钱学森力学手稿

2

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西安交通大学出版社
XI'AN JIAOTONG UNIVERSITY PRESS

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出版前言

2011年12月11日是西安交通大学杰出校友钱学森先生的百年诞辰。为缅怀钱学森学长，学习他的科学思想和卓越风范，展示其丰功伟绩和人格魅力，西安交通大学举办了“纪念钱学森诞辰100周年”系列活动：作为制片方之一，参与西部电影集团摄制传记故事片《钱学森》；与中央电视台合作，出品纪录片《实验班的故事——沿着钱学森走过的路》；扩建钱学森生平业绩展馆，向校内外开放；举办钱学森科学与教育思想研讨会；出版发行《钱学森力学手稿》、《钱学森年谱（初编）》、《钱学森第六次产业革命思想探微丛书》等。

钱学森先生在美国深造和工作期间留下大量珍贵手稿，这些手稿真实展示了钱学森先生博大精深的学识、开拓求实的精神和严谨奋进的作风，是钱老勇攀科学高峰和严谨治学的集中体现。这里，我们将部分原稿整理汇集成册，出版《钱学森力学手稿》，作为钱老百年诞辰的献礼。

《钱学森力学手稿》共10卷，包含两部分内容。第一部分是草稿，包括扁壳、球壳和圆柱壳屈曲分析的公式推导和数值演算。在研究圆柱壳轴压屈曲问题时，为了求得圆柱壳体的临界压力，在有关的五百多页草稿中，对多达二十多种可能的屈曲模

态逐一进行公式推演和数值计算，最终才找到满意的并在论文中采用的屈曲模态。仔细观察草稿中的数据列表，每个数字有效位数都长达八位，在手摇机械式计算机作为主要计算工具的年代，这串串数字凝聚着多少现今难以想象的艰辛劳动。

第二部分是手稿，以航空航天工程为核心，涵盖空气动力学、固体力学、火箭技术、工程控制论和物理力学等领域的部分学术论文手稿、打印稿和讲义。

《钱学森力学手稿》是在西安交通大学校领导的大力支持下，由西安交通大学航天航空学院沈亚鹏教授整理完成。图书出版过程中得到了西安交通大学党委宣传部、校友关系发展部、图书馆、航天航空学院等的积极协助，在此深表感谢。

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Section 1

The Buckling of Spherical Shells by External Pressure

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genhe Lab.
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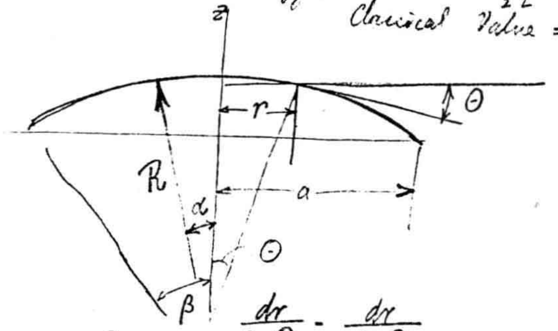
na, Calif.

Bending Energy / unit area $= \frac{D}{2}\left[(k_1+k_2)^2 - 2(1-\mu)(k_1 k_2 - \tau^2)\right]$

Extensional Energy / unit area $= \frac{Et}{2}\left[(\varepsilon_1+\varepsilon_2)^2 - 2(1-\mu)(\varepsilon_1 \varepsilon_2 - \frac{1}{4}\gamma^2)\right]$

Classical Value $= \phi = \frac{\sigma}{E(\frac{t}{R})} = \frac{1}{\sqrt{3(1-\mu^2)}} \cong 0.6$



The original element

$$= \frac{dr}{\cos\Theta_0}$$

The deflected element

$$= \frac{dr}{\cos\Theta}$$

($\frac{1}{4}$)

$$\varepsilon = \frac{\frac{dr}{\cos\Theta} - \frac{dr}{\cos\Theta_0}}{\frac{dr}{\cos\Theta_0}} = \frac{\cos\Theta_0}{\cos\Theta} - 1$$

