中国矿业大学新世纪教材建设工程资助教材

Graph Theory and its Alogrithms

图论及其算法

苗连英　王萃琦　主编

中国矿业大学出版社
China University of Mining and Technology Press

图论及其算法

Graph Theory and its Alogrithms

主　编　苗连英　王萃琦

副主编　姚香娟　段滋明
　　　　付乳燕　田　记

中国矿业大学出版社

内 容 提 要

本书共分九章，主要内容包括图的基本概念、树、图的连通性、Euler 环游和 Hamilton 回路、图的匹配、图的独立集和团、图的染色、平面图、网络流等。每章自成体系，不仅包含相关基础理论，还介绍了一些最新研究成果。另外，每章都穿插介绍了与章节内容紧密相关的若干算法等一些扩展阅读。本书注重理论与应用相结合，深入浅出，清晰易懂，并配有适当的例题和习题。

本书主要使用英文编写，穿插部分中文，可用做普通高等学校数学、计算机科学、信息科学、管理科学等专业本科生的双语教学教材，也可供高校教师、图论研究人员参考使用。

图书在版编目(CIP)数据

图论及其算法：英文／苗连英，王萃琦主编．—徐州：中国矿业大学出版社，2012.3

ISBN 978-7-5646-1194-1

Ⅰ.①图… Ⅱ.①苗…②王… Ⅲ.①图论算法—高等学校—教材—英文 Ⅳ.①O157.5

中国版本图书馆 CIP 数据核字(2011)第 176708 号

书　　名　图论及其算法

主　　编　苗连英　王萃琦

责任编辑　王美柱　褚建萍

出版发行　中国矿业大学出版社有限责任公司

（江苏省徐州市解放南路　邮编 221008）

营销热线　(0516)83885307　83884995

出版服务　(0516)83885767　83884920

网　　址　http://www.cumtp.com　**E-mail**:cumtpvip@cumtp.com

印　　刷　徐州中矿大印发科技有限公司

开　　本　787×1092　1/16　**印张** 14　**字数** 349 千字

版次印次　2012 年 3 月第 1 版　2012 年 3 月第 1 次印刷

定　　价　25.00 元

前　言

图论是数学的重要分支，内容十分丰富，在许多学科领域都有广泛应用。

图论起源于 Königsberg 七桥问题。1736 年，L. Euler 解决了这一问题，并在他的论文中首次提出图的概念，该论文被公认为图论历史上第一篇学术论文，标志着图论的萌芽。从 19 世纪中叶到 1936 年，涌现出许多著名的图论问题，如四色问题（1852 年）和 Hamilton 问题（1856 年）。1847 年，G. R. Kirchhoff 在解决电网络方程组问题时引入了树的概念，标志着人们开始以图论为工具来解决工程技术中的实际问题。1878 年，图论这个词首次出现在英国《自然》杂志中。20 世纪 30 年代，涌现出许多重要的理论和研究成果，为图论的进一步发展奠定了坚实的基础。1936 年，匈牙利数学家 D. Köning 完成了第一本图论著作《有限图与无限图的理论》，标志着图论这一研究分支的形成。1936 年至今，由于计算机技术和科学的飞速发展，大大促进了图的理论研究与应用。图的理论与方法已经渗透到物理、化学、运筹学、计算机科学、电子学、信息论、控制论、生物遗传学、社会科学以及经济管理等众多领域。图论与其他数学分支如拓扑学、几何学、概率论、代数学、数学分析等的联系也越来越密切，标志着图论日益成熟。

由于图论的重要性，几乎所有高校都开设了该课程，有的是作为某些特定专业的必修课，有的是作为全校选修课。本书编者在中国矿业大学已为数学、计算机等专业的本科生和研究生多次开设此课程，部分采用了双语教学的授课形式。由于一直找不到合适的双语教学教材，便萌生了在此基础上编写教材的想法和计划。

本书注重理论与应用的紧密结合，在汲取图的经典理论的基础上加入了一些最新成果，以便读者了解该方向的最新动态；每章都穿插了与章节内容紧密相关的若干算法等一些扩展阅读，以培养读者利用图论解决实际问题的能力。本书力求简明扼要，通俗易懂，图文并茂，可读性强，各章具有相对独立性，读者可以根据自己的需要有所侧重地进行学习。并附有大量练习题，便于学生加强对所学知识的理解和掌握。本书主要使用英文编写，穿插部分中文，既有利于提高读者的专业英语水平，也便于读者对难度较大内容的理解。

