will become evident later, a final zero is written after a final positive digit.

As in other place systems, zero functions as a place-holder. We restrict it to initial and final place, with the exceptions below. Note that we define "digit" to exclude zero, $d \geq 1$. The use of zero rounds out all numerical categories to an integral number of ordered *pairs* $p_i$, each consisting of a *positive place* $x_i$ followed by a *negative place* $y_i$, $p_i = x_i y_i$, the subscript being the ordinal number of the pair. The null category, ego, is written 00.

A numerical kinship category, then, has the canonical form

$$C = x_1y_1,\ x_2y_2,\ \ldots,\ x_{Np}y_{Np},$$

where the *number of pairs* $Np$ may be one or more, $Np \geq 1$, the value of the first and last places may be zero or more, $x_1 \geq 0 \leq y_{Np}$, and the value of any intervening places must be one or more, $x_{i>1} \geq 1 \leq y_{i<Np}$.

With $d_i = 1$ and the *number of digits* $Nd$ also one, we have the lineal primary categories:

10 = parent;
01 = child.

With $Nd = 2$, we have the non-lineal primary categories:

11 = (half-) sibling;
0110 = spouse.

With $Nd \geq 3$, however, there will be one or more 1's which are neither the initial nor the final digit, a fact which invariably signals a step-category. Thus, for three and four digits, we have the following "primary" step-categories:

1110 = step-parent;
0111 = step-child;
1111 = step-sibling;
011110 = co-spouse.

If we avoid "inside 1's", the minimal corresponding non-step-categories are:

1210 = sibling's spouse;
0121 = spouse's sibling;
1221 = sibling's spouse's sibling;
012210 = spouse's sibling's spouse.

Membership in the familiar major groupings of kin types may be determined by inspection of the overall form of a numerical kinship category, as we demonstrate by listing the following *generalized categories*, in which the digit variable $\hat{1} \geq 1$ replaces the 1's of the previous examples:

00 = ego;
$\hat{1}0$ = ascendent;
$0\hat{1}$ = descendent;
$\hat{1}\hat{1}$ = collateral;
$0\hat{1}\hat{1}0$ = affinal;
$\hat{1}\hat{1}\hat{1}0$ = collateral-affinal;
$0\hat{1}\hat{1}\hat{1}$ = affinal-collateral;

$\hat{1}\hat{1}\hat{1}\hat{1}$ = collateral-affinal-collateral;

$0\hat{1}\hat{1}\hat{1}\hat{1}0$ = affinal-collateral-affinal.

The consanguineal categories, with no affinal transitions, are the one-pair categories, if we agree that ego is reflexively consanguineal. In general, each additional pair means an affinal transition, so that the *number of affinal transitions Na* is one less than the number of pairs, $Na = Np - 1$. The *number of collateral transitions Nc* is the number of pairs less the *number of pairs with zero Nz*, $Nc = Np - Nz$.

In one-pair categories, of course, $Nz = 1$ indicates lineality, considering ego reflexively lineal, with the empty place giving the direction. In multi-pair categories, initial or final zeros, or $x_1 = 0$ and $y_{Np} = 0$, indicate initial and final affinal transitions, respectively. Conversely, $x_1 \geq 1$ and $y_{Np} \geq 1$ indicate initial and final collateral transitions.

We have already pointed out that "inside 1's" are diagnostic of step-relations. The non-step-categories of three and four digits may be represented by partially generalized categories with $\tilde{2} \geq 2$:

$\hat{1}\tilde{2}\hat{1}0$ = non-step-collateral-affinal;

$0\hat{1}\tilde{2}\hat{1}$ = non-step-affinal-collateral;

$\hat{1}\tilde{2}\tilde{2}\hat{1}$ = non-step-collateral-affinal-collateral;

$0\hat{1}\tilde{2}\tilde{2}\hat{1}0$ = non-step-affinal-collateral-affinal.

