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Waves and Structures in Nonlinear Nondispersive Media

General Theory and Applications to Nonlinear Acoustics

非线性非分散介质中的波与结构

非线性声学的一般理论及应用

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Preface

The book is aimed at natural science undergraduates, as well as at graduate and post-graduate students studying the theory of nonlinear waves of various physical nature. It may also be useful as a handbook for engineers and researchers who encounter the necessity of taking nonlinear wave effects into account in their work.

Evolution of sufficiently intense waves is determined by nonlinear processes, in which the progress is substantially influenced by dispersion (a dependence of the phase velocity on its frequency). Media without dispersion, where the phase velocity does not depend on the frequency, are the simplest ones with respect to their physical properties and are the most common in nature. But nonlinear interactions of the Fourier spectral components in such media are particularly complex and diverse. Here, practically all "virtual" energy-exchange processes between waves of different frequencies become resonant ones and occur with a high efficiency. An avalanche-like increase of the number of spectral components of the field takes place, which, within the space-time representation, corresponds to formation of structures with strongly pronounced nonlinear properties. Examples of such structures are discontinuities of a function describing the wave field or discontinuities of its derivative, steep shock fronts of various types and multidimensional cellular structures.

Nonlinear structures can be stable only in strong fields, under the conditions of competition with effects of absorption, dispersion, etc, which contribute to the decay of such structures. These objects have properties of quasiparticles. For instance, shock fronts undergo inelastic collisions. Thus, in nondispersive media, nonlinearity provides both a possibility of interactions between stable structures and their very existence. Solitons are other well-known objects in nonlinear physics, which are, generally speaking, stable only in idealized conservative systems. At the same time, quasi stability of shock-front structures or sawtooth waves occurs in real dissipative systems.

Structures of different physical nature are described by similar mathematical models. These models are used not only in the wave theory, but also to describe various non-wave objects, *viz.*: forest-fire fronts, density of a flow of non-interacting particles, etc. Because of the universality of such nonlinear models, it is necessary to analyze them on the basis of general principles of mathematical physics, irrespective of the nature of the described phenomena.

On the other hand, nondispersive waves and structures are widely used in science and technology. A review of these applications, from the authors' viewpoint, is what "brightens up" the theory and may be of interest to many readers.

The theory of nonlinear waves and structures is a very extensive and constant developing field of physics (especially radiophysics and mathematical physics). It has many specific applications. Among them there are both the well-known problems of acoustics, electrodynamics and plasma physics (see, e.g., [1–5]), and the less-known problems, such as surface-growth description [6, 7], dynamics of turbulence [8, 9] and development of a gravitational instability of the large-scale distribution of matter in the Universe [10–14]. A wide range of phenomena arising here have led to the development of a variety of mathematical methods, which are effective in addressing various kinds of nonlinear fields and waves (see, e.g., [15–17]). It is clear that within a single monograph, it is not possible to give an exhaustively comprehensive overview of the whole problem. For this reason, the authors limited themselves to a discussion of the "hydrodynamic" type of nonlinear waves in nondispersive medium. First of all, the properties of solutions to such standard nonlinear wave equations in nondispersive media as the simple wave equation, the Burgers equation and the Kardar-Parisi-Zhang equation have been studied in detail. Apart from the importance of these equations for the theory and applications, an analysis of these solutions allows us to trace stages of development of typical nonlinear processes and, above all, nonlinear distortion of profiles, the gradient catastrophe and emergence of shock waves. In order for the theory of nonlinear waves in nondispersive media not to look too abstract, the presentation is based on illustrative geometric interpretations of both the equations themselves and their solutions, as well as on a comprehensive discussion of the physical meaning of these solutions and the methods used to obtain them.

The monograph consists of two parts. The first part is devoted to a detailed description of the concepts and analysis methods of nonlinear waves and structures in nondispersive media. The second part focuses on an in-depth description of the nonlinear theory as applied only to one type of waves — high intensity acoustic waves. This object, on the one hand, is the most straightforward and, on the other hand, has important practical applications.

