GLOBAL SUPPLY CHAIN MANAGEMENT

(ICGSCM' 02)

Edited by: Jian Chen

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Jian Chen

Global Supply Chain Management
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PART VI SUPPLY CHAIN AND E-MARKET OPTIMIZATION

PART VII SUPPLY CHAIN COORDINATIONS

Finding Near-Optimal Policies for Multi-Echelon Inventory Systems A Branch and Bound Approach*

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ABSTRACT

Inventory coordination among supply chain members is very important for effective supply chain management. In this paper, a supply chain with a supplier and several retailers is considered, where each facility incurs a fixed cost as well as a variable cost when it places an order. The problem is to find an optimal echelon inventory policy for the system so that its expected total cost, which consists of echelon inventory holding costs, echelon backorder penalty costs, fixed and variable order costs of all the facilities, is minimized. For such a problem, the optimal policy is not known. As an alternative, we develop a stochastic branch and bound approach to find near-optimal echelon (s, S) polices. In the approach, a lower bound of the expected cost for a partial solution in which some parameters of a policy are given is obtained by Lagrangian relaxation that relaxes inter-facility constraints of the system. The relaxed problem can be decomposed into a set of subproblems, one for each facility. Each subproblem is a single location inventory problem which can be efficiently solved by using an existing approach. Numerical experiments show that the approach can obtain near optimal policies in a reasonable computation time.

1. INTRODUCTION

In recent years, supply chain management has attracted much attention of both academic and industrial communities. Driven by extensive global competitions, many companies have recognized that the coordination of operations across supply chains is critical for them to further reduce costs while improving the responsiveness to changes in the market place.

One important issue of supply chain management is inventory coordination among supply chain members. In the past, stocks at different members are managed individually based on their own demands and supply data. Each member optimizes its own inventory policy independently without coordination with other members. This may lead to undesired results since all members in a supply chain are interconnected, an output from one member may be an input to another member. Therefore, inventory policies across a supply chain should be well coordinated to achieve the global optimization that minimizes a system-wide cost.

Theoretically, supply chains can be modeled as multi-echelon inventory systems at inventory planning level. For a special class of multi-echelon inventory systems with facilities-in-series structure, if fixed order costs are only charged at the most upstream facility (stock) of the systems, Clark and Scarf ([4]) have shown that optimal inventory policies for the systems are an echelon inventory policy that determines order placement for each stock based on its echelon inventory position. The optimal policy can be computed by decomposing a multi-echelon problem into a set of separate single-location problems, one for each facility, which can be recursively solved. Rosling ([9]) and Chen ([3]) extend this result to assembly systems and systems with batch ordering policies, respectively. However, because of stochastic nature and inherent complexity, no efficient algorithm exists for determining fully optimal inventory policies for most of the multi-echelon inventory systems, especially for those with order costs charged in each facility ([5]).

Since it is very difficult to find full optimal policies, researchers turned to finding close-to-optimal, but not necessarily full optimal polices with a relatively simple structure in recent years. A good review of the research results in this aspect was written by Federgruen ([5]). Most approximation approaches start with an exact formulation of an inventory planning problem as a dynamic program or Markov decision problem. The exact model is then replaced by an approximate one through relaxations, restrictions, projections, or cost approximations. For arborescence distribution systems in which every facility has a unique supplier, approximation approaches by relaxing the nonnegativity constraints of shipments from the supplier to its retail outlets or by restricting inventory polices to a class of regular-interval critical-number polices or the class of (s, S)-policies have been proposed ([5]). However, these approximation approaches are still very complicated in computation.

In literature, some researches also use simulation optimization methods to optimize the parameters of a given inventory policy ([2]). One method is the perturbation analysis method ([7]), but it can only find a policy of local optimum because of its gradient-based nature. Others combine simulation evaluation with meta-heuristics such as genetic algorithms, tabu search, and simulated annealing. However, they cannot tell us whether an optimal policy is obtained and how good a policy found is. The branch and bound approach has been widely used for solving combinatorial optimization problems. Although it is time consuming for large problems, but a clever implementation of the method can solve many realistic problems in a reasonable computation time. Moreover, a well-designed time-truncated branch and bound approach can obtain near-optimal solutions in a short time and a solution obtained can be evaluated by comparing its cost with a lower bound. Recently, a stochastic version of the branch and bound approach is proposed and applied to various stochastic

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optimization problems ([8]).

In this paper, we consider a supply chain with a supplier and several retailers, where each facility incurs a fixed cost as well as a variable cost when it places an order. The problem is to find an optimal echelon inventory policy for the system so that its expected total cost, which consists of echelon inventory holding costs, echelon backorder penalty costs, fixed and variable order costs of all the facilities, is minimized. For such a problem, the optimal policy is not known, let alone an efficient algorithm to find it. As an alternative, we develop a stochastic branch and bound approach to find near-optimal echelon (s, S) polices. In the approach, a lower bound of the expected cost for a partial solution in which some parameters of a policy are given is obtained by Lagrangian relaxation that relaxes inter-facility constraints of the system. The relaxed problem can be decomposed into a set of subproblems, one for each facility. Each subproblem is a single location inventory problem which can be efficiently solved by using an existing approach. Numerical experiments show that the approach can obtain near optimal policies in a reasonable computation time.

2. MODEL

We consider a two echelon supply chain consisting of a supplier, N retailers, and a single product. The demand of each retailer is stationary subject to a normal distribution. It is assumed that the demands for any two retailers and in any two periods are independent. The delivery time from the supplier to retailer i (i = 1, 2, ..., N) is a constant l_i and the replenishment lead time from an outside source to the supplier (or the production lead time of the supplier) is a constant L. The outside source has an ample capacity, an order placed by the supplier is always received L periods after it is placed. Excess demand at each retailer is backordered. The cost of the supplier consists of echelon inventory holding cost, echelon backorder penalty cost, fixed and variable order costs, while the cost of each retailer consists of inventory holding cost, backorder penalty cost, fixed and variable costs for receiving shipments from the supplier. It is assumed that all cost coefficients are constant over time for both the supplier and the retailers, but the following model and results can be extended to a supply chain with variable cost coefficients. It is assumed that the supplier has timely access to the inventory levels and demands of the retailers, so that an echelon inventory policy, which determines order placement for each facility (stock) based on its inventory position, can be applied to the supply chain.