The extensional energy

$$\frac{Et}{2}\int_0^a \left(\frac{\cos\Theta_0}{\cos\Theta} - 1\right)^2 2\pi r \frac{dr}{\cos\Theta_0}$$

$$= \frac{Et}{2}\int_0^\beta \left(\frac{\cos\Theta_0}{\cos\Theta} - 1\right)^2 2\pi R \sin\alpha \; R\, d\alpha$$

$$= \frac{ER^3}{2}\left(\frac{t}{R}\right)\cdot 2\pi \int_0^\beta \left(\frac{\cos\Theta_0}{\cos\Theta} - 1\right)^2 \sin\alpha\, d\alpha$$

The curvature in meridional direction $= \frac{d\Theta}{d\bar{s}} = \frac{d\Theta}{d\alpha} \Big/ \frac{d\bar{s}}{d\alpha}$

$$d\bar{s} = \frac{dr}{\cos\Theta} = \frac{d[R\sin\alpha]}{\cos\Theta} = \frac{R\cos\alpha\, d\alpha}{\cos\Theta}$$

Change in curvature $= \dfrac{\frac{d\Theta}{d\alpha}}{\frac{R\cos\alpha}{\cos\Theta}} - \frac{1}{R} = \frac{1}{R}\left[\frac{\cos\Theta}{\cos\alpha}\frac{d\Theta}{d\alpha} - 1\right]$

change in curvature in other direction

$$=\left(\frac{1}{\frac{R\sin\alpha}{\sin\Theta}}\right)-\frac{1}{R}=\frac{1}{R}\left[\frac{\sin\Theta}{\sin\alpha}-1\right]$$

The bending energy

$$=\frac{Et^3}{24}\int_0^\beta 2\pi R^2\sin\alpha\,d\alpha\,\frac{1}{R^2}\left[\left(\frac{\cos\Theta}{\cos\alpha}\frac{d\Theta}{d\alpha}-1\right)^2+\left(\frac{\sin\Theta}{\sin\alpha}-1\right)^2\right]$$

$$=\frac{R^3E}{2}\,\frac{\left(\frac{t}{R}\right)^3}{12}(2\pi)\int_0^\beta\sin\alpha\left[\left(\frac{\cos\Theta}{\cos\alpha}\frac{d\Theta}{d\alpha}-1\right)^2+\left(\frac{\sin\Theta}{\sin\alpha}-1\right)^2\right]d\alpha$$

The potential energy $\quad p\cdot\int_0^a 2\pi r z\,dr$

$$=p\left[\left[2\pi\frac{r^2}{2}z\right]_0^a-\int_0^a 2\pi\frac{r^2}{2}\frac{dz}{dr}\,dr\right]$$

$$=-p\pi\int_0^\alpha R^2\sin^2\alpha\,\tan\Theta\,R\cos\alpha\,d\alpha$$

$$=-pR^3\pi\int_0^\alpha\sin^2\alpha\,\tan\Theta\,\cos\alpha\,d\alpha$$

$$E\left(\frac{t}{R}\right)\int_0^{\beta}\left(\frac{\cos\alpha}{\cos\theta}-1\right)^2\sin\alpha\,d\alpha+\left[\frac{E\left(\frac{t}{R}\right)^3}{12}\int_0^{\beta}\left\{\left(\frac{\cos\theta}{\cos\alpha}\frac{d\theta}{d\alpha}-1\right)^2+\left(\frac{\sin\theta}{\sin\alpha}-1\right)^2\right\}\sin\alpha\,d\alpha\right] \tag{85}$$

$$+\ p\int_0^{\beta}\sin^2\alpha\cos\alpha\tan\theta\,d\alpha$$

The differential equation

$$E\left(\frac{t}{R}\right)\left[2\sin\alpha\left(\frac{\cos\alpha}{\cos\theta}-1\right)\frac{\cos\alpha}{\cos^2\theta}\sin\theta\right]$$

$$+\ \frac{E\left(\frac{t}{R}\right)^3}{12}\left[2\sin\alpha\left\{-\left(\frac{\cos\theta}{\cos\alpha}\frac{d\theta}{d\alpha}-1\right)\frac{\sin\theta}{\cos\alpha}\frac{d\theta}{d\alpha}+\left(\frac{\sin\theta}{\sin\alpha}-1\right)\frac{\cos\theta}{\sin\alpha}\right\}\right.$$

$$\left.-\frac{d}{d\alpha}\left\{2\sin\alpha\left(\frac{\cos\theta}{\cos\alpha}\frac{d\theta}{d\alpha}-1\right)\frac{\cos\theta}{\cos\alpha}\right\}\right]$$