## THE CATEGORY DIAGRAM

The standard *category diagram* may be read as a simple two-dimensional representation of the consanguineal categories, or all one-pair categories $x_1y_1$. (Fig. 1)

| | | | | | | |
|---|---|---|---|---|---|---|
| 00 | 10 | 20 | 30 | 40 | 50 | 60 |
| 01 | 11 | 21 | 31 | 41 | 51 | 61 |
| 02 | 12 | 22 | 32 | 42 | 52 | 62 |
| 03 | 13 | 23 | 33 | 43 | 53 | 63 |
| 04 | 14 | 24 | 34 | 44 | 54 | 64 |
| 05 | 15 | 25 | 35 | 45 | 55 | 65 |
| 06 | 16 | 26 | 36 | 46 | 56 | 66 |

Fig. 1. Category Diagram. (Consanguineal)

Since $x$ is necessarily positive and $y$ necessarily negative, we are confined to the IVth, or lower right, quadrant of the infinite Cartesian plane. The lineal categories $Nd \leq 1$ are *on the axes*: ego 00 at their intersection, the *origin*, the upper left *square*; ascendents $\hat{1}0$ on the *x-axis*, in the $x_1$th square to the right along the top *row*; descendents $0\hat{1}$ on the *y-axis*, in the $y_1$th square down the leftmost *column*. Collateral categories $\hat{1}\hat{1}$ are *in the quadrant*, at the intersection of $x_1$th column and $y_1$th row.

In general, a consanguineal category $x_1y_1$ is located in the square $x_1y_1$, where the value of $x_1$ may be read off the ascendent categories in the top row and the value of $y_1$ off the descendent categories in the leftmost column.

The category diagram may also be read as an $n$-dimensional representation of multi-pair categories of $n$-digits. Note that the alternate sign principle implies that we are confined to only one of the $2^n$ hyper-quadrants marked off by the $n$ axes of an $n$-dimensional space.

In the two-dimensional case we considered the category $x_1y_1$ to be represented by the square $x_1y_1$. In the $n$-dimensional case we consider that it is represented by the *path* from the origin to $x_1y_1$: a null path for ego 00, a path to the right for ascendents $\bar{1}0$, a path down for descendents $0\bar{1}$, and a path to the right to the *lowest common ascendent LCA*, in the square $x_10$ followed by a path down for collaterals $\bar{1}\bar{1}$. In other words, we reinterpret the expression $x_1y_1$ as an ordered pair of instructions for a move $x_1$ squares to the right followed by a move $y_1$ squares down, with 0 an instruction for a *null move*, the path thus traced standing for the category.

Now, to take the simplest multi-pair categories, affinals $0\bar{1}\bar{1}0$, we may treat $0y_1x_20$ as calling for a null move to the right, then a move $y_1$ squares down, then a move $x_2$ squares to the right, then a null move down. In other words, for any given $x_1 = x_2$ and $y_1$, $0y_1x_20$ is the inverse of $x_1y_1$, the path along the other two sides of a rectangle, reaching the same square. It is, in fact, what we discuss below as the *negative reciprocal.* The point at the moment is that more than one path leads to a given square. An affinal path passes through the *highest common descendent HCD*, the implied (potential) child or children in the link, in the square $0y_1$.

It is obvious that this procedure may be extended to cover categories of more than two digits. A collateral-affinal category $x_1y_1x_20$ calls for a path $x_1$ to the right, then $y_1$ down, then $x_2$ to the right. An affinal-collateral category $0y_1x_2y_2$ calls for a path $y_1$ down, then $x_2$ to the right, then $y_2$ down.

In general, a category $x_1y_1x_2y_2 \ldots\ldots x_{Np}y_{Np}$ is shown by the path traced by the indicated sequence of $n$ pairs of moves $x_i$ to the right followed by $y_i$ down, with the value zero for a null move limited to first and last place, $x_1$ and $y_{Np}$.

If step-categories are to be excluded, the digit 1 will appear only as shown in the pairs $1y_1$, 01, 10, $x_{Np}1$, with "zero or nothing" on its outer flank.

| 00 | 10 | 20 | 30 | 40 | 50 | 60 |
|---|---|---|---|---|---|---|
| 01 | **0110** | **0120** | **0130** | **0140** | **0150** | **0160** |
| 02 | **0210** | **0220** | **0230** | **0240** | **0250** | **0260** |
| 03 | **0310** | **0320** | **0330** | **0340** | **0350** | **0360** |
| 04 | **0410** | **0420** | **0430** | **0440** | **0450** | **0460** |
| 05 | **0510** | **0520** | **0530** | **0540** | **0550** | **0560** |
| 06 | **0610** | **0620** | **0630** | **0640** | **0650** | **0660** |

Fig. 2. Category Diagram (Affinal).