本书是在参考 J. A. Bondy 和 U. S. R. Murty 所著的 *Graph Theory with Applications* 和 *Graph Theory* 两本书的基础上编写而成的，保留了前者的应用部分和后者的一些理论内容，因此本书在理论和应用方面均较全面。

本书具体编写分工为:第1、2章由付乳燕编写,第3、4章由姚香娟编写,第5、6章由王萃琦编写,第7、8章由段滋明编写,第9章由田记编写,由苗连英统校定稿。

本书在编写和出版过程中得到了中国矿业大学教务处和中国矿业大学理学院的大力支持和资助,在此表示衷心的感谢。

由于编者水平所限,加之时间仓促,书中错误之处在所难免,衷心希望广大读者批评指正。

编 者

2012年1月

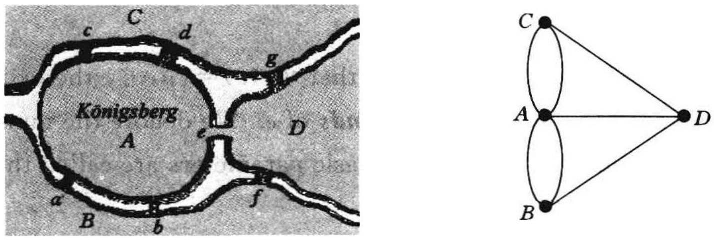
Contents

Chapter 1 Graphs and subgraphs

1.1 Graphs and Their Representation

1.1.1 Introduction

Graph theory had closely relations with mathematical games at the beginning. In previous Königsberg City (now Kaliningrad), there was a river Pregel with two small islands in it. Seven bridges linked the two islands with the bank, as shown in Fig. 1-1(a). It was such a problem: Can we travel across each of the seven bridges once and only once and finally return to the starting point? This is the famous *Seven Bridge Problem*.



(a) (b)

Fig. 1-1 The Seven Bridge Problem

In 1736, Euler solved the problem. Euler used four points to stand for the two islands and two banks, respectively, and used line between the two points to stand for the bridge, as shown in Fig. 1-1(b). Therefore the question is: In this graph, can you find a closed route that travels every line once and only once.

It is not difficult to understand. If such a route exists, for each point, corresponding to each line entering, there should be a line leaving. Therefore, the number of lines incident with each point should be even. But in Fig. 1-1(b), the number of lines incident with each point is odd. So the Seven Bridge Problem has no solution.

In 1857, Hamilton invented a game. The 20 vertices of the dodecahedron stand for 20 cities. The problem is: Can we start from one city, travel every city once and only once and return to the original. This is the so-called *'Problem of Arounding the World'* as shown in Fig. 1-2.

Although graph theory originates from mathematic games, it also has actual backgrounds. Since the 20th century, graph theory has been applied in many scientific

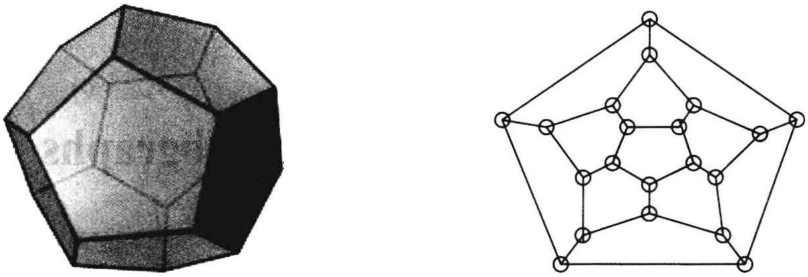


Fig. 1-2 Problem of arounding the world

fields, such as physics, chemical, calculator science, electron science, control theory, information theory, network theory, society research, economical manage, etc.

1.1.2 Definitions

Definition 1.1 A ***graph*** G is an ordered pair $(V(G), E(G))$ consisting of a set $V(G)$ of ***vertices*** and a set $E(G)$, disjoint from $V(G)$, of ***edges***, together with an ***incidence function*** Ψ_G that associates with each edge of G an unordered pair of (not necessarily distinct) vertices of G.

If e is an edge and u and v are vertices such that $\Psi_G(e)=\{u,v\}$, then e is said to ***join*** u and v, and the vertices u and v are called the ***ends*** of e. We denote the numbers of vertices and edges in G by $v(G)$ and $e(G)$; these two basic parameters are called the ***order*** and ***size*** of G, respectively.