The inventory position of each facility is reviewed periodically. For each facility, the sequence of events occurred in each period is defined as follows: 1) orders and shipments arrive, 2) order decision and placement, 3) demand occurs, 4) demand is fulfilled or backordered, 5) inventory holding and backorder penalty costs are charged. The index t will be used to represent the period currently considered. The cost data of the supply chain are listed in the following:

K^s: fixed cost for the supplier to place an order to the outside source;

K_i^r: fixed cost for retailer i to receive a shipment from the supplier;

c^s: variable cost rate for the supplier to place an order to the outside source;

c_i^r: variable cost rate for retailer i to receive a shipment from the supplier;

h^s: echelon inventory holding cost rate of the supplier per period;

h_i^r: inventory holding cost rate of retailer i per period;

p^s: echelon backorder penalty cost rate of the supplier per period, $p^s \geq c^s$;

p_i^r: backorder penalty cost rate of retailer i per period, $p_i^r \geq c_i^r$.

Other parameters include:

l_i: delivery lead time for a shipment from the supplier to retailer i;

L: lead time for inventory replenishment of the supplier;

d_{it}^r: demand of retailer i in period t, d_{it}^r, $i = 1, ..., N$, t = 1, 2, ... are independent identical random variables with normal distribution;

μ_i^r: mean value of demand d_{it}^r, μ_i^r does not depend on t;

σ_i^{r2}: variance of demand d_{it}^r, σ_i^{r2} does not depend on t.

The state and decision variables are defined as:

I_{it}^r: inventory level of retailer i at the end of period t;

I_t^s: echelon inventory level of the supplier at the end of period t;

z_{it}: shipment from the supplier to retailer i, initiated in period t;

δ_{it}^r: 0-1 variable for a shipment to retailer i, $\delta_{it}^r = 1$ if $z_{it} > 0$, otherwise $\delta_{it}^r = 0$;

y_t: order size of the supplier in period t;

δ_t^s: 0-1 variable for order placement of the supplier, $\delta_t^s = 1$ if $y_t > 0$, otherwise $\delta_t^s = 0$.

For all inventory variables I defined above, $I^+ = \max(0, I)$ denotes the on-hand inventory and $I^- = \max(0, -I)$ denotes the backorder level.

Let T denote the time horizon (the number of periods) considered for inventory planning, with the above notations, an echelon inventory model of the supplier can be formulated as:

$$J^s = \sum_{t=1}^{T} (h^s I_t^{s+} + p^s I_t^{s-} + K^s \delta_t^s + c^s y_t)/T, \tag{1}$$

$$\text{s.t.} \quad I_t^s = I_{t-1}^s + y_{t-L} - \sum_{i=1}^{N} d_{it}^r, \quad t = 1, 2, ..., T, \tag{2a}$$

$$(1-\delta_t^s)\, y_t = 0, \quad t = 1, 2, ..., T, \tag{2b}$$

$$y_t \geq 0, \ \delta_t^s \in \{0, 1\}, \quad t = 1, 2, ..., T, \tag{2c}$$

where J^s is the average cost of the supplier per period, constraints (2a) are echelon inventory equilibrium equations, and (2b) imposes the constraints that $\delta_t^s = 1$ if $y_t > 0$ and $\delta_t^s = 0$ otherwise.

Similarly, an (echelon) inventory model of retailer i can be formulated as:

$$J_i^r = \sum_{t=1}^{T} (h_i^r I_{it}^{r+} + p_i^r I_{it}^{r-} + K_i^r \delta_{it}^r + c_i^r z_{it})/T, \tag{3}$$

$$\text{s.t.}\quad I_{it}^r = I_{i,t-1}^r + z_{i,t-l_i} - d_{it}^r,\ t = 1, 2, ..., T, \tag{4a}$$

$$(1-\delta_{it}^r)\, z_{it} = 0,\ t = 1, 2, ..., T, \tag{4b}$$

$$z_{it} \ge 0,\ \delta_{it}^r \in \{0, 1\},\ t = 1, 2, ..., T, \tag{4c}$$

$$i = 1, 2, ..., N,$$

where J_i^r is the average cost of retailer i per period, constraints (4a) are inventory equilibrium equations, and (4b) imposes the constraints that $\delta_{it}^r = 1$ if $z_{it} > 0$, otherwise $\delta_{it}^r = 0$.

The inter-facility constraints between the supplier and the retailers are:

$$\sum_{i=1}^{N} I_{it}^r + \sum_{i=1}^{N}\sum_{\tau=0}^{l_i} z_{i,t-\tau} \le I_t^s + y_{t-L},\ t = 1, 2, ..., T. \tag{5}$$

Constraints (3) imply that when the supplier ships goods to the retailers, the total size of shipments must not exceed its on-hand inventory.

The inventory planning problem of the supply chain we consider in this paper can now be formulated as:

P:

$$\text{Min } EJ = EJ^s + \sum_{i=1}^{N} EJ_i^r,$$

s.t. (2), (4) for $i = 1, 2, ..., N$, and (5),

where E denotes the mathematical expectation operator. The objective of the problem is to minimize the expected total cost of the supply chain per period in terms of echelon inventory holding cost, echelon backorder penalty cost, fixed and variable order and shipment cost subject to physical flow equilibrium constraints.

3. BRANCH AND BOUND APPROACH

A stochastic version of the branch and bound (SBB) method is proposed by Norkin, Pflug and Ruszczynski for stochastic global optimization ([8]). It adopts the idea of the branch and bound method for deterministic optimization, but instead of deterministic bounds, it uses stochastic upper and lower bound estimates of the optimal value of subproblems to guide the partitioning (branch) process. In this section, we briefly introduce the method.

Consider a stochastic optimization problem of the following form:

$$\min_{x \in X \cap D} [F(x) = Ef(x, \theta(\omega))], \tag{6}$$

where X is a compact set in an n-dimensional Euclidean space R^n, D is a closed subset of R^n implicitly defined by some constraints, $\theta(\omega)$ is an m-dimensional random variable defined on a probability space (Ω, Σ, P), $f: X \times R^m \rightarrow R$ is continuous in the first argument and measurable in the second argument, $f(x, \theta(\omega)) \le \bar{f}(\omega)$ for all $x \in X$, and $E\bar{f} < \infty$, $F(x)$ is an objective to be minimized.

In SBB, the original compact set X is sequentially subdivided into compact subsets $Z \subseteq X$ generating a partition Π of X, such that $\cup_{Z \in \Pi} Z = X$. The original problem is then subdivided into subproblems:

$$\min_{x \in Z \cap D} [F(x) = Ef(x, \theta(\omega))],\ Z \in \Pi. \tag{7}$$

SBB iteratively performs the following three operations:

- partitioning a set into smaller subsets,
- stochastic estimation of the objective within the subsets
- removal of some subsets.