It will be seen that there is a unique path, as well as a unique number, corresponding to every possible category, and a unique category corresponding to every possible path, as well as to every possible number.

Note that tilting the diagram $-45°$ restores the conventional semantic and visual orientation of genealogical space to the vertical component of the axes, up (and to the right) for ascent, down (and to the right) for descent.

Tilting it 45° in the opposite direction, clockwise, makes it a blueprint of the device at the IBM exhibit at the New York World's Fair that demonstrated binomial distribution, or the normal curve. Balls inserted at the origin on top and free to fall through the sides of the squares faced a succession of left-right "Choices", the random resulting paths leading to a statistically predictable distribution tapering from the center out at the bottom, approximating Pascal's triangle. This isomorphism allows simple calculation of the number of paths, and categories, of various kinds by well-known formulas, as discussed below.

All categories "in" a given square, that is, whose paths reach the square, share the same values of four properties which we now define. This convergence is thus not "noisy", in the information theory sense, but, rather, meaningful.

## THE POSITIVE AND NEGATIVE SUMS

The *positive sum* $Sx$ is the sum of all digits $x_i$ in a category, and the *negative sum* $Sy$ is the sum of all digits $y_i$. All categories in a given square have the same values of $Sx$ and $Sy$, which are, of course, the values of $x_1$ and $y_1$ of the collateral category occupying the square. (All squares on the axes contain only a single lineal category.)

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |

Fig 3. Positive Sum $Sx$.

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 | 6 | 6 | 6 |

Fig. 4. Negative Sum $Sy$.

All categories of a given $Sx$ are thus crossed by a vertical rook's move, all those of a given $Sy$ by a horizontal rook's move. We thus have a coordinate system in which each square is identified by a coordinate pair $SxSy$, the values read off from the lineal categories as for collateral categories.

## THE ABSOLUTE SUM AND (RELATIVE) DIFFERENCE

The *absolute sum* $Sxy$ of a category is the sum of its positive sum $Sx$ and negative sum $Sy$, or the length of its path in one-square moves. All squares of a given $Sxy$ are crossed by a bishop's move from one axis to the other. The value may be read off from either of the lineal categories at the ends of the diagonal, if the axes are extended sufficiently.

The absolute sum is the measure employed in the Korean "inch" system of kinship terminology, where an uncle or aunt 21 is "three inches" removed, a great-uncle or great-aunt 31 or a first cousin 22 "four inches", a first cousin once removed 32 or 23 "five inches", and so forth, the system being restricted to consanguineals and excluding affinals.

$Sxy$ combines with the *number of digits* $Nd$ in the formula which assigns a category to the $n$th "order" (Radcliffe-Brown 1930) or to the class of $n$-ary "relatives" (Murdock 1949), $n = Sxy - Nd + 1$. In conventional notation based on four primary relations, order is simply the number of terms. This formula corrects for the fact that our notation adds $Nd - 1$ implicit *LCA*'s and *HCD*'s, one for each sibling link and marriage link, respectively, namely the last *unit* in each digit but the last. First cousin 22 is $4 - 2 + 1 = 3$, tertiary.[1]

| 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |

Fig. 5. Absolute Sum $Sxy$.

| 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| -1 | 0 | 1 | 2 | 3 | 4 | 5 |
| -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| -4 | -3 | -2 | -1 | 0 | 1 | 2 |
| -5 | -4 | -3 | -2 | -1 | 0 | 1 |
| -6 | -5 | -4 | -3 | -2 | -1 | 0 |

Fig. 6. (Relative) Difference $Dxy$.