Two examples of graphs should serve to clarify the definition. For notational simplicity, we write uv for the unordered pair $\{u,v\}$.

Example 1.1

$$G = (V(G), E(G))$$

where

$$V(G) = \{u,v,w,x,y\} \quad E(G) = \{a,b,c,d,e,f,g,h\}$$

and Ψ_G is defined by

$$\Psi_G(a) = uv \quad \Psi_G(b) = uu \quad \Psi_G(c) = vw \quad \Psi_G(d) = wx$$

$$\Psi_G(e) = vx \quad \Psi_G(f) = wx \quad \Psi_G(g) = ux \quad \Psi_G(h) = xy$$

Example 1.2

$$H = (V(H), E(H))$$

where

$$V(H) = \{v_0, v_1, v_2, v_3, v_4, v_5\}$$

$$E(H) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$$

and Ψ_H is defined by

$$\Psi_H(e_1) = v_1v_2 \quad \Psi_H(e_2) = v_2v_3 \quad \Psi_H(e_3) = v_3v_4 \quad \Psi_H(e_4) = v_4v_5 \quad \Psi_H(e_5) = v_5v_1$$

$$\Psi_H(e_6) = v_0v_1 \quad \Psi_H(e_7) = v_0v_2 \quad \Psi_H(e_8) = v_0v_3 \quad \Psi_H(e_9) = v_0v_4 \quad \Psi_H(e_{10}) = v_0v_5$$

1.1.3 Drawings of Graphs

Graphs are so named because they can be represented graphically, and it is this graphical representation which helps us understand many of their properties. Each vertex is indicated by a point, and each edge by a line joining the points representing its ends. Diagrams of G and H are shown in Fig. 1-3. (For clarity, vertices are represented by small circles.)

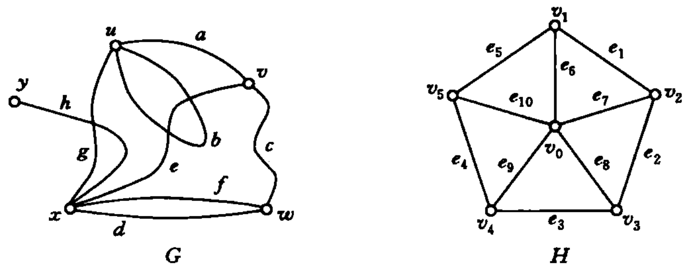


Fig. 1-3 Diagrams of the graphs G and H

There is no single correct way to draw a graph; the relative positions of points representing vertices and the shapes of lines representing edges usually have no significance. In Fig. 1-3, the edges of G are depicted by curves, and those of H by straight-line segments. A diagram of a graph merely depicts the incidence relation holding between its vertices and edges.

Most of the definitions and concepts in graph theory are suggested by this graphical representation. The ends of an edge are said to be ***incident*** with the edge, and ***vice versa***. Two vertices which are incident with a common edge are ***adjacent***, as are two edges which are incident with a common vertex, and two distinct adjacent vertices are ***neighbors***. The set of neighbors of a vertex v in a graph G is denoted by $N_G(v)$.

An edge with identical ends is called a ***loop***, and an edge with distinct ends a ***link***. Two or more links with the same pair of ends are said to be ***parallel edges***. In the graph G of Fig. 1-3, the edge b is a loop, and all other edges are links; the edges d and f are parallel edges.

Throughout the book, the letter G denotes a graph. Moreover, when there is no scope for ambiguity, we omit the letter G from graph-theoretic symbols and write, for example, V and E instead of $V(G)$ and $E(G)$. In such instances, we denote the numbers of vertices and edges of G by n and m, respectively.

A graph is ***finite*** if both its vertex set and edge set are finite. In this book, we mainly study finite graphs, and the term 'graph' always means 'finite graph'. The graph with no vertices (and hence no edges) is the ***null graph***. Any graph with just one vertex is referred to as ***trivial***. All other graphs are ***nontrivial***. We admit the null graph solely for mathematical convenience. Thus, unless otherwise specified, all graphs under discussion should be taken to be nonnull.

A graph is ***simple*** if it has no loops or parallel edges. The graph H in Example 1. 2 is simple, whereas the graph G in Example 1. 1 is not. Much of graph theory is concerned with the study of simple graphs.