Let $F^*(Z \cap D)$ denote the optimal value of subproblem (7), a real-value function L and U defined on a collection of compact subsets $Z \subseteq X$ with $Z \cap D \ne \emptyset$ provide a lower bound and a upper bound, respectively, if the following conditions hold:

1. For every compact subset $Z \subseteq X$ with $Z \cap D \ne \emptyset$,
   $L(Z) \le F^*(Z \cap D) \le U(Z)$,

   and for every singleton $z \in X \cap D$,
   $L(\{z\}) = U(\{z\}) = F(\{z\})$.

2. There are random variables $\xi_k(Z, \omega)$, $\eta_k(Z, \omega)$, $k = 1, 2, \ldots$, defined on some probability space (Ω, Σ, P) such that for all compact subsets $Z \subseteq X$ with $Z \cap D \ne \emptyset$ and for every k
   $E\xi_k(Z, \omega) = L(Z)$,
   $E\eta_k(Z, \omega) = U(Z)$,

   and $\xi_k(Z, \omega)$, $\eta_k(Z, \omega)$ satisfy a generalized Lipschitz property ([8]).

3. There exists a selection mapping s which assigns to each compact subset $Z \subseteq X$ with $Z \cap D \ne \emptyset$ a point
   $s(Z) \in Z \cap D$

   such that
   $F(s(Z)) \le U(Z)$.

With random variables $\xi_k(Z, \omega)$, $\eta_k(Z, \omega)$ for the estimation of the lower and upper bounds, the stochastic branch and bound algorithm can be described in the following. For brevity, the argument ω for random variables is skipped.

Algorithm SBB ([8]):

Initialization. Form an initial partition as $\Pi_1 = \{X\}$. Observe independent random variables $\xi_1(X)$, $\eta_1(X)$ and put $L_1(X) = \xi_1(X)$, $U_1(X) = \eta_1(X)$. Set $k = 1$.

Partitioning. Select a record subset
$Y_k \in \arg\min \{ L_k(Z): Z \in \Pi_k \}$

and an approximate solution $x^k = s(X_k) \in X_k \cap D$,
$X_k \in \arg\min \{ U_k(Z): Z \in \Pi_k \}$.

Construct a partition of the record set, $\Psi(Y_k) = \{ Y_k^i, i = 1, 2, \ldots \}$ such that $Y_k = \cup_i Y_k^i$. Define a new full partition
$\Pi_k{}' = (\Pi_k \setminus Y_k) \cup \Psi(Y_k)$.

Deletion. Clean partition $\Pi_k{}'$ of non-feasible subsets by defining
$\Pi_{k+1} = \Pi_k{}' \setminus \{ Z \in \Pi_k{}', Z \cap D = \emptyset \}$

Bound estimation. For all $Z \in \Pi_{k+1}$ observe random variable $\xi_{k+1}(Z)$, independently observe $\eta_{k+1}(Z)$ and recalculate stochastic estimates

$$L_{k+1}(Z) = (1-\frac{1}{k+1})L_k(\bar{Z}) + \frac{1}{k+1}\xi_{k+1}(Z), \quad (8)$$

$$U_{k+1}(Z) = (1-\frac{1}{k+1})U_k(\bar{Z}) + \frac{1}{k+1}\eta_{k+1}(Z), \quad (9)$$

where $\bar{Z}$ is such that $Z \subseteq \bar{Z} \in \Pi_k$.
Set $k := k+1$ and go to **Partitioning**

When a uniform bound σ^2 is known for the variances of all random variables $\xi_k(Z, \omega)$ and $\eta_k(Z, \omega)$ for all subsets obtained from X by partition, $k = 1, 2, \ldots$, another deletion rule is also proposed for SBB ([8]). That is, after M steps (M is a large number), the algorithm is terminated, M independent observations $\eta_{M_i}(X_M(\omega))$ are made for the subset $X_M(\omega)$ taken from the final partition $\Pi_M(\omega)$, $i = 1, \ldots, M$, and a new estimate for $U(X_M(\omega))$ is calculated:

$$\bar{U}_M(X_M(\omega)) = \frac{1}{M}\sum_{i=1}^{M}\eta_{M_i}(X_M(\omega)). \quad (10)$$

Then, for some accuracy $\varepsilon \in (0, 1)$, all sets $Z \in \Pi_M(\omega)$ such that

$$L_M(Z) > \bar{U}_M(X_M(\omega)) + 2c_M \quad (11)$$

are deleted, where $c_M = \sigma^2/(M\varepsilon)$.

The almost sure convergence of algorithm SBB is proved and its random accuracy estimates derived ([8]).

4. STOCHASTIC BOUNDS

In the branch and bound method, stochastic lower bound estimates and upper bound estimates are used for branching, deleting non-prospective sets, and for estimating the cost of the current solution. In this section, the bound estimates are derived for our problem.

Lower Bound

Lagrangian relaxation has been frequently used to obtain a lower bound for deterministic mixed integer programming problems ([6]). It is also used for deriving a lower bound for our stochastic inventory planning problem.

In order to do so, the inter-facility constraints (5) are first relaxed by

$$E\{\sum_{i=1}^{N} I_{it}^r + \sum_{i=1}^{N}\sum_{\tau=0}^{l_i} z_{i,t-\tau}\}/T \le E\{I_t^s + y_{t-L}\}/T. \quad (12)$$

The constraints (12) are then relaxed by using Lagrange multipliers $\{\lambda_t, t = 1, 2, \ldots, T\}$.

Let $\lambda = \{\lambda_t, t = 1, 2, \ldots, T\}$, $y = \{y_t, t = 1, 2, \ldots, T\}$, $z_i = \{z_{it}, t = 1, 2, \ldots, T\}$, $z = \{z_i, i = 1, 2, \ldots, T\}$. The relaxed problem is:

RP:

$$\min_{\{y,z\}} L(\lambda, \{y, z\}) = EJ + \sum_{t=1}^{T}\lambda_t [E\{\sum_{i=1}^{N} I_{it}^r + \sum_{i=1}^{N}\sum_{\tau=0}^{l_i} z_{i,t-\tau}\}/T - E\{I_t^s + y_{t-L}\}/T] \quad (13)$$

s.t. (2) and (4) for $i = 1, 2, \ldots, N$.