(1) It is interesting that Murdock writes "for our purposes it will be sufficient to class all who are more remote than tertiary relatives as "*distant relatives*" (p. 95) in a work which devotes approximately two hundred pages to kinship. Consider that order mounts up so rapidly that second cousin 33 is already quinary

The absolute sum $Sxy$ combines with the positive sum $Sx$ and the negative sum $Sy$ in the formula for the *number of categories Nc* in a given square, which may be computed from any category in the square by $Nc = \frac{Sxy!}{Sx!\,Sy!}$, where $n$ factorial $n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n$. The number of categories per square for $Sx \leq 6$ and $Sy \leq 6$ are shown in Fig. 8.

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 0 | 1 | 2 | 3 | 4 | 5 |
| 2 | 1 | 0 | 1 | 2 | 3 | 4 |
| 3 | 2 | 1 | 0 | 1 | 2 | 3 |
| 4 | 3 | 2 | 1 | 0 | 1 | 2 |
| 5 | 4 | 3 | 2 | 1 | 0 | 1 |
| 6 | 5 | 4 | 3 | 2 | 1 | 0 |

Fig. 7. Absolute Difference *Dhl.*

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 3 | 6 | 10 | 15 | 21 | 28 |
| 1 | 4 | 10 | 20 | 35 | 56 | 84 |
| 1 | 5 | 15 | 35 | 70 | 126 | 210 |
| 1 | 6 | 21 | 56 | 126 | 252 | 462 |
| 1 | 7 | 28 | 84 | 210 | 462 | 924 |

Fig. 8. Number of Categories *Nc.*

The *(relative) difference Dxy* of a category is the positive sum $Sx$ minus the negative sum $Sy$, $Dxy = Sx - Sy$. It is thus the sum of the digits taking account of their signs, while $Sxy$ is the sum of their absolute values. All squares of a given $Dxy$ are crossed by a bishop's move away from both axes, and the value, which may be negative, is given by the single lineal category at the bounded end of the diagonal. The relative difference corresponds, of course, to the generation with respect to ego. The *main diagonal* of the category diagram, $Dxy = 0$, or ego's generation, is important as an axis of symmetry.

Computation of this useful measure may be simplified for multi-digit categories by several readily discoverable heuristic aids such as cancellation of mirror-image symmetry *abccba,* symmetrical repetition *abcabc,* "twin" pairs *aa,* and so forth. In most practical work digits will tend to have low values, and the chances of such patterns occurring will be far higher than, say, in telephone numbers.

The absolute sum and relative difference together provide an alternate system of coordinate pairs $SxyDxy$ which is rotated 45° from the other. Conversion from $SxSy$ to $SxyDxy$ is obvious: $Sxy = Sx + Sy$ and $Dxy = Sx - Sy$. The reverse is also possible: $Sx = \frac{Sxy + Dxy}{2}$ and $Sy = \frac{Sxy - Dxy}{2}$. Note that a negative $Dxy$ in effect inverts the operation in the numerator.

## THE HIGHER AND LOWER SUMS AND ABSOLUTE DIFFERENCE

Three other measures specify properties which are symmetrical with respect to the main diagonal.

The *higher sum* $Hxy$ is the larger of $Sx$ and $Sy$, the *lower sum* $Lxy$ the smaller. All squares of a given $Hxy$ are crossed by horizontal and vertical moves outward from the square $Sx = Sy = Hxy$, in other words, by a right-angle parallel to that formed by the axes and $Hxy$ squares from it. All squares of a given $Lxy$ are crossed by horizontal and vertical rook's moves from the square $Sx = Sy = Lxy$ to the axes, in other words, by a right angle that combines with the axes to form a square figure.

The lower sum, or, strictly, the lower digit, is the value of $n$ in $n$th *collaterality*. The higher sum seems to function as a measure of *propinquity* in certain cultures, a given number of consecutive generations of ascent or descent constituting a cut-off point for a given kind or degree of relatedness.

Subtracting $Lxy$ from $Hxy$ gives the *absolute difference* $Dhl$, the number of generations from ego's. This is, of course, the absolute value of $Dxy$. We may then say that English cousinship terminology is of the form $(Lxy - 1)$th cousin $Dhl$ times removed.[1] Squares of a given $Dhl$ naturally occupy diagonals parallel to and $Dhl$ squares either side of the main diagonal. (Note that in counting diagonal distance one must count stepwise.)