Remark A set V, together with a set E of two-element subsets of V, defines a simple graph (V,E), where the ends of an edge uv are precisely the vertices u and v. Indeed, in any simple graph we may dispense with the incidence function Ψ by renaming each edge as the unordered pair of its ends. In a diagram of such a graph, the labels of the edges may then be omitted.

1. 1. 4 Special Families of Graphs

Some graphs with simple structures are thought to deserve special names. ***A vertex-graph*** is an edgeless graph having exactly one vertex [Fig. 1-4(a)]. ***A loop-graph*** consists of a single loop with its one end [Fig. 1-4(b)], and ***a link-graph*** consists of a single link with its two ends [Fig. 1-4(c)].

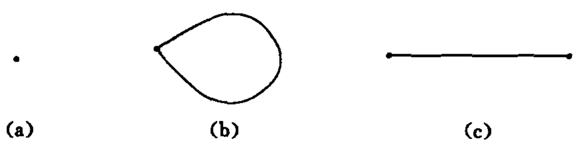


Fig. 1-4 (a) A vertex-graph; (b) a loop-graph; (c) a link-graph

Definition 1. 2 A ***path*** is a simple graph whose vertices can be arranged in a linear sequence in such a way that two vertices are adjacent if they are consecutive in the sequence, and are nonadjacent otherwise.

Definition 1. 3 A ***cycle*** on three or more vertices is a simple graph whose vertices can be arranged in a cyclic sequence in such a way that two vertices are adjacent if they are consecutive in the sequence, and are nonadjacent otherwise; a cycle on one vertex consists of a single vertex with a loop, and a cycle on two vertices consists of two vertices joined by a pair of parallel edges.

The ***length*** of a path or a cycle is the number of its edges. A path or cycle of length k is called a ***k-path*** or ***k-cycle***, respectively; the path or cycle is ***odd*** or *even* according to the parity of k. A 3-cycle is often called a ***triangle***, a 4-cycle a ***quadrilateral***, a 5-cycle a ***pentagon***, a 6-cycle a ***hexagon***, and so on.

Fig. 1-5 depicts a 3-path and a 5-cycle.

Definition 1. 4 A ***complete graph or clique*** is a simple graph in which any two vertices are adjacent, an ***empty graph*** one in which no two vertices are adjacent (that is, one whose edge set is empty).

We frequently use K_n for the complete graph of ordern. Then K_n is a loopless graph

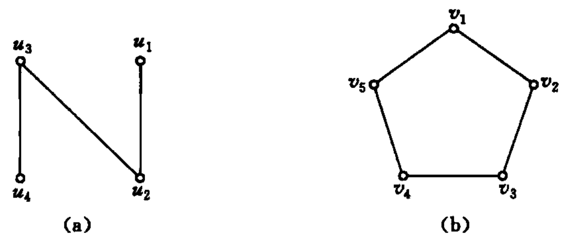


Fig. 1-5 (a) A path of length three; (b) a cycle of length five

with exactly n vertices and $\frac{1}{2}n(n-1)$ edges, each pair of vertices being joined by a single link.

Definition 1.5 A graph is ***bipartite*** if its vertex set can be partitioned into two subsets X and Y so that every edge has one end in X and one end in Y; such a partition (X,Y) is called a ***bipartition*** of the graph, and X and Y its ***parts***.

We denote a bipartite graph G with bipartition (X,Y) by $G[X,Y]$. If $G[X,Y]$ is simple and every vertex in X is joined to every vertex in Y, then G is called a ***complete bipartite graph***. A ***star*** is a complete bipartite graph $G[X,Y]$ with $|X|=1$ or $|Y|=1$.

Fig. 1.6 shows diagrams of a complete graph, a complete bipartite graph, and a star.

A simple graph is ***r-partite graph***, if

$$V = V_1 \cup V_2 \cup \cdots \cup V_r, \quad V_i \cap V_j = \varnothing (i \neq j)$$

and no edge joins two vertices in the same class. The symbol $K_{n_1,n_2,\cdots,n_r}$ denotes a complete r-partite graph which has n_i vertices in the ith class and contains all edges joining vertices in distinct classes.