Notice that $I_{it}^r = I_{it}^{r+} - I_{it}^{r-}$ and $I_t^s = I_t^{s+} - I_t^{s-}$, L can be written as:

$$L(\lambda, \{y, z\}) = E\{\sum_{t=1}^{T}[(h^s-\lambda_t) I_t^{s+} + (p^s+\lambda_t) I_t^{s-} + K^s\delta_t^s + (c^s-\lambda_{t+L})y_t]\}/T + \sum_{i=1}^{N} E\{\sum_{t=1}^{T}[(h_i^r+\lambda_t) I_{it}^{r+} + (p_i^r-\lambda_t) I_{it}^{r-} + K_i^r\delta_{it}^r + (c_i^r + \sum_{\tau=0}^{l_i}\lambda_{t+\tau}) z_{it}]\}/T$$

$$= L^s(\lambda, y) + \sum_{i=1}^{N} L_i^r(\lambda, z_i),$$

where

$$L^s(\lambda, y) = E\{\sum_{t=1}^{T}[(h^s-\lambda_t) I_t^{s+} + (p^s+\lambda_t) I_t^{s-} + K^s\delta_t^s + (c^s-\lambda_{t+L})y_t]\}/T,$$

$$L_i^r(\lambda, z_i) = E\{\sum_{t=1}^{T}[(h_i^r+\lambda_t) I_{it}^{r+} + (p_i^r-\lambda_t) I_{it}^{r-} + K_i^r\delta_{it}^r + (c_i^r + \sum_{\tau=0}^{l_i}\lambda_{t+\tau}) z_{it}]\}/T.$$

The relaxed problem can be decomposed into $N+1$ subproblems, one for each facility:

RP^s:

$$\min_{y} L^s(\lambda, y), \quad (14)$$

s.t. (2).

RP_i^r:

$$\min_{z_i} L_i^r(\lambda, z_i), \quad (15)$$

s.t. (4).

These subproblems are single location, multi-period inventory problems with positive set-up cost. The optimal solutions for them have been proved to be (s, S) policies, which can be computed by using a dynamic programming algorithm ([5]). The reorder point s and the order-up-to level S of these policies may change over time (period).

Let $\phi(\lambda) = \min_{\{y,z\}} L(\lambda, \{y, z\})$ be the optimal value of *RP*. The Lagrangian dual problem of *P* is:

DP:

$$\max_{\lambda\in\Theta} \phi(\lambda), \quad (16)$$

where

$$\Theta = \{\{\lambda_t\} \mid \lambda_t \ge 0, h^s-\lambda_t \ge 0, p^s+\lambda_t \ge c^s-\lambda_{t+L}, p_i^r-\lambda_t \ge 0,\ p_i^r-\lambda_t \ge c_i^r + \sum_{\tau=0}^{l_i}\lambda_{t+\tau}\}.$$

The dual function is concave, which can be maximized by using the subgradient method ([6]). The optimal value of the dual problem provides a lower bound for the optimal cost of the original problem *P*.

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Because the dynamic programming algorithm for solving the relaxed subproblems with finite time horizon is time-consuming somehow, in the rest of this paper, we confine ourselves to an infinite horizon model of the above inventory planning problem, i.e., we take $T = +\infty$. In this case, all Lagrange multipliers λ_t, $t = 1, \ldots$ can be taken identical, and an (s, S) policy with parameters s and S constant over period is optimal for the relaxed subproblems. The policy can be efficiently computed by using an algorithm of Zheng and Federgruen ([11]).

Let $\lambda_t = \lambda$, $t = 1, \ldots$ (here, we abuse the symbol λ to represent a scalar). The dual problem becomes a single-dimensional concave maximization problem, which can be solved by simply using dichotomy search.

The above discussion has developed a method for computing a lower bound for our problem at the root node of SBB. At other nodes, the lower bound can be computed similarly except that for some relaxed subproblems either one parameter or two parameters of their (s, S) policy is given. For the first case, Zheng and Federgruen's algorithm can still be used to optimize the other parameter to minimize the cost of the subproblem after a slight modification. For the second case, the cost of the subproblem can be analytically evaluated ([1]).

To reduce the time spending on the computation of the lower bound in SBB, optimal multipliers and subproblem solutions at a parent node are used as a starting point for the computation of the lower bound at its son nodes.

Upper Bound

In industry, for a multi-echelon inventory system, its echelon (s, S) policy is usually designed stock-by-stock, whose parameters for each stock are computed based on the corresponding single location model and the echelon lead-time of the stock. The derived policy usually provides a good solution ([10]). In view of this, one initial upper bound of our problem (i.e., an upper bound at the root node of SBB) is calculated in the following way:

$$s_i^r = \mu_i^r \times (l_i+1) + z_i^r \times \sigma_i^r \times \sqrt{l_i + 1}\,, \quad (17)$$

where z_i^r is the z-value for the stock of retailer i determined by the ratio $\frac{p_i^r - c_i^r}{p_i^r + h_i^r}$, i.e., $\text{Prob}\{x \le z\} = \frac{p_i^r - c_i^r}{p_i^r + h_i^r}$, where x is a normal distribution random variable with mean value 0 and standard deviation 1.

$$S_i^r = \max(Q_i^r, \mu_i^r \times (l_i+1)) + z_i^r \times \sigma_i^r \times \sqrt{l_i + 1}\,, \quad (18)$$

where $Q_i^r = \sqrt{\frac{2K_i^r \times \mu_i^r}{h_i^r}}$ is the economic order quantity for retailer i calculated based on its mean demand.

$$s^s = \sum_{i=1}^{N} \mu_i^r \times (L + l_i + 1) + z^s \times \sqrt{\sum_{i=1}^{N} \sigma_i^{r\,2} (L + l_i + 1)}\,, \quad (19)$$

where z^s is the z-value for the echelon stock of the supplier determined by the ratio $\frac{p^s - c^s}{p^s + h^s}$.

$$S^s = \max(Q^s, \sum_{i=1}^{N} \mu_i^r \times (L + l_i + 1)) + z^s \times \sqrt{\sum_{i=1}^{N} \sigma_i^{r\,2} (L + l_i + 1)}\,, \quad (20)$$

where $Q^s = \sqrt{\frac{2K^s \times \mu^s}{h^s}}$ is the economic order quantity for the supplier calculated based on its mean demand $\mu^s = \sum_{i=1}^{N} \mu_i^r$.