A coordinate pair $HxyLxy$ identifies two squares symmetrical with respect to the main diagonal and $Dhl$ squares from it.

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 2 | 2 | 3 | 4 | 5 | 6 |
| 3 | 3 | 3 | 3 | 4 | 5 | 6 |
| 4 | 4 | 4 | 4 | 4 | 5 | 6 |
| 5 | 5 | 5 | 5 | 5 | 5 | 6 |
| 6 | 6 | 6 | 6 | 6 | 6 | 6 |

Fig. 9. Higher Sum *Hxy*. (Propinquity)

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 2 | 2 | 2 | 2 | 2 |
| 0 | 1 | 2 | 3 | 3 | 3 | 3 |
| 0 | 1 | 2 | 3 | 4 | 4 | 4 |
| 0 | 1 | 2 | 3 | 4 | 5 | 5 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |

Fig. 10. Lower Sum *Lxy*. (Collaterality)

(1) Compare the simplicity of this rule, once the conceptual infrastructure has been established, with that of the rule given by Roark (1961).

## CORRELATIONS

We have pointed out that, while a coordinate pair *SxSy* or *HxyDxy* identifies a single square of the category diagram, a coordinate pair *HxyLxy* or *SxyDhl* identifies two squares which are symmetrically located with respect to the main diagonal is symmetrically located with respect to itself, and is thus uniquely specified.(1)

Such a *correlation* between two squares implies two kinds of correlation, *negation* and *reciprocity*, between the categories of one square and their respective *inverses* in the other, each on a one-to-one basis. When the correlation is between a main diagonal square and itself, each category finds its inverses in the same square. Every category finds its *negative reciprocal* in its own square.

The *negative* $(-1)C$ of a category $C$ is the product of its *scalar multiplication* by $-1$, inverting the sign of each digit. Since our notation indicates sign value by an alternate sign place system, this simply involves the dropping or adding of initial and final zeroes:

$$(-1)d_1....d_n = 0d_1....d_n0;$$
$$(-1)0d_1....d_n0 = d_1....d_n;$$
$$(-1)d_1....d_n0 = 0d_1....d_n;$$
$$(-1)0d_1....d_n = d_1....d_n0.$$

The negative of a collateral category NN.... is an affinal category ON.... and vice versa. A category $C$ and its negative $(-1)C$ together constitute a *negative correlation* $(\pm 1)C$.

Where there is no basis of choice, we arbitrarily use the collateral term of the correlation in this expression. Of course, $(-1)(-1)C = C$. Note that only the identity element ego 00 is its own negative.

The path of a category will exactly match the path of its negative if the diagram is folded over along the main diagonal, the two paths of the correlation together forming an ink blot pattern.

The *reciprocal* $C^{-1}$ of a category $C$ is its right-to-left inverse, the expression read backwards:

$$d_1....d_n{}^{-1} = d_n....d_1;$$
$$0d_1....d_n0^{-1} = 0d_n....d_10;$$
$$d_1....d_n0^{-1} = 0d_n....d_1;$$
$$0d_1....d_n{}^{-1} = d_n....d_10.$$

A *reciprocal correlation* $C^{\pm 1}$ may be written with either term of the correlation as $C$, since $(C^{-1})^{-1} = C$. When the choice is arbitrary, we write the term which is higher when read as a decimal fraction ".$C$".

(1) Note that coodinate pairs *HxyDxy* and *LxyDxy* also specify single squares, while *HxySxy* and *LxySxy* also specify correlations.

In our alternate sign place system, turning an expression head for tail automatically inverts all signs. With the exception that the necessary inversion of lineal links is not automatic, the same simple method of finding the reciprocal applies in the notations of Radcliffe-Brown and Romney. The operation is considerably more cumbersome in the language-based conventional notation, despite the fact that the principle is simply that of retracing one's steps.

All categories $Dhl = 0$, on the main diagonal, find their reciprocals in their own squares, with all categories of symmetrical form $x_i = y_{n-i+1}$ such as 22 or 1331 being their own reciprocals. All other categories, of course, have reciprocals in the correlated square across the main diagonal.