$$+\ p\left[\sin^2\alpha\cos\alpha\sec^2\theta\right]=0$$

$$2E\left(\frac{t}{R}\right)\left[\left(\frac{\cos\alpha}{\cos\theta}-1\right)\frac{\sin\alpha\cos\alpha}{\cos\theta}\tan\theta\right]$$

$$+\ \frac{E\left(\frac{t}{R}\right)^3}{6}\left[\sin\alpha\left\{\left(\frac{\sin\theta}{\sin\alpha}-1\right)\frac{\cos\theta}{\sin\alpha}-\left(\frac{\cos\theta}{\cos\alpha}\frac{d\theta}{d\alpha}-1\right)\frac{\sin\theta}{\cos\alpha}\frac{d\theta}{d\alpha}\right\}\right.$$

$$-\left\{\sec^2\alpha\cos\theta\left(\frac{\cos\theta}{\cos\alpha}\frac{d\theta}{d\alpha}-1\right)-\tan\alpha\sin\theta\left(\frac{\cos\theta}{\cos\alpha}\frac{d\theta}{d\alpha}-1\right)\frac{d\theta}{d\alpha}\right.$$

$$\left.\left.+\tan\alpha\cos\theta\left(\frac{-\sin\theta}{\cos\alpha}\left(\frac{d\theta}{d\alpha}\right)^2+\frac{\cos\theta\tan\alpha}{\cos\alpha}\left(\frac{d\theta}{d\alpha}\right)+\frac{\cos\theta}{\cos\alpha}\frac{d^2\theta}{d\alpha^2}\right)\right\}\right]$$

$$+\ p\left[\sin^2\alpha\cos\alpha\sec^2\theta\right]=0.$$

$$2E\left(\frac{t}{R}\right)\left[\frac{\sin\alpha\cos\alpha}{\cos\Theta}\tan\Theta\left(\frac{\cos\alpha}{\cos\Theta}-1\right)\right]$$ 186)

$$+\frac{E\left(\frac{t}{R}\right)^3}{6}\left[\cos\Theta\left(\frac{\sin\Theta}{\sin\alpha}+\cot^2\alpha\right)-\frac{\cos^2\Theta}{\cos\alpha}\left(2\tan^2\alpha+1\right)\frac{d\Theta}{d\alpha}+\frac{\sin\Theta\cos\Theta\tan\alpha}{\cos\alpha}\left(\frac{d\Theta}{d\alpha}\right)^2\right.$$

$$\left.-\frac{\cos^2\Theta\tan\alpha}{\cos\alpha}\frac{d^2\Theta}{d\alpha^2}\right]+\sin^2\alpha\cos\alpha\sec^2\Theta\,p=0.$$

The other boundary condition: if the support is a hinge support is

$$\tan\alpha\cos\Theta\left(\frac{\cos\Theta}{\cos\alpha}\frac{d\Theta}{d\alpha}-1\right)=0 \quad \text{at} \quad \alpha=\beta$$

or

$$\frac{\cos\Theta}{\cos\alpha}\frac{d\Theta}{d\alpha}=1 \quad \text{at} \quad \alpha=\beta$$

[no change in curvature]

the other boundary condition is $\Theta=0$ at $\alpha=0$

If we neglect the bending energy, then

$$2E\frac{t}{R}\left[\sin\Theta\left(\frac{\cos\alpha}{\cos\Theta}-1\right)\right]+\sin\alpha\,p=0.$$

Let Θ_1 be the value of Θ at $\alpha=\beta$.