Another way to design an echelon (s, S) policy for an industrial supply chain is to set the order-up-to-level S as the sum of the reorder point s and the economic order quantity Q for each stock ([10]). That is, $S = s + Q$. Applying to our problem, we have

$$S_i^r = Q_i^r + s_i^r,\ i = 1, \ldots, N,\ S^s = s^s + Q^s. \quad (21)$$

We evaluate the cost of the supply chain for both policies. The lower cost is taken as the initial upper bound for our branch and bound algorithm.

5. THE ALGORITHM

In our implementation of the branch and bound algorithm, rather than estimating the lower and upper bounds according to (8) and (9), we calculate or estimate the bounds in the following way: For each partial solution, the lower bound proposed in last section is analytically calculated based on theoretical results for single location infinite horizon inventory models ([1]). As for the upper bound, as soon as a complete solution is found, M independent simulations (M is a given number) are performed to evaluate its cost for original problem P. We use a finite horizon model of T periods to approximate the infinite horizon model of the problem at each simulation, where T is taken sufficiently large. The deletion rule (11) of SBB is then accordingly modified to

$$L(Z) > \overline{U}_M(X) + e(T) + c_M(T), \quad (22)$$

where Z is a partial solution corresponding to a compact subset in the solution space of problem P, $L(Z)$ is the lower bound analytically calculated for the partial solution, X is the best solution of problem P found so far, $\overline{U}_M(X)$ is the average cost of the problem at solution X for the M simulations, $e(T) = |EJ(T) - EJ(\infty)|$, $J(T)$ and $J(\infty)$ are the costs of problem P with finite horizon of T periods and with infinite horizon, respectively, $c_M(T) = \sigma(T)^2/(M\varepsilon)$, $\sigma(T)^2$ is the variance of random variable $J(T)$.

Because $e(T)$ and $c_M(T)$ are difficult to estimate for our problem, in our implementation, the deletion rule (22) is further modified in an ad hoc way to:

$$L(Z) > \overline{U}_M(X). \quad (23)$$

The application of this ad hoc deletion rule may cause the loss of optimal solutions, but the probability of the loss should be small when we take M and T sufficiently large. We can also adaptively set the number of simulations M to reduce the probability. That is, when the upper bound of a currently obtained complete solution is compared with the lower bound of a partial solution, if the two solutions are close in the branch and bound tree, M is taken a larger value to improve the estimation accuracy of the upper bound. Otherwise, M is taken a smaller value. Because as more and more variables of a solution are given their values, the gap between the estimated lower bound and the exact cost of the solution becomes smaller and smaller. With this adaptive strategy, even if we do not obtain optimal solutions, we can find high quality near-optimal solutions.

Our algorithm enumerates (s, S) policy parameters for each of the supplier and the retailers within finite discrete sets $S \in \{S_{min}, S_{min}+1, \ldots, S_{max}-1, S_{max}\}$, $s \in \{s_{min}, s_{min}+1, \ldots, S-1, S\}$, where the bounds S_{min}, S_{max} and s_{min} are estimated based on the cost coefficients, the demand and the lead time of each facility. The enumeration starts from the supplier and then to retailer 1, 2, …, and N. For each facility, parameter S is first enumerated and then parameter s. The branching of the algorithm follows the depth-first rule.

6. NUMERICAL RESULTS

In this section, we test the performance of our approach by a set of problems with different features. 8 problems are tested. The main features of these problems are given in Table 1. For each problem, the mean value of the demand of each retailer in each period is randomly generated from uniform distribution U[20, 100], while its standard deviation is generated by the mean value multiplied by a ratio randomly generated from uniform distribution U[0.1, 0.3]. The cost coefficients of the supplier are taken as $K^s = 1000$, $c^s = 0$, $h^s = 1$, $p^s = 5$, while the cost coefficients of each retailer are taken as $K_i^r = 500$, $c_i^r = 0$, $h_i^r = 2$, $p_i^r = 10$.

Table 1: Data for tested problems

Problem Number	Number of Retailers	Lead time L	Lead time l_i
1	2	0	0
2	2	1	0
3	2	0	1
4	2	1	1
5	3	0	0
6	3	1	0
7	3	0	1
8	3	1	1

In our test, T is taken as 1000 and M is taken as 20. For each instance, the algorithm is terminated after a run of one hour in a Pentium III PC with CPU speed 700 Mb. After the termination, the cost of the best solution found is re-evaluated by 100 replications of simulation to improve its accuracy. The computational results are shown in Table 2, where C_{best} is the best cost obtained, U_{root} is the upper bound at the root node, PI is the percentage improvement of the solution obtained by our algorithm over the industrial solutions designed according to (17-20) and (21) on cost, and L_{root} is the lower bound obtained by our algorithm at the root node.

Table 2: Results for tested problems

Prob. No.	C_{best}	U_{root}	PI	L_{root}
1	1542.16	1711.83	9.91	1263.48
2	1279.46	1363.87	6.19	1109.11
3	1315.83	1406.31	6.43	1069.34
4	1130.71	1199.87	5.76	983.82
5	1989.57	2182.38	8.83	1569.42
6	1722.02	1946.38	11.53	1453.55
7	2125.16	2254.45	5.73	1716.89
8	1842.85	1970.44	6.48	1551.91

From this table, we can see that after a reasonable time of computation, our algorithm can found a solution with the cost 7.6% lower than that of the industrial solutions on average. This amount of improvement implies a lot of money savings for the supply chains in a long term run.

We also run our algorithm for several of the above instances for a long time. It shows that our algorithm takes quite a long time to terminate. One reason for this slow convergence is the stochastic nature of the algorithm. Another reason is that the cost function of the inventory planning problem is flat near its optimum as pointed by Glasserman and Tayur ([7]). That is, near the optimum, the cost does not vary much. The third reason is that our lower bound obtained at the root node is not very tight. A tighter lower bound may improve the convergence of our algorithm. However, computation time is not very critical for inventory planning since it is usually done off-line.

The long run of the algorithm for these instances also shows that after a reasonable time of computation, the upper bound will not decrease more for a long time and tend to be steady. This implies that the current best solution is near to the optimum or is located in the flat area of the cost function.

7. CONCLUSION

In this paper, a stochastic branch and bound approach is developed for find near-optimal policies for multi-echelon inventory systems. Numerical results show that the approach can obtain near optimal policies in a reasonable computation time. Further work is to provide random accuracy estimates for the approach.

