Dynamics of Markets

Econophysics and Finance

Joseph L. McCauley

市场动力学 经济物理学和金融

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Mainly for my stimulating partner
Cornelia,
who worked very hard and effectively helping me to improve this text,
but also for our youngest son,
Finn.

Preface

This book emphasizes what standard texts and research in economics and finance ignore: that there is as yet no evidence from the analysis of real, unmassaged market data to support the notion of Adam Smith's stabilizing Invisible Hand. There is no empirical evidence for stable equilibrium, for a stabilizing hand to provide self-regulation of unregulated markets. This is in stark contrast with the standard model taught in typical economics texts (Mankiw, 2000; Barro, 1997), which forms the basis for the positions of the US Treasury, the European Union, the World Bank, and the IMF, who take the standard theory as their credo (Stiglitz, 2002). Our central thrust is to introduce a new empirically based model of financial market dynamics that prices options correctly and also makes clear the instability of financial markets. Our emphasis is on understanding how markets really behave, not how they hypothetically "should" behave as predicted by completely unrealistic models.

By analyzing financial market data we will develop a new model of the dynamics of market returns with nontrivial volatility. The model allows us to value options in agreement with traders' prices. The concentration is on financial markets because that is where one finds the very best data for a careful empirical analysis. We will also suggest how to analyze other economic price data to find evidence for or against Adam Smith's Invisible Hand. That is, we will explain that the idea of the Invisible Hand is falsifiable. That method is described at the end of Sections 4.9 and 7.5.

Standard economic theory and standard finance theory have entirely different origins and show very little, if any, theoretical overlap. The former, with no empirical basis for its postulates, is based on the idea of equilibrium, whereas finance theory is motivated by, and deals from the start with, empirical data and modeling via nonequilibrium stochastic dynamics.

However, mathematicians teach standard finance theory as if it would be merely a subset of the abstract theory of stochastic processes (Neftci, 2000). There, lognormal pricing of assets combined with "implied volatility" is taken as the standard model.

The "no-arbitrage" condition is regarded as the foundation of modern finance theory and is sometimes even confused with the idea of Adam Smith's Invisible Hand (Nakamura, 2000). Instead of following the finance theorists and beginning with mathematical theorems about "no-arbitrage," we will use the empirically observed market distribution to deduce a new dynamical model. We do not need the idea of "implied volatility" that is required when using the lognormal distribution, because we will deduce the empirical volatility from the observed market distribution. And, if a market perfectly satisfies a no-arbitrage condition, so is it, and if not, then so is it as well. We ask what markets are doing empirically, not what they would do were they to follow our wishes expressed as mathematically convenient model assumptions. In other words, we present a physicist's approach to economics and finance, one that is completely uncolored by any belief in the ideology of neo-classical economic theory or by pretty mathematical theorems about Martingales. One strength of our empirically based approach is that it exposes neo-classical expectations of stability as falsified, and therefore as a false basis for advising the world in financial matters.

But before we enter the realm of economics and finance, we first need to emphasize the difference of socio-economic phenomena with natural phenomena (physics, chemistry, cell biology) by bringing to light the underlying basis for the discovery of mathematical laws of nature. The reader finds this presented in Chapter 1 where we follow Wigner and discuss invariance principles as the fundamental building blocks necessary for the discovery of physical law.

Taking the next step, we review the globally dominant economic theory critically. This constitutes Chapter 2. We show that the neo-classical microeconomic theory is falsified by agents' choices. We then scrutinize briefly the advanced and very impressive mathematical work by Sonnenschein (1973a, b), Radner (1968), and Kirman (1989) in neo-classical economics. Our discussion emphasizes Sonnenschein's inadequately advertised result that shows that there is no macroeconomic theory of markets based on utility maximization (Keen, 2001). The calculations made by Radner and Kirman show that equilibrium cannot be located by agents, and that liquidity/money and therefore financial markets can not appear in the neo-classical theory.