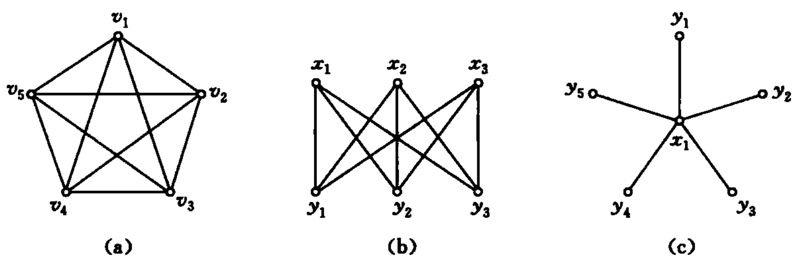


Fig. 1-6 (a) A complete graph; (b) a complete bipartite graph; (c) a star

Definition 1.6 A graph is ***connected*** if, for every partition of its vertex set into two nonempty sets X and Y, there is an edge with one end in X and one end in Y; a graph is ***disconnected*** if its vertex set can be partitioned into two nonempty subsets X and Y so that no edge has one end in X and one end in Y.

Connection is an equivalence relation on the vertex set V. Thus there is a partition of V into nonempty subsets $V_1, V_2, \cdots, V_\omega$ such that two vertices u and v are connected if and only if both u and v belong to the same set V_i. The subgraphs $G[V_1], G[V_2], \cdots, G[V_n]$ are called the ***components*** of G. We denote the number of components of G by $\omega(G)$.

Examples of connected and disconnected graphs are displayed in Fig. 1-7.

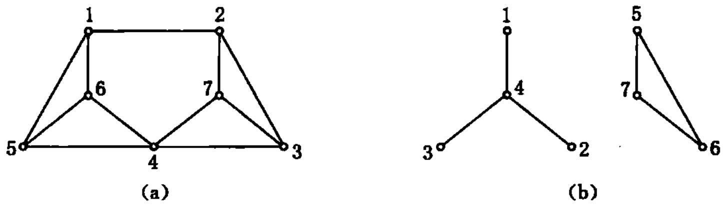


Fig. 1-7 (a) A connected graph; (b) a disconnected graph

For the sake of clarity, we observe certain conventions in representing graphs by diagrams: we do not allow an edge to intersect itself, nor let an edge pass through a vertex that is not an end of the edge; clearly, this is always possible. However, two edges may intersect at a point that does not correspond to a vertex, as in the drawings of the first two graphs in Fig. 1-6.

Definition 1.7 A graph which can be drawn in the plane in such a way that edges meet only at points corresponding to their common ends is called a ***planar graph***, and such a drawing is called a ***planar embedding*** of the graph.

For instance, the graphs G and H of Example 1.1 and Example 1.2 are both planar, even though there are crossing edges in the particular drawing of G shown in Fig. 1-3. The first two graphs in Fig. 1-6, on the other hand, are not planar, as proved later.

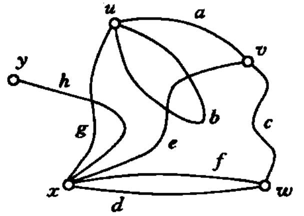
1.1.5 Incidence and Adjacency Matrices

Although drawings are a convenient means of specifying graphs, they are clearly not suitable for storing graphs in computers, or for applying mathematical methods to study their properties. For these purposes, we consider two matrices associated with a graph, its incidence matrix and its adjacency matrix.

Definition 1.8 The ***incidence matrix*** of G is the $n \times m$ matrix $M_G := (m_{ve})$, where m_{ve} is the number of times (0, 1, or 2) that vertex v and edge e are incident.

Definition 1.9 The ***adjacency matrix*** of G is the $n \times n$ matrix $A_G := (a_{uv})$, where a_{uv} is the number of edges joining vertices u and v, each loop counting as two edges.

Incidence and adjacency matrices of the graph G of Fig. 1-3 are shown in Fig. 1-8.



G

	a	b	c	d	e	f	g	h
u	1	2	0	0	0	0	1	0
v	1	0	1	0	1	0	0	0
w	0	0	1	1	0	1	0	0
x	0	0	0	1	1	1	1	1
y	0	0	0	0	0	0	0	1

M

	u	v	w	x	y
u	2	1	0	1	0
v	1	0	1	1	0
w	0	1	0	2	0
x	1	1	2	0	1
y	0	0	0	1	0

A

Fig. 1-8 Incidence and adjacency matrices of a graph

Remark 1 Because most graphs have many more edges than vertices, the adjacency matrix of a graph is generally much smaller than its incidence matrix and thus requires less storage space. When dealing with simple graphs, an even more compact representation is possible. For each vertex v, the neighbors of v are listed in some order. A list $(N(v): v \in V)$ of these lists is called an ***adjacency list*** of the graph. Simple graphs are usually stored in computers as adjacency lists.

