

Howard Baer  
Xerxes Tata

# Weak Scale Supersymmetry

From Superfields to Scattering Events

弱尺度超对称

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## From Superfields to Scattering Events

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# WEAK SCALE SUPERSYMMETRY

## From Superfields to Scattering Events

Supersymmetry has been proposed as a new symmetry of nature. Supersymmetric models of particle physics predict new superpartner matter states for each known particle in the Standard Model. The existence of such superpartners will have wide-ranging implications, from the early history of the Universe to what is observed at high energy accelerators such as CERN's LHC.

In this text, the authors develop the concepts of supersymmetry from first principles, and show how it can be incorporated into a theoretical framework for describing unified theories of elementary particles. They develop the technical tools of supersymmetry using four-component spinor notation familiar to high energy experimentalists and phenomenologists. The text takes the reader from an abstract formalism to a straightforward recipe for writing supersymmetric gauge theories of particle physics, and ultimately to the calculation of cross sections and decay rates necessary for practical applications to experiments both at colliders and for cosmology.

This advanced text is a comprehensive, practical, and accessible introduction to supersymmetry for experimental and phenomenological particle physicists and graduates studying supersymmetry. Some familiarity with the Standard Model and tree-level calculations in quantum field theory is required. Exercises and worked examples that supplement and clarify the material are interspersed throughout.

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## Preface

Supersymmetry (SUSY) is a lovely theoretical construct, and has captured the imagination of many theoretical physicists. It allows for a new synthesis of particle interactions, and offers a new direction for the incorporation of gravity into particle physics. The supersymmetric extension of the Standard Model also ameliorates a host of phenomenological problems in the physics of elementary particles, if superpartners exist at the TeV scale. These new states may well be discovered in experiments at high energy colliders or in non-accelerator experiments within the next few years!

There are several excellent books that explore the theoretical structure of supersymmetry. These advanced texts are rather formal, and focus more on the theoretical structure rather than on the implications of supersymmetry. This makes them somewhat inaccessible to a large number of our experimental as well as phenomenological particle physics colleagues, working on the search for the new particles predicted by supersymmetry. Our goal in this book is to provide a comprehensive (and comprehensible) introduction to the theoretical structure of supersymmetry, and to work our way towards an exploration of its experimental implications, especially for collider searches. Although we have attempted to orient this book towards experimentalists and phenomenologists interested in supersymmetry searches, we hope that others will also find it interesting. In particular, we hope that it will provide theorists with an understanding and appreciation of some of the experimental issues that one is confronted with in the search for new physics.

We use the language of four-component relativistic spinors throughout this text, rather than the sometimes more convenient approach using two-component spinors. Although this makes some of the manipulations, especially in Chapters 5–6, appear to be somewhat more cumbersome, we felt that the use of four-spinors, which is familiar to most “practical particle physicists,” would make up for this. For this reason, and also because we did not want to adopt a schizophrenic approach using two-component spinors for some things, and four-component spinors for others, we have eschewed the use of two-component spinors throughout.

After a review of the Standard Model (mainly to set up notation) and an examination of the motivations for weak scale supersymmetry, the text naturally divides into three parts. The first part (Chapters 3–7) of the book introduces supersymmetry, and details how to construct globally supersymmetric relativistic quantum field theories. We provide a “master formula” for the Lagrangian of a general, globally supersymmetric non-Abelian gauge theory that can serve as the starting point for the construction of supersymmetric models of particle physics. The inclusion of supersymmetry breaking is discussed in Chapter 7.

The second part of the book applies these lessons and develops the so-called Minimal Supersymmetric Standard Model, the MSSM, which is (almost) the direct supersymmetrization of the Standard Model. The physical particles of the MSSM are identified, and their various couplings, which are necessary for exploration of the broad phenomenological implications of the theory, are calculated. An assortment of implications of the MSSM are examined in Chapter 9, including the SUSY flavor and  $CP$  problems, renormalization group running, cosmological dark matter, and more. We discuss local supersymmetry (which, we show, includes general relativity) in Chapter 10, and in the following chapter present an overview of some of the specific mechanisms by which Standard Model superpartners may acquire supersymmetry breaking masses and couplings.

