北京邮电大学高等数学双语教学组◎编

高等数学（下）

Advanced Mathematics

(II)

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内 容 提 要

本书为《高等数学》双语教材的第二部分，主要内容包括微分方程及其简单应用、解析几何、多元函数的微分及其应用、多元函数的积分及其应用，以及曲线、曲面积分。

本书的每一个部分都经过了精细的筛选，力求做到重点突出、层次分明、叙述清楚、深入浅出、简明易懂。全书例题较为丰富，并且每一节之后均配有一定数量的习题。习题分为两个部分，第一部分主要是对基本知识和基本方法的训练，第二部分则主要强调对基本知识和方法的灵活运用能力。本书适用于高等学校理工科各专业学生的双语教学，同时也可作为其他专业的教材和参考教材。

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前　　言

关于高等数学

高等数学(微积分)是一门研究运动和变化的数学，产生于 16 至 17 世纪，受当时科学家们在研究力学问题时对相关数学的需要而逐渐发展起来的。高等数学中微分处理的是求已知函数的变化率的问题，例如，曲线的斜率、运动物体的速度和加速度等；积分处理的则是在当函数的变化率已知时，如何求原函数的问题，例如，通过物体当前的位置和作用在该物体上的力来预测该物体的未来位置，计算不规则平面区域的面积，计算曲线的长度等。现在，高等数学已经成为高等院校学生尤其是工科学生最重要的数学基础课程之一，学生在这门课程上学习情况的好坏对其后续课程能否顺利学习有着至关重要的影响。

关于本书

虽然国内一些出版社已经影印出版了多种国外编写的“高等数学”的优秀英文教材，但这些教材的内容都过于简单，不符合教育部给出的“高等数学”本科教学内容的要求，因此并不十分适合我国现有高等院校尤其是重点理工科高等院校的教学实际情况。国内编写的双语“高等数学”教材几乎没有，这些因素促使我们下决心编写一本适合我国重点理工科高等院校特别是我校特色专业学生学习的英文“高等数学”教材，以满足我校乃至全国理工科高等院校对“高等数学”课程的双语教学要求。

本书的所有作者都在我校主讲了多年的双语“高等数学”课程，获得了丰富的教学经验，了解学生在学习双语“高等数学”课程中所面临的问题与困难。与此同时，主要成员都有出国留学或学术访问经历，英文水平良好。本书函数、空间解析几何及微分部分由张文博、王学丽和朱萍三位副教授编写，级数、微分方程及积分部分则由艾文宝教授和袁健华副教授编写，全书由孙洪祥教授审阅校对。本书对高等数学中首次出现的数学术语用中文和英文同时标出，以方便学生学习及在国内考研。此外，本书在内容编排和讲解上适当吸收了欧美国家微积分教材的一些优点。由于作者水平有限，加上时间匆忙，书中出现一些错误在所难免，欢迎并感谢读者通过邮箱(jhyuan_bupt@yahoo.com.cn)给我们指出这些错误，以便我们及时纠正。

致谢

本书在编写过程中得到北京邮电大学、北京邮电大学理学院和国际学院的教改项目资金支持，作者在此表示衷心感谢。同时也借此机会，感谢所有在写书过程中支持和帮助过我们的同事和朋友。

致学生的话

高等数学的学习没有捷径可走，它需要你付出艰苦的努力。只要你能勤奋学习并持之以恒，定能取得成功。希望你能喜欢这本书，并预祝你取得成功！

Preface

What is advanced mathematics?

Advanced mathematics that we refer to contains mainly calculus. Calculus is the mathematics of motion and change. It was first invented to meet the mathematical needs of the scientists of the 16th and 17th centuries, needs that were mainly mechanical in nature. Differential calculus deals with the problem of calculating rates of change. It enables people to define slopes of curves, to calculate velocities and accelerations of moving bodies, etc.. Integral calculus deals with the problem of determining a function from information about its rate of change. It enables people to calculate the future location of a body from its present position and a knowledge of the forces acting on it, to find the areas of irregular regions in the plane, to measure the lengths of curves, and so on. Now, advanced mathematics becomes one of the most important courses for the college students in natural science and engineering.

About this book

Although several excellent textbooks in English language concerned with advanced mathematics that came from abroad have been published in China, their contents do not amply meet the requirements presented by the Ministry of Education for advanced mathematics. So they do not amply accord with the teaching requirements of domestic universities. Furthermore, few textbooks in English language concerned with advanced mathematics, which were written by domestic authors, can be found in locality. All these factors inspire us to write out this book, in order to satisfy the increasing bilingualteaching demand of universities in China. All the authors have been teaching this course by bilingual languages for many years. They have much experience in both course-teaching and bilingual-teaching. They know the obstacles encountered by Chinese students in learning this course in English. Moreover, all the authors had experience visiting abroad. The contents of functions, space analytic geometry and differential calculus were jointly written by Associate Professors Wenbo Zhang, Xueli Wang and Ping Zhu, while the contents of series, differential equations and integration calculus jointly by Professor Wenbao Ai and

Associate Professor Jianhua Yuan. The whole book was proofread by Professor Hongxiang Sun. Because of the careful checking and proofing by us, the authors believe this book to be almost error-free. For any errors remaining, the authors would be grateful if they were sent to: jhyuan bupt@yahoo. com. cn.