Next, in Chapter 3, we introduce probability and stochastic processes from a physicist's standpoint, presenting Fokker–Planck equations and Green functions for diffusive processes parallel to Ito calculus. Green functions are later used to formulate market dynamics and option pricing.

With these tools in hand we proceed to Chapter 4 where we introduce and discuss the standard notions of finance theory, including the Nobel Prize winning Modigliani–Miller argument, which says that the amount of debt doesn't matter. The most important topic in this chapter is the analysis of the instability and lack of equilibrium of financial markets, based on the example provided by the standard lognormal pricing model. We bring to light the reigning confusion in economics over the notion of equilibrium, and then go on to present an entirely new interpretation of Black's idea of value. We also explain why an assumption of microscopic randomness cannot, in and of itself, lead to universality of macroscopic economic rules.

Chapter 5 presents standard portfolio selection theory, including a detailed analysis of the capital asset pricing model (CAPM) and an introduction to option pricing based on empirical averages. Synthetic options are also defined. We present and discuss the last part of the very beautiful Black–Scholes paper that explains how one can understand bondholders (debt owners) as the owners of a firm, while stockholders merely have options on the company's assets. Finally, for the first time in the literature, we show why Black and Scholes were wrong in claiming in their original path finding 1973 paper that the CAPM and the delta hedge yield the same option price partial differential equation. We show how to solve the Black–Scholes equation easily by using the Green function, and then end the chapter by discussing Enron, an example where the ratio of debt to equity did matter.

The main contribution of this book to finance theory is our (Gunaratne and McCauley) empirically based theory of market dynamics, volatility and option pricing. This forms the core of Chapter 6, where the exponential distribution plays the key role. The main idea is that an (x, t)-dependent diffusion coefficient is required to generate the empirical returns distribution. This automatically explains why volatility is a random variable but one that is perfectly correlated with returns x. This model is not merely an incremental improvement on any existing model, but is completely new and constitutes a major improvement on Black–Scholes theory. Nonuniqueness in extracting stochastic dynamics from empirical data is faced and discussed. We also show that the "risk neutral" option pricing partial differential equation is simply the backward Kolmogorov equation corresponding to the Fokker–Planck equation describing the data. That is, all information required for option pricing is included in the Green function of the market Fokker–Planck equation. Finally, we show how to price options using stretched exponential densities.

In Chapter 7 we discuss liquidity, reversible trading, and replicating, self-financing hedges. Then follows a thermodynamic analogy that leads us back to a topic introduced in Chapter 4, the instability of financial markets. We explain in this chapter why empirically valid thermodynamic analogies cannot be achieved in economic modeling, and suggest an empirical test to determine whether any market can be found that shows evidence for Adam Smith's stabilizing Invisible Hand.

In Chapter 8, after introducing affine scaling, we discuss the efficient market hypothesis (EMH) in light of fractional Brownian motion, using Ito calculus to formulate the latter. We use Kolmogorov's 1962 lognormal model of turbulence to show how one can analyze the question: do financial data show evidence for an information cascade? In concluding, we discuss Levy distributions and then discuss the results of financial data analyses by five different groups of econophysicists.

We end the book with a survey of various ideas of complexity in Chapter 9. The chapter is based on ideas from nonlinear dynamics and computability theory. We cover qualitatively and only very briefly the difficult unanswered question whether biology might eventually provide a working mathematical model for economic behavior.

