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优秀学术文集

（自然科学版）

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序　言

“学术是根本”是我校的办学理念。围绕这一理念，近年来，我校从制度着手，通过加强学术方面的制度建设，努力营造一个良好的学术发展环境：通过对《湛江师范学院科研项目管理办法》的修订，增加了科研参与学校“一体化全员育人”体系建设的规定，让更多的教师，尤其是青年教师，还有学生都有参加项目研究的机会，极大增强了教师和学生们的学术意识和学术兴趣，通过对《湛江师范学院科研成果奖励办法》的修订，使得奖励在资源有限的情况下能够更公平和合理，以最大限度激发教师们的学术积极性；通过对《湛江师范学院重点学科管理办法》的修订，规范了学校的学术发展方向，让资源得到更有效的配置；而《湛江师范学院举办学术会议管理办法》的出台，则让学校的学术交流更加活跃和开放……。

合理的规章制度和有力的贯彻推行让我校的学术发展日益蓬勃，成效也是显著的，单学术论文一项，2004年到2006年，累计发表学术论文近2500篇，年均论文发表数超过800篇。相对于2004年以前的年均500多篇，在数量上有了一个较大的飞跃。而质量上的提高也是明显的，发表在核心刊物的论文比例从2004年前的25%提高到了35%；三年内被三大索引收录的论文有89篇，年均近30篇，比2004年前的每年不到20篇增加了三分之一。

对学术的重视使我校收获了丰硕的学术成果，基此，我们编印了这部《2004——2006湛江师范学院优秀学术文集》(分社会科学、自然科学版)。这部文集，共收录了我校教师从2004年到2006年发表的部分优秀论文178篇，其中社会科学类论文92篇，自然科学类论文86篇；收录的原则是：社会科学类论文为权威A或权威B级别，自然科学类论文为权威A级别。这本汇编，可以看作是对我校过去三年学术成果的一个总结，同时更是对未来的一个激励，鞭策更多的湛师人如书中作者一样，兢兢业业地从事学术研究，多出成果，出高水平的成果。

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A note on the exponential diophantine equation $a^x + b^y = c^z$

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Abstract: Let a, b, c be fixed coprime positive integers. In this paper we prove that if $b \equiv 3 \pmod 4$, $a \equiv -1 \pmod{b^{2l}}$, $a^2 + b^{2l-1} = c$ and c is odd, where l is a positive integer, then the equation $a^x + b^y = c^z$ has only the positive integer solution $(x, y, z) = (2, 2l-1, 1)$.

Key words: Exponential diophantine equations; primitive divisors of Lucas numbers.

1. Introduction. Let $\mathbf{Z}$, $\mathbf{N}$ be the sets of all integers and positive integers respectively. Let a, b, c be fixed coprime positive integers. Recently, using the theory of linear forms in logarithms, Terai [7] proved that if b is a prime with $b \equiv 3 \pmod 4$, $a \equiv -1 \pmod{b^{2l}}$, $a^2 + b^{2l-1} = c$ and c is odd, where $l \in \{1, 2\}$, then the equation

$$a^x + b^y = c^z, \quad x, y, z \in \mathbf{N} \tag{1}$$

has only the solution $(x, y, z) = (2, 2l-1, 1)$. In this paper, by means of different approach, we shall show that the conditions b is a prime and $l \in \{1, 2\}$ can be eliminated from the above-mentioned result. We prove a general result as follows:

Theorem. *Let l be a positive integer. If $b \equiv 3 \pmod 4$, $a \equiv 1 \pmod{b^{2l}}$, $a^2 + b^{2l-1} = c$ and c is odd, then* (1) *has only the solution $(x, y, z) = (2, 2l-1, 1)$.*

2. Preliminaries.

Lemma 1 ([2, 3]). *The equation $X^2 + 3^{2m+1} = Y^n$, $X, Y, m, n \in \mathbf{Z}$, $X > 0$, $Y > 0$, $\gcd(X, Y) = 1$, $m \geq 0$, $n > 1$ has only the solution $(X, Y, m, n) = (10, 7, 2, 3)$ with n an odd prime.*

Let D be a positive integer, and let $\mathrm{h}(-4D)$ denote the class number of positive binary quadratic forms of discriminant $-4D$.