Remark 2 When G is a bipartite graph, as there are no edges joining pairs of vertices belonging to the same part of its bipartition, a matrix of smaller size than the adjacency matrix may be used to record the numbers of edges joining pairs of vertices. Suppose that $G[X, Y]$ is a bipartite graph, where $X := \{x_1, x_2, \cdots, x_r\}$ and $Y := \{y_1, y_2, \cdots, y_s\}$. We define the ***bipartite adjacency matrix*** of G to be the $r \times s$ matrix $B_G = (b_{ij})$, where b_{ij} is the number of edges joining x_i and y_j.

1.1.6 Vertex Degrees

Definition 1.10 The ***degree*** of a vertex v in a graph G, denoted by $d_G(v)$, is the number of edges of G incident with v, each loop counting as two edges.

In particular, if G is a simple graph, $d_G(v)$ is the number of neighbors of v in G. A vertex of degree zero is called an ***isolated vertex***. We denote by $\delta(G)$ and $\Delta(G)$ the minimum and maximum degrees of the vertices of G, and by d(G) their ***average degree***, $\frac{1}{n}\sum_{v \in V} d(v)$.

Theorem 1.1 For any graph G,

$$\sum_{v \in V} d(v) = 2m \tag{1.1}$$

Proof Consider the incidence matrix M of G. The sum of the entries in the row corresponding to vertex v is precisely $d(v)$. Therefore $\sum_{v \in V} d(v)$ is just the sum of all the entries in M. But this sum is also $2m$, because each of the m column sums of M is 2, each edge having two ends. □

Corollary 1.2 In any graph, the number of vertices of odd degree is even.

Proof Consider equation (1.1) modulo 2. We have

$$d(v) \equiv \begin{cases} 1 \pmod 2 & \text{if } d(v) \text{ is odd} \\ 0 \pmod 2 & \text{if } d(v) \text{ is even} \end{cases}$$

Thus, modulo 2, the left-hand side is congruent to the number of vertices of odd degree, and the right-hand side is zero. The number of vertices of odd degree is therefore congruent to zero modulo 2. □

Proposition 1.3 Let $G[X,Y]$ be a bipartite graph without isolated vertices such that $d(x) \geqslant d(y)$ for all $xy \in E$, where $x \in X$ and $y \in Y$. Then $|X| \leqslant |Y|$, with equality if and only if $d(x) = d(y)$ for all $xy \in E$.

Proof The first assertion follows if we can find a matrix with $|X|$ rows and $|Y|$ columns in which each row sum is one and each column sum is at most one. Such a matrix can be obtained from the bipartite adjacency matrix B of $G[X, Y]$ by dividing the row corresponding to vertex x by $d(x)$, for each $x \in X$. (This is possible since $d(x) \neq 0$.) Because the sum of the entries of B in the row corresponding to x is $d(x)$, all row sums of the resulting matrix B' are equal to one. On the other hand, the sum of the entries in the column of B' corresponding to vertex y is $\sum 1/d(x)$, the sum being taken over all edges xy incident to y, and this sum is at most one because $1/d(x) \leqslant 1/d(y)$ for each edge xy, by hypothesis, and because there are $d(y)$ edges incident to y.

The above argument may be expressed more concisely as follows.

$$|X| = \sum_{x \in X} \sum_{\substack{y \in Y \\ xy \in E}} \frac{1}{d(x)} = \sum_{x \in X} \sum_{\substack{xy \in E \\ y \in Y}} \frac{1}{d(x)} \leqslant \sum_{x \in X} \sum_{\substack{xy \in E \\ y \in Y}} \frac{1}{d(y)} = \sum_{y \in Y} \sum_{\substack{x \in X \\ xy \in E}} \frac{1}{d(y)} = |Y|$$

Furthermore, if $|X| = |Y|$, the middle inequality must be an equality, implying that $d(x) = d(y)$ for all $xy \in E$. □

Definition 1.11 A graph G is ***k-regular*** if $d(v) = k$ for all $v \in V$; ***a regular graph*** is one that is k-regular for some k.

For instance, the complete graph on n vertices is $(n-1)$-regular, and the complete bipartite graph with k vertices in each part is k-regular. For $k = 0, 1$ and 2, k-regular graphs have very simple structures and are èasily characterized. By contrast, 3-regular graphs can be remarkably complex. These graphs, also referred to as ***cubic*** graphs, play a prominent role in graph theory.