$$2E\frac{t}{R}\left[\sin\Theta_1\left(\frac{\cos\beta}{\cos\Theta_1}-1\right)\right]+\sin\beta\,p=0.$$

$$\cancel{\left(E\frac{t}{R}\right)p=-\left(\frac{\cos\beta}{\cos\Theta_1}-1\right)\frac{\sin\Theta_1}{\sin\beta}}$$

$$\frac{p}{E\left(\frac{t}{R}\right)}=-2\frac{\sin\Theta_1}{\sin\beta}\left(\frac{\cos\beta}{\cos\Theta_1}-1\right)$$

$$\frac{\partial p}{\partial\Theta_1}=0 \quad \text{gives} \quad \cos\beta=\cos^3\Theta_1\ ;$$

$$\therefore \quad \frac{p}{E\left(\frac{t}{R}\right)} = -2\frac{\left[1-(\cos\beta)^{2/3}\right]^{\frac{1}{2}}}{\sin\beta}\left[(\cos\beta)^{2/3}-1\right] \qquad 187)$$

$$= 2\,\frac{\left[1-(\cos\beta)^{2/3}\right]^{3/2}}{\sin\beta}$$

When β is small, $1-(\cos\beta)^{2/3} = \frac{1}{3}\beta^2$

$$\frac{p}{E\left(\frac{t}{R}\right)} \approx 2\frac{1}{(3)^{3/2}}\beta^2 = 0.385\,\beta^2$$

When $\beta = 0.1$ $\quad \frac{p}{E\left(\frac{t}{R}\right)} \approx 0.00385$ $\quad p \approx 0.00385\,E\left(\frac{t}{R}\right)$

The critical stress $= \frac{p\,r}{2t} = 0.001925\,E$

Experiments $= \frac{0.25\times 0.60}{900}E = 0.000167\,E$ for $\frac{R}{t} = 900$.

If $\beta = 0.0294$, the values will agree.

The necessary radius to make a 1.5" radius dent will be

51"

---

from the relation

$$\sin\Theta\left(\frac{\cos\alpha}{\cos\Theta}-1\right)+\frac{1}{2}\sin\alpha\left(\frac{p}{E\frac{t}{R}}\right)=0 \qquad \overline{}\; k$$

$$\sin\Theta\cos\alpha - \sin\Theta\cos\Theta + \frac{1}{2}k\cdot\sin\alpha\cos\Theta = 0.$$

$$\left(\cos\Theta\cos\alpha - \cos 2\Theta - \frac{1}{2}k\cdot\sin\alpha\sin\Theta\right)\frac{d\Theta}{d\alpha} - \sin\Theta\sin\alpha + \frac{k}{2}\cdot\cos\alpha\cos\Theta = 0.$$

$$\frac{d\Theta}{d\alpha} = \frac{\sin\Theta\sin\alpha - \frac{1}{2}k\cos\alpha\cos\Theta}{\cos\Theta\cos\alpha - \cos 2\Theta - \frac{1}{2}k\sin\alpha\sin\Theta}$$

If Θ, α are small, then

$$\tfrac{1}{2}\Theta(\Theta^2-\alpha^2)+\tfrac{1}{2}k\alpha=0$$

or
$$\Theta^3-\alpha^2\Theta+k\alpha=0$$

$$\therefore \quad \Theta=\left(-\frac{k\alpha}{2}+\sqrt{\frac{k^2\alpha^2}{4}-\frac{\alpha^6}{27}}\right)^{\frac{1}{3}}+\left(-\frac{k\alpha}{2}-\sqrt{\frac{k^2\alpha^2}{4}-\frac{\alpha^6}{27}}\right)^{\frac{1}{3}}$$

$$=\alpha^{\frac{1}{3}}\left[\left(-\frac{k}{2}+\sqrt{\frac{k^2}{4}-\frac{\alpha^4}{27}}\right)^{\frac{1}{3}}+\left(-\frac{k}{2}-\sqrt{\frac{k^2}{4}-\frac{\alpha^4}{27}}\right)^{\frac{1}{3}}\right]$$

etc.

$$\left(\frac{\cos\alpha}{\cos\Theta}-1\right)=-\frac{k}{2}\frac{\alpha}{\Theta}$$

$$\frac{d\Theta}{d\alpha}=\frac{2\Theta\alpha-k}{3\Theta^2-\alpha^2-k\Theta\alpha}$$