Lemma 2. *Let k be an odd integer with $\gcd(D, k) = 1$. If $D > 3$, then every solution (X, Y, Z) of the equation*

$$X^2 + DY^2 = k^Z, \quad X, Y, Z \in \mathbf{Z},$$
$$\gcd(X, Y) = 1, \quad Z > 0$$

can be expressed as

$$Z = Z_1 t, \quad t \in \mathbf{N},$$
$$X + Y\sqrt{-D} = \lambda_1 (X_1 + \lambda_2 Y_1 \sqrt{-D})^t,$$
$$\lambda_1 \lambda_2 \in \{1, -1\},$$

where X_1, Y_1, Z_1 are positive integers satisfying

$$X_1^2 + DY_1^2 = k^{Z_1}, \quad \gcd(X_1, Y_1) = 1,$$
$$\mathrm{h}(-4D) \equiv 0 \pmod{Z_1}.$$

Proof. This lemma is the special case of [6, Theorems 1 and 2] for $D_1 = 1$ and $D_2 < 3$. □

Lemma 3 ([5, Theorems 12.10.1 and 12.14.3]). *For any positive integer D, we have*

$$\mathrm{h}(-4D) < \frac{4\sqrt{D}}{\pi} \log(2e\sqrt{D}).$$

Let α, β be algebraic integers. If $\alpha + \beta$ and $\alpha\beta$ are nonzero coprime integers and α/β is not a root of unity, then (α, β) is called a Lucas pair. Further, let $A = \alpha + \beta$ and $C = \alpha\beta$. Then we have

$$\alpha = \frac{1}{2}(A + \lambda\sqrt{B}), \quad \beta = \frac{1}{2}(A - \lambda\sqrt{B}), \quad \lambda \in \{1, -1\},$$

where $B = A^2 - 4C$. We call (A, B) the parameters of the Lucas pair (α, β). Two Lucas pairs (α_1, β_1) and (α_2, β_2) are equivalent if $\alpha_1/\alpha_2 = \beta_1/\beta_2 = \pm 1$. Given a Lucas pair (α, β), one defines the corresponding sequence of Lucas numbers by

$$L_s(\alpha, \beta) = \frac{\alpha^s - \beta^s}{\alpha - \beta}, \quad s = 0, 1, 2, \cdots.$$

For equivalent Lucas pairs (α_1, β_1) and (α_2, β_2), we have $L_s(\alpha_1, \beta_1) = \pm L_s(\alpha_2, \beta_2)$ for any $s \geq 0$. A prime p is called a primitive divisor of $L_s(\alpha, \beta)$ ($s > 1$) if

$$p \mid L_s(\alpha, \beta) \ \text{ and } \ p \nmid BL_1(\alpha, \beta) \cdots L_{s-1}(\alpha, \beta).$$

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A Lucas pair (α, β) such that $L_s(\alpha, \beta)$ has no primitive divisors will be called a s-defective Lucas pair. Further, a positive integer s is called totally non-defective if no Lucas pair is s-defective.

Lemma 4 ([8]). *Let s satisfy $4 < s \le 30$ and $s \neq 6$. Then, up to equivalence, all parameters of s-defective Lucas pairs are given as follows*:

(i) $s = 5$, $(A, B) = (1,5), (1,-7), (2,-40), (1,-11), (1,-15), (12,-76), (12,-1364)$.
(ii) $s = 7$, $(A, B) = (1,-7), (1,-19)$.
(iii) $s = 8$, $(A, B) = (2,-24), (1,-7)$.
(iv) $s = 10$, $(A, B) = (2,-8), (5,-3), (5,-47)$.
(v) $s = 12$, $(A, B) = (1,5), (1,-7), (1,-11), (2,-56), (1,-15), (1,-19)$.
(vi) $s \in \{13, 18, 30\}$, $(A, B) = (1,-7)$.

Lemma 5 ([1]). *If $s > 30$, then s is totally non-defective.*

3. Proof of theorem. Let (x, y, z) be a solution of (1) with $(x, y, z) \neq (2, 2l-1, 1)$. Since $a \equiv -1 \pmod b$ and $c \equiv a^2 \equiv 1 \pmod b$, we see from (1) that x must be even. Since $b \equiv 3 \pmod 4$ and c is odd, we see from $a^2 + b^{2l-1} = c$ that a is even and $c \equiv 3 \pmod 4$. Hence, by (1), we get $y \equiv z \pmod 2$. Further, since $c \equiv 3 \pmod 4$, we conclude that $y \equiv z \equiv 1 \pmod 2$ by (1). It implies that y and z are both odd. Hence, by Lemma 1, we may assume that b is not a power of 3.