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Single-Period Two-Product Inventory Model with Substitution and Information Updating

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ABSTRACT

In this paper, we study a single-period two-product inventory model with stochastic demands, downward substitution, and information updating. The downward substitution is that demands of a lower class product can be satisfied by using the stocks of products of higher class. We consider the case that the retailer could place his order twice before the demands are realized. In view of the added value of information, the retailer will continue to collect the information of demands after he places the first order, and update the initial forecast, and then place the second order to adapt the changing environment. A general profit maximization model is built to describe this case. Then we prove that the objective function of the problem is concave with respect to the order quantities of the first order. Finally, we present some computational results to show that the information can improve the retailer's profit.

1. INTRODUCTION

With the rapid development of economy, the competition among the enterprises is fiercer than before. And the field of competition has now shifted to the management of supply chains. The facts also show that a well-managed supply chain is crucial to the success of an enterprise. The inventory management is an important part of supply chain management.

In this paper, we study a single-period two-product inventory model with stochastic demands, proportional revenues and costs, full downward substitution, and information updating. The downward substitution means that demands from products of lower class can be satisfied by the stocks of products of higher class, however the products of lower class can not satisfy the demands for products of higher class. The downward substitution structure exists in real life, such as the products with higher capabilities or more functions can satisfy the demands for the product with lower capabilities or less functions. There are many examples in the semiconductor industry, the steel industry, and so on. For example, the integrated circuits with higher performance characters (e.g., speed) can substitute the integrated circuits with lower performance characters in the semiconductor industry; the higher capacity memory chips can satisfy the demands for the lower capacity memory chips in the computer manufacturing factory[1]; the steel beams with greater strength can be used to satisfy the demands for the beams with lesser strength in the steel industry[2]; the petrol with higher quality can substitute the petrol with lower quality at the petrol station.

We assume that there are two products, and product 1 has more functions or higher capacity than product 2, in other words, demands from product 2 can be satisfied by using the stocks of product 1, however demands from product 1 can not be satisfied by using the stocks of product 2. We also assume that the demands for each product are stochastic. The order, holding, penalty, and salvage costs are proportional to the quantity, and the revenue earned is also linear in the quantity sold.

In view of the added value of information, we assume that the retailer could place his order twice before the demands are realized. After placing the first order, the retailer will continue to collect the information of demands, and update the initial forecast, and then place the second order to adapt the changing environment. A general profit maximization model is built to describe this case, and the objective is to maximize the total profit by choosing the order quantity for each product when the retailer is placing the order. We prove that the objective function of the problem is concave with respect to the order quantities of the first order. Then we present some computational results to show that the information can improve the retailer's profit.

Because the retailer can use products for substitution from the higher class when there is shortage in the lower class, the sever level may be improved with substitution. So the substitution problem attracts many researchers' attention. The analysis in single-period two-product substitution problems including McGillivray and Silver[3], Pasternack and Drezner[4], Gerchak, Tripathy and Wang[5]. Within a newsboy type framework, Ernst and Kouvelis study the problem that the retailer sells products not only as independent items, but also as part of multi-product packets[6]. Their model includes two products that are not direct substitutes for each other, sold either independently or as part of a packet, and then they present the optimal conditions. Balakrishnan and Geunes address a dynamic requirements-planning problem for two-stage multi-product manufacturing systems with bill-of-materials flexibility, i.e., with options to use substitute components or subassemblies produced by an upstream stage to meet demand in each period at the downstream stage[7]. They model the problem, and describe a dynamic programming solution method to find the production and the substitution quantities that satisfy given multi-period downstream demands at minimum total setup, production, conversion, and holding costs. Computational results show that substitution can save, on average, 8.7% of manufacturing costs. Zheng studies multi-period multi-product inventory models with full downward substitution, proportional costs and revenues[8]. He develops a general profit maximization model for the problem, considering the order, holding, penalty, salvage costs, and shows that the profit function is concave and submodular with respect to the order quantities, then presents an optimal allocation policy. Bassok, Anupindi and Akkela study a single-period multi-product inventory problem with full downward substitution, proportional costs and revenues[9]. They develop a general tow-stage profit maximization model for the problem, considering the order, holding, penalty, salvage and substitution costs, and show that the profit function is concave. They also present a greedy algorithm for the allocation products to demands and show that the greedy allocation policy is optimal. Finally, they present a computational study to illustrate that the profit can be improved by using the substitution policy. Chen, Cai and Yan study a single-period two-product inventory model with stochastic demands and full downward substitution[10]. They develop a general profit maximization model for this problem, show that it is concave, and obtain the optimal condition for the order quantities. For the optimal quantities, they study the impact of the parameters, and give some interesting properties, which may be helpful for the retailer to make the decision. Then they compare the model with newsboy model, with respect to the optimal order quantities, the expected profit and the sever level, and prove that the profits and the sever level can be improved by using the substitution policy.

Because information updating and sharing are important in a supply chain, many works are concerned with the added value of information and supply contracts. Lau and Lau present a model for designing the pricing and return-credit strategy for a monopolistic manufacturer of single-period commodities[11]. Pasternack explores how to coordinate a supply chain with appropriate pricing scheme and return policy[12]. He demonstrates that coordination can be achieved by allowing the retailer to return all surplus products at a partial refund. Donohue studies the problem of developing supply contracts that encourage proper coordination of forecast information and production decisions between a manufacturer and distributor of high fashion, seasonal products with two production modes[13]. He presents the optimal conditions of the contract prices.

The rest of the paper is organized as follows. In section 2, we develop a general profit maximization model for the single-period two-product substitution problem with information updating, and prove that the objective function is concave with respect to the order quantities of the first order. In section 3, we present a number of computational results to show the added value of information. Finally, we conclude in section 4 with a summary and the directions for the future research.

2. THE MODEL

In this section, we develop a general profit maximization model for single-period two-product substitution problem with information updating. We give the optimal solution of the second stage, and then show that the profit function is concave with respect to the quantities of the first order.

2.1 Notation and Assumptions

There are two products, product 1 and product 2, and product 1 can substitute product 2. Respectively, we assume the following sequence events:

1. At the beginning of the period, instant T0, the first order is placed for each product.

2. The retailer continues to collect the information of demands, and then updates the initial forecast.

3. At instant T1(>T0), the second order is placed for each product to adapt the changing environment.

4. The order is delivered.

5. Demands for all products are realized.

6. Demands are satisfied.

7. Excess stock of each product is salvaged.

Denote demand for product i as D_i, with probability distribution function $f_i(\cdot)$ and cumulative distribution function $F_i(\cdot)$ respectively. Let $f(\cdot)$ and $F(\cdot)$ be the joint probability distribution function and the joint cumulative distribution function of demands for product 1 and product 2. And with new information, I, denote them as demand for product i as D_i^1, $f_i^1(\cdot)$, $F_i^1(\cdot)$, $f^1(\cdot)$, and $F^1(\cdot)$ after information updating. For each unit of product i, the purchase cost of the first order is c_i, but the purchase cost of the second order is w_i, the return price of the second order is b_i, the selling price is p_i, h_i is the inventory holding cost, π_i is the shortage penalty, and s_i is the salvage value for any surplus at the end of the period. Let v_i be the effective per unit salvage value of product i, i.e. $v_i = s_i - h_i$. Denote $r_i = p_i + \pi_i$. Furthermore, we make assumptions as follows.