1.1.7 Isomorphisms

Definition 1.12 Two graphs G and H are ***identical***, written $G = H$, if $V(G) = V(H)$, $E(G) = E(H)$, and $\psi_G = \psi_H$.

If two graphs are identical, they can clearly be represented by identical diagrams. However, it is also possible for graphs that are not identical to have essentially the same diagram. For example, the graphs G and H in Fig. 1-9 can be represented by diagrams which look exactly the same, as the second drawing of H shows; the sole difference lies in the labels of their vertices and edges. Although the graphs G and H are not identical, they do have identical structures, and are said to be isomorphic.

Definition 1.13 Two graphs G and H are ***isomorphic***, written $G \cong H$, if there are bijections $\theta: V(G) \to V(H)$ and $\Phi: E(G) \to E(H)$ such that $\psi_G(e) = uv$ if and only if $\psi_H(\Phi(e)) = \theta(u)\theta(v)$; such a pair of mappings is called an ***isomorphism*** between G and H.

In order to show that two graphs are isomorphic, one must indicate an isomorphism between them. The pair of mappings (θ, Φ) defined by

$$\theta = \begin{pmatrix} a & b & c & d \\ w & z & y & x \end{pmatrix} \qquad \Phi = \begin{bmatrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ f_3 & f_4 & f_1 & f_6 & f_5 & f_2 \end{bmatrix}$$

is an isomorphism between the graphs G and H in Fig. 1-9.

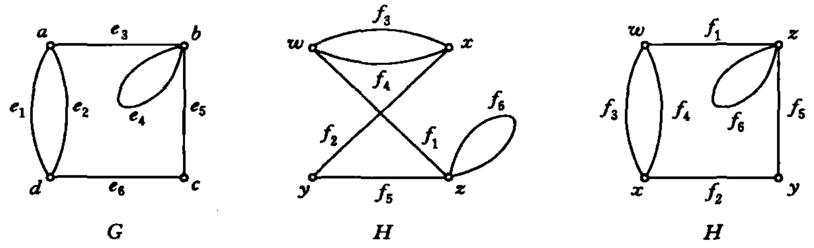


Fig. 1-9 Isomorphic graphs

Remark 1 In the case of simple graphs, the definition of isomorphism can be stated more concisely, because if (θ, Φ) is an isomorphism between simple graphs G and H, the mapping Φ is completely determined by θ; indeed, $\Phi(e)=\theta(u)\theta(v)$ for any edge $e=uv$ of G. Thus one may define an isomorphism between two simple graphs G and H as a bijection $\theta: V(G)\rightarrow V(H)$ which preserves adjacency (that is, the vertices u and v are adjacent in G if and only if their images $\theta(u)$ and $\theta(v)$ are adjacent in H). Consider, for example, the graphs G and H in Fig. 1-10.

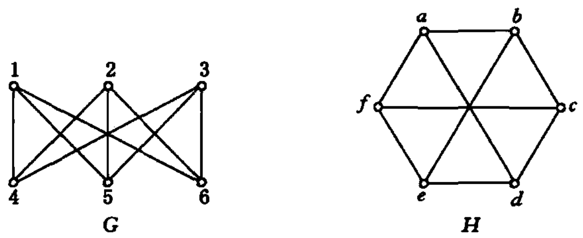


Fig. 1-10 Isomorphic simple graphs

The mapping

$$\theta := \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ b & d & f & c & e & a \end{pmatrix}$$

is an isomorphism between G and H, as is

$$\theta' := \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & c & e & d & f & b \end{pmatrix}$$

Remark 2 Isomorphic graphs clearly have the same numbers of vertices and edges. On the other hand, equality of these parameters does not guarantee isomorphism. For instance, the two graphs shown in Fig. 1-11 both have eight vertices and twelve edges, but they are not isomorphic. To see this, observe that the graph G has four mutually nonadjacent vertices, v_1, v_3, v_6, and v_8. If there were an isomorphism θ between G and H, the vertices $\theta(v_1), \theta(v_3), \theta(v_6)$, and $\theta(v_8)$ of H would likewise be mutually nonadjacent. But it can readily be checked that no four vertices of H are mutually nonadjacent. We deduce that G and H are not isomorphic.

Remark 3 Up to isomorphism, there is just one complete graph on n vertices, denoted K_n.