Acknowledgments

The authors wish to thank all the persons who have been involved in producing the book. Our special thanks go to the Science School and the International School of BUPT for their financial support on producing the book by teaching-reform grants.

To the student

Learning advanced mathematics requires a lot of hard work and effort on your part. No one else can do this for you, and there are no shortcuts. However, if you work consistently and diligently through this book you will succeed. Enjoy this book, and good luck for you!

School of Science, BUPT

Contents

Chapter 7

Differential Equations

Usually, a function reflects the quantitative aspects of some phenomenon. In many practical problems, however, it is impossible to establish the concerned function directly. But we may be able to establish a relation between the unknown function and its derivatives (or differentials) according to some special properties of the problem and some related knowledge. This kind of relation is called the differential equation [微分方程]. After we set up a required differential equation, we need to solve this equation to obtain the concerned functun, or as we say, to integrate the differential equation.

7.1 Basic Concepts of Differential Equations

7.1.1 Examples of Differential Equations

Example 7.1.1 Let a plane curve passes through a point (1,2) in the xOy plane, and let its tangent slope at any point (x, y) on the curve be $2x$. Find the equation of the curve.

Solution By the geometric meaning of derivatives, the desired curved $y=f(x)$ should satisfy

$$\frac{\mathrm{d}y}{\mathrm{d}x}=2x \quad \text{or} \quad \mathrm{d}y=2x\mathrm{d}x \tag{7.1.1}$$

This equation involves the derivative (or differential) of the unknown function $y=f(x)$. In order to find the unknown $f(x)$, integrating on both sides of (7.1.1) with respect to x obtains

$$y=\int 2x\mathrm{d}x=x^2+C, \tag{7.1.2}$$

where C is an arbitrary constant.

Equation (7.1.2) represents a family of curves. Since the desired curve passes through the point (1,2), that is

$$y=2 \quad \text{at} \quad x=1. \tag{7.1.3}$$

Substituting the condition into the expression (7.1.2) yields

$$2=1+C,$$

so that

$$C=1.$$

Therefore, the equation of the desired curve is

$$y=x^2+1. \tag{7.1.4}$$

Example 7.1.2 Suppose that a particle of mass m falls freely from a height of H (Figure. 7.1.1), with initial velocity V_0. If we neglect the resistance of air, find the particle's height $h(t)$ at any time t while it is falling.

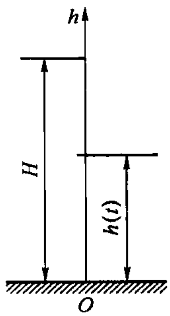

Figure 7.1.1

Solution Let the initial time when the partial starts to fall be $t=0$, and let the height of the partial at any time t in the process of falling be $h=h(t)$. By Newton's law, h should satisfy the following equation

$$m\frac{d^2h}{dt^2}=-mg,$$

or

$$\frac{d^2h}{dt^2}=-g. \tag{7.1.5}$$

In order to find $h(t)$, integrating both sides of (7.1.5) we have

$$\frac{dh}{dt}=-gt=+C_1. \tag{7.1.6}$$

Integrating again we obtain

$$h=-\frac{1}{2}gt^2+C_1t+C_2, \tag{7.1.7}$$

where C_1 and C_2 are both arbitrary constants.

By the given problem the function $h(t)$ should satisfy the following two supplementary conditions, which are called **initial conditions**:

$$h|_{t=0}=H; \quad V|_{t=0}=\left.\frac{dh}{dt}\right|_{t=0}=V_0. \tag{7.1.8}$$

Substituting the conditions into the expression (7.1.7) we have

$$C_1=V_0, \quad C_2=H.$$

Therefore the desired function $h(t)$ is

$$h(t)=-\frac{1}{2}gt^2+V_0t+H, \tag{7.1.9}$$

which is familiar with physics.

7.1.2 Basic Concepts

Definition 7.1.3 (Differential equation). A differential equation [微分方程] is one which connects the independent variable x, unknown function and its derivatives.

Symbolically, a differential equation may be written as follows:

$$F(x,y,y',y'',\cdots,y^{(n)})=0.$$

If the unknown function y is of one variable, the differential equation is called ordinary equation[常微分方程].

Definition 7.1.4(The order of an equation). The **order** of the differential equation **[微分方程的阶]**is the order of the highest derivative that appears.