Assumption 1: There are two products, product 1 can substitute for product 2, and the per unit selling price plus the per unit penalty cost of product 2 is not less than the per unit effective salvage value of product 1, i.e. $r_2 \ge v_1$.

Assumption 1 states that it is profitable for the retailer to satisfy the demands for product 2 using the stocks of product 1 when there is shortage in product 2.

Assumption 2: For each product, the per unit selling price is not less than the purchase cost, and the per unit purchase cost is not less than the effective salvage value, i.e. $r_i = p_i + \pi_i \ge p_i \ge c_i \ge v_i > 0$, for $i = 1, 2$.

Assumption 2 states that each product will indeed be used to supply demand for that product, instead of being held as inventory and exchanged for salvage value, and there is incentive for placing orders.

Assumption 3: The per unit selling price, penalty cost, purchase cost and effective salvage value of product 1 are not less than that of product 2, i.e. $p_1 \ge p_2$, $\pi_1 \ge \pi_2$, $c_1 \ge c_2$, $v_1 \ge v_2$.

Assumption 3 states that it is more profitable to satisfy unmet demand of product 1 than of product 2, and it is not optimal to substitute product 1 for product 2 whenever there is excess inventory of product 2.

Assumption 4: if a unit of product 1 supplies the demand for product 2, the price charged is p_2 (instead of p_1).

Assumption 5: For each product, the per unit purchase cost of the second order is greater than that of the first order, and both of them are greater than the return price, i.e. $w_1 \ge c_1 \ge b_1$, $w_2 \ge c_2 \ge b_2$.

Assumption 5 ensures that the retailer will order proper quantities in his first order.

Assumption 6: The per unit purchase cost of the second order, the return price of product 1 are not less than that of product 2, and the return price of product 1 is greater than the purchase cost of product 2 of the second order, i.e. $w_1 \ge w_2$, $b_1 \ge b_2$, $b_1 > w_2$.

Assumption 5 ensures that the retailer will order proper quantities in his second order.

2.2 The Profit Function

Let $P(Q_1,Q_2)$ be the expected profits when the order quantity of product i in the first order is Q_i. From the assumptions, we can observe that the retailer will always supply demand for product i using on-hand product i units as much as possible, and always supply the unmet demand for product 2 using the excess inventory of product 1. So the problem can be expressed as the following maximization problem, a two-stage dynamic programming model:

$$\max_{Q_1,Q_2} P(Q_1,Q_2) = -c_1 Q_1 - c_2 Q_2 + E_I\left[P_1(Q_1,Q_2,I)\right] \tag{1}$$

$$P_1(Q_1,Q_2,I) = \max_{\overline{Q}_1(I),\overline{Q}_2(I)} P_2(Q_1,Q_2,\overline{Q}_1(I),\overline{Q}_2(I),I) \tag{2}$$

$$\begin{aligned}
&P_2(Q_1,Q_2,\overline{Q}_1(I),\overline{Q}_2(I),I) = \\
&\quad - w_1\left[\overline{Q}_1(I)-Q_1\right]^+ - w_2\left[\overline{Q}_2(I)-Q_2\right]^+ \\
&\quad + b_1\left[Q_1-\overline{Q}_1(I)\right]^+ + b_2\left[Q_2-\overline{Q}_2(I)\right]^+ \\
&\quad + p_1\left[\int_{-\infty}^{\overline{Q}_1(I)} x f_1^I(x)dx + \int_{\overline{Q}_1(I)}^{\infty} \overline{Q}_1(I) f_1^I(x)dx\right] \\
&\quad + p_2\left[\int_{-\infty}^{\overline{Q}_1(I)} \int_{-\infty}^{\overline{Q}_1(I)+\overline{Q}_2(I)-x} y f^I(x,y)dydx\right. \\
&\quad + \int_{-\infty}^{\overline{Q}_1(I)} \int_{\overline{Q}_1(I)+\overline{Q}_2(I)-x}^{\infty} \left(\overline{Q}_1(I)+\overline{Q}_2(I)-x\right) f^I(x,y)dydx \\
&\left. + \int_{\overline{Q}_1(I)}^{\infty} \left(\int_{-\infty}^{\overline{Q}_2(I)} y f^I(x,y)dy + \int_{\overline{Q}_2(I)}^{\infty} \overline{Q}_2(I) f^I(x,y)dy\right)dx\right] \\
&+ v_1\left[\int_{-\infty}^{\overline{Q}_2(I)} \int_{-\infty}^{\overline{Q}_1(I)-x} \left(\overline{Q}_1(I)-x\right) f^I(x,y)dxdy\right. \\
&\left. + \int_{\overline{Q}_2(I)}^{\infty} \int_{-\infty}^{\overline{Q}_1(I)+\overline{Q}_2(I)-y} \left(\overline{Q}_1(I)+\overline{Q}_2(I)-x-y\right) f^I(x,y)dxdy\right] \\
&+ v_2 \int_{-\infty}^{\overline{Q}_2(I)} \left(\overline{Q}_2(I)-y\right) f_2^I(y)dy \\
&- \pi_1 \int_{\overline{Q}_1(I)}^{\infty} \left(x-\overline{Q}_1(I)\right) f_1^I(x)dx \\
&- \pi_2\left[\int_{-\infty}^{\overline{Q}_1(I)} \int_{\overline{Q}_1(I)+\overline{Q}_2(I)-x}^{\infty} \left(x+y-\overline{Q}_1(I)-\overline{Q}_2(I)\right) f^I(x,y)dydx\right. \\
&\left. + \int_{\overline{Q}_1(I)}^{\infty} \int_{\overline{Q}_2(I)}^{\infty} \left(y-\overline{Q}_2(I)\right) f^I(x,y)dydx\right]
\end{aligned} \tag{3}$$

Where $\overline{Q}_1(I)$ and $\overline{Q}_2(I)$ are the final order quantity of product 1 and product 2 in the second order with the new information I. $P_2(Q_1,Q_2,\overline{Q}_1(I),\overline{Q}_2(I),I)$ is the expected profits in the second stage with the order quantity of product 1 and product 2 in the first order Q_1,Q_2, and $\overline{Q}_1(I),\overline{Q}_2(I)$ in the second order, and new information I. $P_1(Q_1,Q_2,I)$ is the optimal expected profits in the second stage with the order quantity of product 1 and product 2 in the first order Q_1,Q_2 and new information I.