The final third of the book is oriented towards collider physics. We detail the calculations of scattering cross sections and decay rates starting from the couplings of supersymmetric particles that were found in Chapter 8. We focus on technical aspects of these calculations, including methods for dealing with Majorana particles, which the reader may not be familiar with. We also outline methods for simulation of collider scattering events in which supersymmetric matter has been produced. We then discuss what has been learned, and what may be learned, about weak scale supersymmetry from past, present, and future experiments at both hadron and  $e^+e^-$  colliders. In a final chapter, we go beyond the MSSM, but only insofar as to introduce  $R$ -parity violation, which changes the phenomenology considerably. In three appendices, we present formulae for evaluating tree-level scattering cross sections at electron–positron as well as hadron colliders, decay rates of supersymmetric particles, and decay rates of the several Higgs bosons present in all SUSY models. Various exercises are interspersed throughout the text. Some of these are pedagogical in nature, asking the reader to fill in or complete a calculation, while others develop the subject beyond the discussion in the text.

We have not attempted to make a comprehensive list of references to the vast literature on supersymmetry. Where we develop a topic from scratch, we reference only some of the classic papers on the subject. Sometimes, this means that we may not reference earlier pioneering work in favor of more complete studies that may prove more useful to a reader attempting to learn the subject. However, where we

content ourselves with stating a particular result rather than deriving it completely – this is more frequently the case in the latter part of the text where we discuss supersymmetry phenomenology – we provide a reference where the reader may find further details. Thus, except for referencing some classic papers, we generally provide references only to papers where necessary details not presented in the text may be found. We apologize to the reader for this shortcoming, and also to the many researchers whose work has not been explicitly referenced.

Although we hope that the interested reader will work through the entire book, those who are interested only in phenomenology and are willing to accept supersymmetric couplings from the MSSM at face value, can skip Chapters 3–7 altogether. Chapter 10 can also be omitted without essential loss of continuity. Alternatively, the reader who is interested in model-building but does not want to work through the machinery of SUSY may use the “master formula” in Chapter 6 as a starting point, focussing on its use for writing down supersymmetric models. We urge all readers to visit Chapter 3, where many of the extraordinary properties of supersymmetric theories are explicitly illustrated.

We assume that the reader is familiar with tree-level calculations in quantum field theory through QED, as presented, for instance, by the first seven chapters of *Introduction to Quantum Field Theory*, by M. Peskin and D. V. Schroeder. We also assume some familiarity with the Standard Model of particle physics, but just in the unitarity gauge, as presented for instance in *Collider Physics*, by V. Barger and R. J. N. Phillips. No prior knowledge of supersymmetry is assumed. Indeed we have done our utmost to develop this subject from scratch, paying attention both to concepts as well as to technical details that will enable the reader to carry out research in the field. However, while we have emphasized pedagogy in our development of topics to do with supersymmetry, it is not possible to be as detailed on every topic that is necessary for describing the implications of supersymmetry for particle physics. Aside from tree-level quantum field theory and the basics of the Standard Model that we have already mentioned, these might include the ideas of the parton model, collider kinematics, Grand Unification, renormalization group methods, Big Bang cosmology etc. that have become part of the repertoire of many working particle physicists. Although we develop these ideas enough for the reader to be able to follow along, the reader who is interested in their detailed development is urged to consult the references in the text, and also the excellent treatment in the many text books listed in the Bibliography.

In writing this book, we are indebted to an enormous number of people, including teachers, students and colleagues from whom we have learned much. One of us (XT) benefited vastly from S. Weinberg’s lectures on supersymmetry at the University of Texas at Austin in Spring 1982. Much of what we know is the result of collaborations and discussions over the years with many people,

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### **Corrections to this book**

A list of misprints and corrections to this book is posted on the World Wide Web at the URL [www.cambridge.org/97805218578640/](http://www.cambridge.org/97805218578640/). Reports of additional errors or misprints in this book would be appreciated by the authors.