Note that the reciprocal of a lineal category $Sxy = Hxy$ is the same as its negative.

Limiting consideration to categories of three digits or less, exclusive of ego, we find the following generalized reciprocal correlations:

| | |
|---|---|
| Lineal: | $10^{\pm 1} = 10$ and 01; |
| Collateral: | $11^{\pm 1} = 11$; |
| Affinal: | $0110^{\pm 1} = 0110$; |
| Non-lineal, $Nd = 3$: | $1110^{\pm 1} = 1110$ and 0111. |

Most work on kinship restricts the last correlation to $1110^{\pm 1}$, spouses of collaterals and collaterals of spouse. Without any empirical claim as to the sociological boundedness of this correlation, we find it useful to apply the concept unrestrictedly in mathematical operations on categories.

The *Negative reciprocal* $(-1)C^{-1}$ of a category $C$ has the order of digits reversed without inversion of signs:

$$(-1)d_1....d_n{}^{-1} = 0d_n....d_1 0;$$
$$(-1)0d_1....d_n 0^{-1} = d_n....d_1;$$
$$(-1)d_1....d_n 0^{-1} = d_n....d_1 0;$$
$$(-1)0d_1....d_n{}^{-1} = 0d_n....d_1.$$

Two zeros are added or dropped, single zeros keep their places.

Since negation cancels the change of signs in the reciprocal, we see that $-1(-1)(C^{-1})^{-1}=C$. Actually, as this shows, taking the reciprocal is a complex operation combining inversion of the signs of all digits and reversal of their order, while taking the negative reciprocal is just the second of these simple operations.

However, since the reciprocal is of greater sociological significance and facilitates computation in our system, we prefer to consider it a simple operation, providing for it to be simply indicated and simply performed, at the slight cost of the self-cancelling negation in the artificially complex concept of negative reciprocity.

## CATEGORY ADDITION

The operation of *category addition* may be approached through a reconsideration of translation from the traditional or Romney notations into numerical category notation, which involves one of its sub-operations, *concatenation*.

Given a compound term consisting of a sequence of sex-generalized primary relationships, we first substitute on a one-to-one basis from a lexicon of four entries in the form of rewrite rules, where "$f \rightarrow g$" is read "$f$ is rewritten as $g$":

+a, Parent → 10;
−a, Child → 01;
Oa, Sibling → 11;
=a, Spouse → 0110.

After substitution, the following syntactic rewrite rules are applied to remove inside zeros (the first rule, with null effect, is included only to show all posible combinations):

$$xy \rightarrow xy;$$
$$x00y \rightarrow xy;$$
$$y0y' \rightarrow y'' = Syy';$$
$$x0x' \rightarrow x'' = Sxx'.$$

In other words, two digits separated by no zeros or two are written together, and two digits seperated by a single zero are summed. These rules follow naturally from the alternate sign principle, no zeros or two between digits indicating that they have opposite signs, a single zero that they have the same sign.

Note that *cancellation of double zero* may be considered a special case of *summing across zero*, with summing of the first digit and second zero across the first zero and of the first zero and second digit across the second zero. The same interpretation may be given to concatenation of the identity element 00 to another category or, trivially, to itself, although this is more conveniently thought of simply as a null operation.

We illustrate with a concatenation table for the primary categories, in which cancellation is shown by slashes through zeros and summing by underlining:

| | 10 | 01 | 11 | 0110 |
|---|---|---|---|---|
| 10 | *1010* → 20 | 1ØØ1 → 11* | *1011* → 21 | 1ØØ110 → 1110** |
| 01 | 0110* | 0*101* → 02 | 0111** | 0*101*10 → 0210 |
| 11 | 1110** | 1*101* → 12 | 1111** | 1*101*10 → 1210 |
| 0110 | 01*1010* → 0120 | 011ØØ1 → 0111* | 01*1011* → 0121 | 011ØØ110 → 011110** |

Categories marked with a single asterisk are, of course, themselves members of the set of primary relatives. As mentioned before, "parent's child", available in the conventional notation to represent half-sibling, is equated with full sibling in ours as so far presented. "Child's parent" can be tautological with "spouse".