The first and the second term in equation (3) are the purchase costs in the second stage. The third and fourth term are the revenue from returning products to the supplier. The fifth and sixth term are the revenue from supplying the demands, both directly and using substitution. The seventh and eighth term are the net salvage value for excess inventory. The last two terms are the penalty costs for shortage.

2.3 The Solution of the Second Stage

In order to solve the problem, we rewrite equation (3) as follows:

$$
\begin{aligned}
P_2(Q_1,Q_2,\overline{Q}_1(I),\overline{Q}_2(I),I) = &\\
& -w_1[\overline{Q}_1(I)-Q_1]^+ - w_2[\overline{Q}_2(I)-Q_2]^+\\
& +b_1[Q_1-\overline{Q}_1(I)]^+ + b_2[Q_2-\overline{Q}_2(I)]^+\\
& +c_1\overline{Q}_1(I)+c_2\overline{Q}_2(I)\\
& +P_3(\overline{Q}_1(I),\overline{Q}_2(I),I) \qquad (4)
\end{aligned}
$$

Where $P_3(\overline{Q}_1(I),\overline{Q}_2(I),I)$ is the terms in equation (3), except the first four terms, minus $c_1\overline{Q}_1(I)$ and $c_2\overline{Q}_2(I)$.

It is obvious that $P_3(\overline{Q}_1(I),\overline{Q}_2(I),I)$ is independent of Q_1,Q_2. It can be proved that $P_3(\overline{Q}_1(I),\overline{Q}_2(I),I)$ is concave with respect to $\overline{Q}_1(I),\overline{Q}_2(I)$ [10]. With the assumptions $w_1 \ge c_1 \ge b_1$ and $w_2 \ge c_2 \ge b_2$, it can be shown that $P_2(Q_1,Q_2,\overline{Q}_1(I),\overline{Q}_2(I),I)$ is concave with respect to $\overline{Q}_1(I),\overline{Q}_2(I)$, too. So it can be solved by the traditional methods.

Denote $G^I(Q_1,Q_2)=\int_{-\infty}^{Q_1}\int_{-\infty}^{Q_1+Q_2-x} f^I(x,y)dydx$, the joint serve level, then the optimal conditions can be given as follows:

$$
F_1^I(\overline{Q}_1^*(I))+\frac{r_2-v_1}{r_1-r_2}G^I(\overline{Q}_1^*(I),\overline{Q}_2^*(I)) = \begin{cases} \dfrac{r_1-b_1}{r_1-r_2}, & \text{if } Q_1^* < Q_1 \\ \dfrac{r_1-w_1}{r_1-r_2}, & \text{if } Q_1^* > Q_1 \end{cases} \qquad (5)
$$

$$
F_2^I(\overline{Q}_2^*(I))+\frac{r_2-v_1}{r_2-v_2}\left[G^I(\overline{Q}_1^*(I),\overline{Q}_2^*(I))-F^I(\overline{Q}_1^*(I),\overline{Q}_2^*(I))\right] = \begin{cases} \dfrac{r_2-b_2}{r_2-v_2}, & \text{if } Q_2^* < Q_2 \\ \dfrac{r_2-w_2}{r_2-v_2}, & \text{if } Q_2^* > Q_2 \end{cases} \qquad (6)
$$

The optimal solution of the second stage is shown in figure 1.

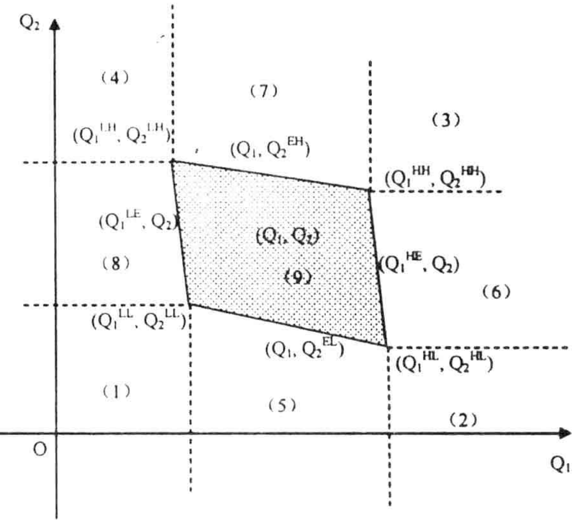

Figure 1 The Solution of the Second Stage

For example, if (Q_1,Q_2) is in the region (4), it means that the order quantity of product 1 in the first order is small, and the order quantity of product 2 in the first order is excessive. Then in the second order, the retailer will increase the order quantity of product 1, and return some of product 2 to the supplier. In this case, the final order quantity of product 1 and product 2, $\overline{Q}_1^{LH}(I)$ and $\overline{Q}_2^{LH}(I)$, satisfies the following condition:

$$
F_1^I(Q_1^*(I))+\frac{r_2-v_1}{r_1-r_2}G^I(Q_1^*(I),Q_2^*(I))=\frac{r_1-w_1}{r_1-r_2}
$$

$$
F_2^I(Q_2^*(I))+\frac{r_2-v_1}{r_2-v_2}\left[G^I(Q_1^*(I),Q_2^*(I)) - F^I(Q_1^*(I),Q_2^*(I))\right]=\frac{r_2-b_2}{r_2-v_2} \qquad (7)
$$

2.4 The Property of the profit function

After obtaining the solution of the second stage, we now can get the form of function $P(Q_1,Q_2)$ in detail. And it can be shown that the retailer's profit, $P(Q_1,Q_2)$, is concave respect to Q_1,Q_2, by check the first partial derivative of the function. So we can obtain the optimal order quantities in the first order for the retailer by the traditional optimization methods.

3. COMPURATIONAL STUDY

The main purpose of this computational study is to demonstrate the added value of new information. Denote PI as the retailer's optimal expected profit with information updating, and PW as the retailer's optimal expected profit without information updating. We will compare PI with PW, and define the percentage gain as:

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