The authors have attempted to communicate all materialls at the following "two levels" of complexity. The first level is intended to introduce beginning investigators (above all undergraduate, graduate and PhD students) to the concepts and methods of the theory of nonlinear waves and structures in nondispersive media. In order to achieve a deeper understanding of the foundations, it is useful to solve the problems given in the end of the chapters in Part I. The second, higher, level is meant for researchers, who already have experience in this field of study and are interested in the state of the art or in specific results. Naturally, it is impossible to reflect the entire diversity of approaches used to study nonlinear fields and waves in a single monograph. This is why the material is presented at a simple, "physical" level of rigor, where possible. Those, who are interested in a more rigorous mathematical foundation of the problems discussed here, are advised to turn to monographs [15, 17], where mathematical foundations of many topics touched upon in this book are thoroughly discussed. An in-depth review of the methods used to solve nonlinear problems, along with profound results of the nonlinear field theory, can be found in book [16]. In monograph [18], and also in textbook [19], the theory of generalized functions necessary for construction of generalized solutions of nonlinear equations is comprehensively elucidated. We recommend those who intend deeper to delve into the nonlinear field theory, without burying themselves in mathematical subtleties, the following thorough monographs and textbooks: [1, 2, 4, 5], which are written by physicists for physicists. Basic concepts of the nonlinear wave theory, along with illustrative physical examples, can be found in the remarkable textbook [14]. To those who are going professionally to engage themselves in the field of nonlinear acoustics, we recommend monograph [3] and the books of problems [20, 21], where a set of problems aiding in mastering various aspects of nonlinear acoustics is given. If one is interested in statistical properties of nonlinear random waves as applied to nonlinear acoustics, astrophysics and turbulence, he or she can pick up necessary information from monograph [10]. We also advise to turn to monograph [8], which covers the foundations of the theory of strong turbulence and its inherent phenomena, such as intermittency and multifractality.

We are grateful to the renowned scientists, fruitful interactions with whom over the years have formed our vision of the problems and methods of the nonlinear science. First of all, they are: academicians A.V. Gaponov-Grekhov, Ya.B. Zeldovich, R.V. Khokhlov, V.I. Arnold and Ya.G. Sinai; corresponding members of the Russian Academy of Sciences M.I. Rabinovich and D.I. Trubetskov; Professors A.N. Malakhov, L.A. Ostrovsky, S.A. Rybak, S.I. Soluyan, A.P. Sukhorukov, A.S. Chirkin and S.F. Shandarin. We are delighted to remember the years of collaboration with international colleagues, among whom are: D. Crighton, U. Frisch, B. Enflo, D. Blackstock, M. Hamilton, L. Cram, E. Aurell, A. Noullez, W.A. Woyczynski and many others.

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Contents

Part I
Foundations of the Theory of Waves in Nondispersive Media

Chapter 1
Nonlinear Equations of the First Order

The basic patterns of nonlinear fields and waves of the hydrodynamic type already can be discerned by the behavior of solutions to the simplest nonlinear partial differential equations of the first order. This chapter discusses solutions of such equations. Those wishing to study the theory of the first-order nonlinear equations more fully are advised to turn to the following literature: [1–4].

1.1 Simple wave equation

The simplest and, at the same time, crucial equation of the nonlinear wave theory of the hydrodynamic type is the *simple wave equation*. In what follows, we will pay tribute to the remarkable mathematician Riemann, who laid the foundations of the nonlinear wave theory, and call this equation the *Riemann equation*. In mathematical literature, this equation is often called the Hopf equation. By using the equation as an example it is most instructive to explain such typically nonlinear effects as the wave steepening and gradient catastrophe.

1.1.1 The canonical form of the equation

The *simple wave* (*Riemann*) *equation* is the following first order partial differential equation:

$$\frac{\partial u}{\partial t} + C(u)\frac{\partial u}{\partial x} = 0 \tag{1.1}$$

with respect to the function $u(x,t)$ which has different geometric, mechanical, economic, etc. meanings in different applications.

By multiplying Eq. (1.1) by $C'(u)$, it is reduced to the equivalent, but simpler in form, canonical Riemann equation:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = 0 \tag{1.2}$$

with respect to the new function $v(x,t) = C(u(x,t))$. Thus, without loss of generality, in what follows, we will limit ourselves to a detailed analysis of the Riemann equation (1.2) with an initial condition $v(x,t=0) = v_0(x)$. The following instructive mechanical interpretation of solutions to the Riemann equation helps better familiarize oneself with peculiarities of solutions to this equation.