For instance, the following equations

$$\frac{dy}{dx}=2x, ydx+xdy=0, \frac{dy}{dx}+2y^2+xy=0$$

are ones of the first order. And the orders of the equations

$$\frac{d^2h}{dt^2}=g, y''+3y'+3y=e^x, y''+(y')^3=x$$

are two.

Definition 7.1.5 (Solution, general solution and initial conditions, particular solution). A solution [解]of an equation is any function $y=f(x)$, which, when put into the equation, converts the equation into an identity.

The general solution [通解]of an equation of the nth order is a family of solutions, which family dependson n arbitrary and mutually independent constants.

If all the arbitrary constants have been determined, then the solution is called a

particular solution [**特解**]of the equation.

The supplementary conditions to determine a particular solution are called the **initial conditions**[**初始条件**]. Usually, they reflet the initial situation of the desired motion or some properties at a given point of the desired curve and can be used to determine the values of the arbitrary constants in the general solution to give a particular solution. For example, solutions (7. 1. 2) and (7. 1. 7) are the general solutions of the equations (7. 1. 1) and (7. 1. 5) respectively; the conditions (7. 1. 3) and (7. 1. 8) are the initial conditions of the equations (7. 1. 1) and (7. 1. 5) respectively; the solutions (7. 1. 4) and (7. 1. 9) are the particular solutions of the equations (7. 1. 1) and (7. 1. 5) respectively.

$h=-\frac{1}{2}gt^2+C_1t$ and $h=-\frac{1}{2}gt^2+C_1+2C_2$ are both solutions of the equation $\frac{d^2h}{dt^2}=-g$, but they are not general solutions because the former contains only one arbitrary constant, and the later seems to contain two arbitrary constants but they may be combined into a constant $C=C_1+2C_2$.

Example 7. 1. 6 Is the function $y=\frac{1}{x+C}$ the general solution of the differential equation

$$y'+y^2=0?$$

Solution Substituting $y'=-\frac{1}{(x+C)^2}$ and $y=\frac{1}{x+C}$ into the given differential equation we get

$$-\frac{1}{(x+C)^2}+\left(\frac{1}{x+C}\right)^2=0.$$

Hence, the function $y=\frac{1}{x+C}$ is the general solution of the differential equation $y'+y^2=0$.

□

7. 1. 3 Geometric Interpretation of the First-Order Differential Equation

Let us explain the geometric meaning of the two definitions for first order equation by means of the example (7. 1. 1).

The differential equation (7. 1. 1) means that for any point $P(x,y)$ in the xOy plane there is one and only one value $2x=y'|_{(x,y)}$, which is the slope of a linear element (or line segment) at P. Geometrically, a given differential equation like (7. 1. 2) describes **a field of linear elements** shown in Figure 7. 1. 2(a).

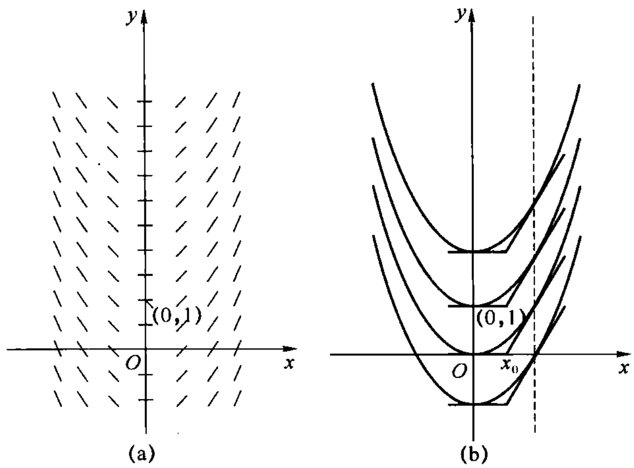

Figure 7.1.2

To find solutions of the equation (7.1.1) is to find those parabolas $y=x^2+C$ such that at any point P, the tangent of the corresponding parabola just coincides with the linear element at the point P. From the Figure 7.1.2(b) we can see that there is an infinite number of these parabolas which depend on the constant C. The family of parabolas represents the general solution. Any given point $P_0(x_0, y_0)$ represents an initial condition, and the particular solution, which satisfies the initial condition $y|_{x=x_0}=y_0$, is such a parabola that passes through the point $P_0(x_0, y_0)$. For instance, the particular solution satisfying the initial condition $y|_{x=1}$ is the parabola $y=x^2$.

In the following sections we will introduce some types of differential equations of first-order and show how to solve them. Because differential types of differential equations are solved by differential solution methods, it is important for the reader to recognize the types of the given equations and remember their solution methods.

Exercises 7.1

1. Find the orders of the following differential equations:

(1) $y'-2y=x+2$; (2) $x^2y''-3xy'+y=x^4e^x$;

(3) $(1+x^2)(y')^3-2xy=0$; (4) $xy'''+\cos^2\frac{dy}{dx}+y=\tan x$;

(5) $x\ln x dy+(y-\ln x)dx=0$; (6) $L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=0$.