If the Θ, α are of same order and are small, the differential equation can be written as

$$\underbrace{E\left(\frac{t}{R}\right)\alpha\Theta(\Theta^2-\alpha^2)+\alpha^2 p}+\frac{E\left(\frac{t}{R}\right)^3}{6}\left\{\left(\frac{\Theta}{\alpha}+\alpha^2\right)-(2\alpha^2+1)\frac{d\Theta}{d\alpha}+\alpha\Theta\left(\frac{d\Theta}{d\alpha}\right)^2-\alpha\frac{d^2\Theta}{d\alpha^2}\right\}=0$$

or approximately

$$\alpha\Theta(\Theta^2-\alpha^2)+k_1\alpha^2+k_2\left\{\frac{\Theta}{\alpha}-\frac{d\Theta}{d\alpha}-\alpha\frac{d^2\Theta}{d\alpha^2}\right\}=0.$$

where $k_1=\dfrac{p}{\left(E\frac{t}{R}\right)}$, $k_2=\dfrac{\left(\frac{t}{R}\right)^2}{6}$

If α, Θ are small, the total energy can be written as 189)

$$E\left(\frac{t}{R}\right)\int_0^\beta \frac{1}{4}(\Theta^2-\alpha^2)^2\alpha\,d\alpha + \frac{E\left(\frac{t}{R}\right)^3}{12}\int_0^\beta\left\{\left(\frac{d\Theta}{d\alpha}-1\right)^2+\left(\frac{\Theta}{\alpha}-1\right)^2\right\}\alpha\,d\alpha + p\int_0^\beta \alpha^2\Theta\,d\alpha$$

Let $\Theta = C\alpha$, then we have

$$E\left(\frac{t}{R}\right)\frac{1}{4}(C^2-1)^2\int_0^\beta \alpha^5\,d\alpha + \frac{E\left(\frac{t}{R}\right)^3}{12}(C-1)^2\int_0^\beta 2\alpha\,d\alpha + pC\int_0^\beta \alpha^3\,d\alpha$$

$$= E\left(\frac{t}{R}\right)\frac{1}{24}(C^2-1)^2\beta^6 + \frac{E\left(\frac{t}{R}\right)^3}{12}(C-1)^2\beta^2 + \frac{pC}{4}\beta^4$$

Differentiate with respect to C,

$$\frac{E\left(\frac{t}{R}\right)}{6}C(C^2-1)\beta^2 + \frac{E\left(\frac{t}{R}\right)^3}{6}\frac{(C-1)}{\beta^2} + \frac{p}{4} = 0.$$

Or $$p = \frac{2}{3}E\left(\frac{t}{R}\right)\left\{C(1-C^2)\beta^2 + \left(\frac{t}{R}\right)^2\frac{(1-C)}{\beta^2}\right\}$$

Minimizing with respect to β^2,

$$C(1-C^2)\beta^2 = \left(\frac{t}{R}\right)^2\frac{(1-C)}{\beta^2}$$

or $$\beta^2 = \left(\frac{t}{R}\right)\sqrt{\frac{1-C}{C(1-C^2)}} = \left(\frac{t}{R}\right)\frac{1}{C^{\frac{1}{2}}(1+C)^{\frac{1}{2}}}$$

$$\therefore\quad p = \frac{2}{3}E\left(\frac{t}{R}\right)\left\{2\left(\frac{t}{R}\right)C^{\frac{1}{2}}(1-C)(1+C)^{\frac{1}{2}}\right\}$$

$$= \frac{4}{3}E\left(\frac{t}{R}\right)^2\left\{C^{\frac{1}{2}}(1+C)^{\frac{1}{2}}(1-C)\right\}$$

Minimizing with respect to C, $\frac{\partial}{\partial C}\left[C-C^2-C^3+C^4\right]=0$

or $1-2C-3C^2+4C^3=0$

Divide through by $c-1$,

$$4c^2+c-1=0$$

$$c = \frac{1}{8}\left[-1 \pm \sqrt{1+16}\right]$$

$$= \frac{1}{8}\left[-1 \pm \sqrt{17}\right] = \begin{matrix} +0.3904 \\ -0.640 \end{matrix}$$

$$c^{\frac{1}{2}}(1+c)^{\frac{1}{2}}(1-c) = 0.3904^{\frac{1}{2}} \times 1.3904^{\frac{1}{2}} \times 0.6096 = 0.625 \times 1.179 \times 0.6096$$

$$= 0.450$$

$$p = 0.60 E \left(\frac{t}{R}\right)^2$$

$$\sigma = 0.30 E \left(\frac{t}{R}\right)$$

$$\beta^2 = \left(\frac{t}{R}\right) \frac{1}{0.625 \times 1.179}$$

$$= 1.357 \left(\frac{t}{R}\right)$$

or $\left(\frac{t}{R}\right) = \frac{1}{900}$, $\beta^2 = 0.001507$

$$\underline{\underline{\beta = 0.0388}}$$

Let us put $\Theta = c\left[\alpha + \left(\frac{1}{c}-1\right)\frac{\alpha^2}{\beta}\right]$, when $\alpha=\beta$

$\Theta = \beta$.