1.1.2 Particle flow

The easiest way to comprehend properties of solutions to the Riemann equation is by using a flow of particles uniformly moving along the x-axis as an example. Let a particle at the point y at the initial moment of time $t = 0$ have the velocity $v_0(y)$. Then the particle's motion is given by the following equations:

$$X(y,t) = y + v_0(y)t, \quad V(y,t) = v_0(y). \tag{1.3}$$

By varying y, we obtain the laws of motion of other particles in the flow. Note that apart from the time t, another argument y, the initial particle position, appears here. Such coordinates, which are rigidly bound to the particles of a flow, are called *Lagrangian coordinates* (a pictorial comparative discussion of flow descriptions in the Lagrangian and Eulerian coordinate systems is given in textbook [4]).

Usually, an observer measures the velocity of a flow at some fixed position with a Cartesian coordinate x. These, more natural for an external observer, coordinates are called *Eulerian*. The mapping from the Lagrangian into Eulerian coordinates is described by the following equation:

$$x = X(y,t). \tag{1.4}$$

In the case of uniformly moving particles, this equation has the following form:

$$x = y + v_0(y)t. \tag{1.5}$$

Let the field $v(x,t)$ of particle velocities in a flow be given as a function of the Eulerian coordinate x and time t. If, in addition to that, the mapping (1.4) of the Lagrangian to Eulerian coordinates is also known, then the dependence of the velocity field on the Lagrangian coordinates is given by the following equation:

$$V(y,t) = v(X(y,t),t). \tag{1.6}$$

In what follows, the fields describing the behavior of particles in the Lagrangian coordinate system will be called the Lagrangian fields, and the fields in the Eulerian coordinate system will be referred to as the Eulerian fields. So $v(x,t)$ is the Eulerian particle-velocity field, and $X(y,t)$ is the Lagrangian field of the Eulerian coordinates of the particles.

From the uniformity of particle motion follows that the velocity $V(y,t)$ of a particle with the Lagrangian coordinate y does not depend on time, i.e. it satisfies the following simplest differential equation:

$$\frac{dV}{dt} = 0, \tag{1.7}$$

and its coordinate obeys a no less obvious equation:

$$\frac{dX}{dt} = V. \tag{1.8}$$

Equations (1.7) and (1.8) are nothing else than *characteristic equations* for the first order partial differential equation (1.2). In order to reconstruct the solution of the Riemann equation from the solutions of the characteristic equation (1.7), (1.8), it is sufficient to find the inverse of function (1.4)

$$y = y(x,t),$$

which maps the Eulerian coordinates to the Lagrangian ones. If this function is known, then, with provision for (1.3) and (1.6), the solution of the Riemann equations takes on the following form:

$$v(x,t) = V(y(x,t),t) = v_0(y(x,t)). \tag{1.9}$$

Let us emphasize that the single-valued inverse function $y(x,t)$ exists, and Eq. (1.9) gives the classical Riemann solution of Eq. (1.2), only if the mapping from the Lagrangian coordinates to the Eulerian ones (1.4), (1.5) is a monotonically increasing function y from $\mathbb{R}$ onto $\mathbb{R}$. In the following chapter we will discuss in detail what happens if this condition is violated. At the moment, let us assume that it is satisfied.

1.1.3 Discussion of the Riemann solution

Let us discuss the characteristic peculiarities of the behavior of the Riemann solution $v(x,t)$ as a function of the x-coordinate and time t. But, before doing that, let us list the main forms of notation for solutions of the Riemann equation. By substituting $y(x,t)$ for y in the equation of uniform motion of a particle (1.5)

$$y(x,t) = x - v_0(y(x,t))t \Rightarrow y(x,t) = x - v(x,t)t \tag{1.10}$$

and by inserting the right-hand side of this expression into Eq. (1.9), we obtain the implicit form of the Riemann solution:

$$v(x,t) = v_0(x - v(x,t)t). \tag{1.11}$$