For those readers who are not trained in advanced mathematics but want an overview of our econophysics viewpoint in financial market theory, here is a recommended "survival guide": the nonmathematical reader should try to follow the line of the argumentation in Chapters 1, 2, 4, 5, 7, and 9 by ignoring most of the equations. Selectively reading those chapters may provide a reasonable understanding of the main issues in this field. For a deeper, more critical understanding the reader can't avoid the introduction to stochastic calculus given in Chapter 3. For those with adequate mathematical background, interested only in the bare bones of finance theory, Chapters 3–6 are recommended. Those chapters, which form the core of finance theory, can be read independently of the rest of the book and can be supplemented with the discussions of scaling, correlations and fair games in Chapter 8 if the reader is interested in a deeper understanding of the basic ideas of econophysics. Chapters 6, 7 and 8 are based on the mathematics of stochastic processes developed in Chapter 3 and cannot be understood without that basis. Chapter 9 discusses complexity qualitatively from the perspective of Turing's idea of computability and von Neumann's consequent ideas of automata and, like Chapters 1 and 2, does not depend at all on Chapter 3. Although Chapter 9 contains no equations, it relies on very advanced ideas from computability theory and nonlinear dynamics.

I teach most of the content of Chapters 2–8 at a comfortable pace in a one-semester course for second year graduate students in physics at the University of Houston. As homework one can either assign the students to work through the derivations, assign a project, or both. A project might involve working through a theoretical paper like the one by Kirman, or analyzing economic data on agricultural commodities (Roehner, 2001). The goal in the latter case is to find nonfinancial economic data that are good enough to permit unambiguous conclusions to be drawn. The main idea is to plot histograms for different times to try to learn the time evolution of price statistics.

As useful background for a graduate course using this book, the students have preferably already had courses in statistical mechanics, classical mechanics or nonlinear dynamics (primarily for Chapter 2), and mathematical methods. Prior background in economic theory was neither required nor seen as useful, but the students are advised to read Bodie and Merton's introductory level finance text to learn the main terminology in that field.

I'm very grateful to my friend and colleague Gemunu Gunaratne, without whom there would be no Chapter 6 and no new model of market dynamics and option pricing. That work was done together during 2001 and 2002, partly while I was teaching econophysics during two fall semesters and also via email while I was in Austria. Gemunu's original unpublished work on the discovery of the empirical distribution and consequent option pricing are presented with slight variation in Section 6.1.2. My contribution to that section is the discovery that γ and ν must blow up at expiration in order to reproduce the correct forward-time initial condition at expiration of the option. Gemunu's pioneering empirical work was done around 1990 while working for a year at Tradelink Corp. Next, I am enormously indebted to my life-partner, hiking companion and wife, former newspaper editor Cornelia Küffner, for critically reading this Preface and all chapters, and suggesting vast improvements in the presentation. Cornelia followed the logic of my arguments, made comments and asked me penetrating and crucial questions, and my answers to her questions are by and large written into the text, making the presentation much more complete. To the extent that the text succeeds in getting the ideas across to the reader, then you have her to thank. My editor, Simon Capelin, has always been supportive and encouraging since we first made contact with each other around 1990. Simon, in the best tradition of English respect and tolerance for nonmainstream ideas, encouraged the development of this book, last but not least over a lively and very pleasant dinner together in Messina in December, 2001, where we celebrated Gene Stanley's 60th birthday. Larry Pinsky, Physics Department Chairman at the University of Houston, has been totally supportive of my work in econophysics, has financed my travel to many conferences and also has created, with the aid of the local econophysics/complexity group, a new econophysics option in the graduate program at our university. I have benefited greatly from discussions, support, and also criticism from many colleagues, especially my good friend and colleague Yi-Cheng Zhang, who drew me into this new field by asking me first to write book reviews and then articles for the econophysics web site www.unifr.ch/econophysics. I'm also very much indebted to Gene Stanley, who has made *Physica A* the primary econophysics journal, and has thereby encouraged work in this new field. I've learned from Doyne Farmer, Harry Thomas (who made me realize that I had to learn Ito calculus), Cris Moore, Johannes Skjeltorp, Joseph Hrgovcic, Kevin Bassler, George Reiter, Michel Dacorogna, Joachim Peinke, Paul Ormerod, Giovanni Dosi, Lei-Han Tang, Giulio Bottazzi, Angelo Secchi, and an anonymous former Enron employee (Chapter 5). Last but far from least, my old friend Arne Skjeltorp, the father of the theoretical economist Johannes Skjeltorp, has long been a strong source of support and encouragement for my work and life.