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# 1

## The Standard Model

The 1970s witnessed the emergence of what has become the Standard Model (SM) of particle physics. The SM describes the interactions of quarks and leptons that are the constituents of all matter that we know about. The strong interactions are described by quantum chromodynamics (QCD) while the electromagnetic and the weak interactions have been synthesized into a single electroweak framework. This theory has proven to be extremely successful in describing a tremendous variety of experimental data ranging over many decades of energy. The discovery of neutral currents in the 1970s followed by the direct observation of the  $W$  and  $Z$  bosons at the CERN  $Sp\bar{p}S$  collider in the early 1980s spectacularly confirmed the ideas underlying the electroweak framework. Since then, precision measurements of the properties of the  $W$  and  $Z$  bosons at both  $e^+e^-$  and hadron colliders have allowed a test of electroweak theory at the  $10^{-3}$  level. QCD has been tested in the perturbative regime in hard collision processes that result in the breakup of the colliding hadrons. In addition, lattice gauge calculations allow physicists to test non-perturbative QCD via predictions for the observed properties of hadrons for which there is a wealth of experimental information.

### 1.1 Gauge invariance

One of the most important lessons that we have learned from the SM is that dynamics arises from a symmetry principle. If we require the Lagrangian density to be invariant under *local* gauge transformations, we are *forced* to introduce a set of gauge potentials with couplings to elementary scalar and fermion matter fields that, apart from an overall scale, are completely determined by symmetry principles. The most familiar example of such a field theory is the electrodynamics of Dirac fermions or complex scalars, where the invariance of the Lagrangian under



spacetime-dependent phase transformations,

$$\psi(x) \rightarrow e^{iq_\psi \alpha(x)} \psi(x),$$

or

$$\phi(x) \rightarrow e^{iq_\phi \alpha(x)} \phi(x),$$

requires us to introduce the vector potential  $A_\mu$ , with a coupling given by,

$$\mathcal{L} = i\bar{\psi}\gamma_\mu D^\mu \psi - m\bar{\psi}\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \quad (1.1a)$$

or

$$\mathcal{L} = (D^\mu \phi)^*(D_\mu \phi) - m^2 \phi^* \phi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}. \quad (1.1b)$$

Here,  $D_\mu$  is the gauge covariant derivative given by  $D_\mu = \partial_\mu + iq_{\psi/\phi}A_\mu(x)$ ,  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  and  $q_{\psi/\phi}$  is any real number identified with the charge of the field. It is easy to check that if, in addition to the local phase transformation of the fields  $\psi$  and  $\phi$ , the vector potential transforms inhomogeneously as

$$A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) - \partial_\mu \alpha(x),$$

the Lagrangians of Eq. (1.1a) and Eq. (1.1b) will be invariant under the set of local gauge transformations. The phase transformations of the fermion or scalar “matter” fields form the group  $U(1)$ . We will thus regard electrodynamics as a gauge theory based on the group  $U(1)$ , which is an Abelian group – i.e. its elements commute with one another. We stress two features of these Lagrangians.

- The coupling of the vector potential (identified with the photon field when the theory is quantized) to matter fields is given by the “minimal coupling principle” where the ordinary derivative is replaced by the gauge-covariant derivative. For fermionic matter, this gives the familiar fermion–antifermion–photon “vector” coupling (proportional to the charge  $q_\psi$ ), while in the case of scalar matter, we have both a three-point derivative coupling to the photon proportional to the charge  $q_\phi$  and a four-point non-derivative scalar–scalar–photon–photon coupling proportional to  $q_\phi^2$ . The point to be made is that *the form of the interactions of the photons with matter is completely fixed by the requirement of local gauge invariance*.
- The photon field is massless because a mass term  $\frac{1}{2}m_\gamma^2 A_\mu A^\mu$  would not be locally gauge invariant. The matter fields may, however, be massive.

Yang and Mills, and independently Shaw (and later Utiyama), generalized this idea to more complicated transformations of matter fields that form a non-Abelian group rather than the group  $U(1)$ . The construction of these Yang–Mills gauge