Then $$\Theta^2 - \alpha^2 = \alpha^2\left\{c^2\left[1+\left(\frac{1}{c}-1\right)\frac{\alpha}{\beta}\right]^2 - 1\right\}$$

$$\frac{d\Theta}{d\alpha} = c\left[1 + 2\left(\frac{1}{c}-1\right)\frac{\alpha}{\beta}\right]$$

Thus put this values to the integral for potential energy,

$$\frac{E(\frac{t}{R})}{4}\int_0^\beta\left\{C^2\left[1+(\tfrac{1}{c}-1)(\tfrac{\alpha}{\beta})\right]^2-1\right\}^2\alpha^5\,d\alpha+\frac{E(\frac{t}{R})^3}{12}\int_0^\beta\left\{\left[(c-1)+2(1-c)(\tfrac{\alpha}{\beta})\right]^2+\left[(c-1)+(1-c)\tfrac{\alpha}{\beta}\right]^2\right\}\alpha\,d\alpha$$

191)

$$+\,pC\int_0^\beta\left\{\alpha^3+(\tfrac{1}{c}-1)\frac{\alpha^4}{\beta}\right\}d\alpha$$

$$=\frac{E(\frac{t}{R})}{4}\int_0^\beta\Big\{(c^2-1)^2+4c^2(\tfrac{1}{c}-1)(c^2-1)(\tfrac{\alpha}{\beta})+2c^2(3c^2-1)(\tfrac{1}{c}-1)^2(\tfrac{\alpha}{\beta})^2$$
$$+4c^4(\tfrac{1}{c}-1)^3(\tfrac{\alpha}{\beta})^3+c^4(\tfrac{1}{c}-1)^4(\tfrac{\alpha}{\beta})^4\Big\}\alpha^5\,d\alpha$$

$$+\frac{E(\frac{t}{R})^3}{12}(c-1)^2\int_0^\beta\left\{2-6(\tfrac{\alpha}{\beta})+5(\tfrac{\alpha}{\beta})^2\right\}\alpha\,d\alpha+pC\int_0^\beta\left\{\alpha^3+(\tfrac{1}{c}-1)\frac{\alpha^4}{\beta}\right\}d\alpha$$

$$=\frac{E(\frac{t}{R})}{4}\left\{\frac{(c^2-1)^2}{6}+\frac{4}{7}c(1-c)(c^2-1)+\frac{1}{4}(3c^2-1)(1-c)^2+\frac{4}{9}c(1-c)^3+\frac{1}{10}(1-c)^4\right\}\beta^6$$

$$+\frac{E(\frac{t}{R})^3}{12}(c-1)^2\frac{\beta^2}{4}+pc\left\{\frac{1}{4}+\frac{1}{5}(\tfrac{1}{c}-1)\right\}\beta^4$$

minimizing with respect to C,

$$\frac{E(\frac{t}{R})}{4}\Big\{\frac{2}{3}c(c^2-1)-\frac{4}{7}(c-1)(4c^2+c-1)+\frac{1}{2}(c-1)(6c^2-3c-1)-\frac{4}{9}(c-1)(4c^2-5c+1)$$
$$+\frac{2}{5}(c-1)^3\Big\}\beta^6$$

$$+\frac{E(\frac{t}{R})^3}{12}\frac{(c-1)\beta^2}{2}+p\left\{\frac{1}{4}-\frac{1}{5}\right\}\beta^4=0$$

$$p=5E(\tfrac{t}{R})\Big[(1-c)\beta^2\Big\{\frac{2}{3}c(c+1)-\frac{4}{7}(4c^2+c-1)+\frac{1}{2}(6c^2-3c-1)-\frac{4}{9}(4c^2-5c+1)$$
$$+\frac{2}{5}(c-1)^2\Big\}$$
$$+(\tfrac{t}{R})^2\frac{(1-c)}{6\beta^2}\Big]$$