I end the Preface by explaining why Erwin Schrödinger's face decorates the cover of this book. Schrödinger was the first physicist to inspire others, with his Cambridge (1944) book *What is Life?*, to apply the methods of physics to a science beyond physics. He encouraged physicists to study the chromosome molecules/fibers that carry the "code-script." In fact, Schrödinger's phrase "code-script" is the origin of the phrase "genetic code." He attributed the discrete jumps called mutations to quantum jumps in chemical bonding. He also suggested that the stability of rules of heredity, in the absence of a large N limit that would be necessary for any macroscopic biological laws, must be due to the stability of the chromosome molecules (which he called linear "aperiodic crystals") formed via chemical bonding à la Heitler–London theory. He asserted that the code-script carries the complete set of instructions and mechanism required to generate any organism via cellular replication, and this is, as he had guessed without using the term, where the "complexity" lies. In fact, *What is Life?* was written parallel to (and independent of) Turing's and von Neumann's development of our first ideas of complexity. Now, the study of complexity includes economics and finance. As in Schrödinger's day, a new fertile research frontier has opened up.

Joe McCauley
Ehrwald (Tirol)
April 9, 2003

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The moving target

1.1 Invariance principles and laws of nature

The world is complicated and physics has made it appear relatively simple. Everything that we study in physics is reduced to a mathematical law of nature. At very small distances nature is governed by relativistic quantum field theory. At very large distances, for phenomena where both light speed and gravity matter, we have general relativity. In between, where neither atomic scale phenomena nor light speed matter, we have Newtonian mechanics. We have a law to understand and explain everything, at least qualitatively, except phenomena involving decisions made by minds. Our success in discovering that nature behaves mathematically has led to what a famous economist has described as "the Tarzan complex," meaning that physicists are bold enough to break into fields beyond the natural sciences, beyond the safe realm of mathematical laws of nature. Where did our interest in economics and finance come from?

From my own perspective, it started with the explosion of interest in nonlinear dynamics and chaos in the 1980s. Many years of work in that field formed the perspective put forth in this book. It even colors the way that I look at stochastic dynamics. From our experience in nonlinear dynamics we know that our simple looking local equations of motion can generate chaotic and even computationally complex solutions. In the latter case the digitized dynamical system is the computer and the digitized initial condition is the program. With the corresponding explosion of interest in "complexity," both in dynamical systems theory and statistical physics, physicists are attempting to compete with economists in understanding and explaining economic phenomena, both theoretically and computationally. Econophysics – is it only a new word, a new fad? Will it persist, or is it just a desperate attempt by fundless physicists to go into business, to work where the "real money" is found? We will try to demonstrate in this text that econophysicists can indeed contribute to economic thinking, both critically and creatively. First, it is important to have a clear picture of just how and why theoretical physics differs from economic theorizing.

Eugene Wigner, one of the greatest physicists of the twentieth century and the acknowledged expert in symmetry principles, thought most clearly about these matters. He asked himself: why are we able to discover mathematical laws of nature at all? An historic example points to the answer. In order to combat the prevailing Aristotelian ideas, Galileo Galilei proposed an experiment to show that relative motion doesn't matter. Motivated by the Copernican idea, his aim was to explain why, if the earth moves, we don't feel the motion. His proposed experiment: drop a ball from the mast of a uniformly moving ship on a smooth sea. It will, he asserted, fall parallel to the mast just as if the ship were at rest. Galileo's starting point for discovering physics was therefore the principle of relativity. Galileo's famous thought experiment would have made no sense were the earth not a local inertial frame for times on the order of seconds or minutes.[1] Nor would it have made sense if initial conditions like absolute position and absolute time mattered.

The known mathematical laws of nature, the laws of physics, do not change on any time scale that we can observe. Nature obeys inviolable mathematical laws only because those laws are grounded in local invariance principles, local invariance with respect to frames moving at constant velocity (principle of relativity), local translational invariance, local rotational invariance and local time-translational invariance. These local invariances are the same whether we discuss Newtonian mechanics, general relativity or quantum mechanics. Were it not for these underlying invariance principles it would have been impossible to discover mathematical laws of nature in the first place (Wigner, 1967). Why is this? Because the local invariances form the theoretical basis for repeatable identical experiments whose results can be reproduced by different observers independently of where and at what time the observations are made, and independently of the state of relative motion of the observational machinery. In physics, therefore, we do not have merely *models* of the behavior of matter. Instead, we know mathematical laws of nature that cannot be violated intentionally. They are beyond the possibility of human invention, intervention, or convention, as Alan Turing, the father of modern computability theory, said of arithmetic in his famous paper proving that there are far more numbers that can be defined to "exist" mathematically than there are algorithms available to compute them.[2]

[1] There exist in the universe only local inertial frames, those locally in free fall in the net gravitational field of other bodies, there are no global inertial frames as Mach and Newton assumed. See Barbour (1989) for a fascinating and detailed account of the history of mechanics.

[2] The set of numbers that can be defined by continued fractions is uncountable and fills up the continuum. The set of algorithms available to generate initial conditions ("seeds") for continued fraction expansions is, in contrast, countable.

How are laws of nature discovered? As we well know, they are only established by repeatable identical (to within some decimal precision) experiments or observations. In physics and astronomy all predictions must in practice be falsifiable, otherwise we do not regard a model or theory as scientific. A falsifiable theory or model is one with few enough parameters and definite enough predictions (preferably of some new phenomenon) that it can be tested observationally and, if wrong, can be proven wrong. The cosmological principle (CP) may be an example of a model that is not falsifiable.[3] A nonfalsifiable hypothesis may belong to the realm of philosophy or religion, but not to science.

But we face more in life than can be classified as science, religion or philosophy: there is also medicine, which is not a completely scientific field, especially in everyday diagnosis. Most of our own daily decisions must be made on the basis of experience, bad information and instinct without adequate or even any scientific basis. For a discussion of an alternative to Galilean reasoning in the social field and medical diagnosis, see Carlo Ginzburg's (1992) essay on Clues in *Clues, Myths, and the Historical Method*, where he argues that the methods of Sherlock Holmes and art history are more fruitful in the social field than scientific rigor. But then this writer does not belong to the school of thought that believes that *everything* can be mathematized. Indeed, not everything can be. As von Neumann wrote, a simple system is one that is easier to describe mathematically than it is to build (the solar system, deterministic chaos, for example). In contrast, a complex system is easier to make than it is to describe completely mathematically (an embryo, for example). See Berlin (1998) for a nonmathematical discussion of the idea that there may be social problems that are not solvable.

1.2 Humanly invented law can always be violated

Anyone who has taken both physics and economics classes knows that these subjects are completely different in nature, notwithstanding the economists' failed attempt to make economics look like an exercise in calculus, or the finance theorists' failed attempt to portray financial markets as a subset of the theory of stochastic processes obeying the Martingale representation theorem. In economics, in contrast with physics, there exist no known inviolable mathematical laws of "motion"/behavior. Instead, economic law is either legislated law, dictatorial edict, contract, or in tribal societies the rule of tradition. Economic "law," like any legislated law or social contract, can always be violated by willful people and groups. In addition, the idea of falsification via observation has not yet taken root. Instead,

[3] The CP assumes that the universe is uniform at large enough distances, but out to the present limit of 170 Mpc h^{-1} we see nothing but clusters of clusters of galaxies, with no crossover to homogeneity indicated by *reliable* data analyses.