Since $a \equiv -1 \pmod{b^{2l}}$ and $a^2 + b^{2l-1} = c$, we have $c \equiv 1 + b^{2l-1} \pmod{b^{2l}}$. Hence, by (1), we get $1 + b^y \equiv 1 \pmod{b^{2l-1}}$ and $y \ge 2l-1$. If $y = 2l-1$, then from (1) we get

$$(2) \qquad 1 + b^{2l-1} \equiv (1 + b^{2l-1})^z \pmod{b^{2l}},$$

whence we obtain

$$(3) \qquad z - 1 \equiv 0 \pmod b.$$

Further, since $y = 2l-1$ and $(x, y, z) \neq (2, 2l-1, 1)$, we have $z > 1$. Therefore, by (3), we get

$$(4) \qquad z - 1 \ge b.$$

If $y > 2l-1$, then from (1) we get

$$(5) \qquad 1 \equiv (1 + b^{2l-1})^z \pmod{b^{2l}}.$$

It implies that $z \equiv 0 \pmod b$ and

$$(6) \qquad z \ge b.$$

Therefore, by (4), (6) holds for any case.

Since $b > 3$ and y is odd, we find from (1) that $(X, Y, Z) = (a^{x/2}, b^{(y-1)/2} z)$ is a solution of the equation

$$(7) \qquad X^2 + bY^2 = c^Z, \quad X, Y, Z \in \mathbf{Z}, \quad \gcd(X, Y) = 1, \quad Z > 0.$$

Since c is odd, by Lemma 2, we obtain

$$(8) \qquad z = Z_1 t, \quad t \in \mathbf{N},$$

$$(9) \quad a^{x/2} + b^{(y-1)/2}\sqrt{-b} = \lambda_1 (X_1 + \lambda_2 Y_1 \sqrt{-b})^t, \quad \lambda_1 \lambda_2 \in \{1, -1\},$$

where X_1, Y_1, Z_1 are positive integers satisfying

$$(10) \qquad X_1^2 + bY_1^2 = c^{Z_1}, \quad \gcd(X_1, Y_1) = 1, \quad \mathrm{h}(-4b) \equiv 0 \pmod{Z_1}.$$

Moreover, since z is odd, we see from (8) that t must be odd.

Let

$$(11) \qquad \alpha = X_1 + Y_1\sqrt{-b}, \quad \beta = X_1 - Y_1\sqrt{-b}.$$

By (10) and (11), we have

$$(12) \qquad \alpha + \beta = 2X_1, \quad \alpha\beta = c^{Z_1}, \quad \frac{\alpha}{\beta} = \frac{1}{c^{Z_1}}\left((X_1^2 - bY_1^2) + 2X_1Y_1\sqrt{-b}\right).$$

Since $\gcd(X_1, Y_1) = \gcd(b, c) = 1$, we observe from (12) that $\alpha + \beta$ and $\alpha\beta$ are nonzero coprime integers and α/β is not a root of unity. Hence, (α, β) is a Lucas pair with parameters $(2X_1, -4bY_1^2)$. Further, let $L_s(\alpha, \beta)$ $(s = 0, 1, 2, \cdots)$ denote the corresponding Lucas numbers. By (9) and (11), we get

$$(13) \qquad b^{(y-1)/2} = Y_1 |L_t(\alpha, \beta)|.$$

We find from (13) that the Lucas number $L_t(\alpha, \beta)$ has no primitive divisors. Therefore, by Lemma 5, we get $t \le 30$. Further, it is easy to remove all cases in Lemma 4 and conclude that $t \le 4$. So we have $t \in \{1, 3\}$.

When $t = 1$, we get from (8) and (10) that $z = Z_1$ and $\mathrm{h}(-4b) \equiv 0 \pmod z$. It implies that $\mathrm{h}(-4b) \ge z$. Further, by (6),

$$(14) \qquad \mathrm{h}(-4b) \ge b.$$

By Lemma 3, we see from (14) that

$$(15) \qquad b < \frac{4\sqrt{b}}{\pi} \log(2e\sqrt{b}),$$

whence we conclude that $b < 19$. Recall that $b \equiv 3 \pmod 4$ and b is not a power of 3. We have $b \in \{7, 11, 15\}$. But, (14) is impossible, since $\mathrm{h}(-4 \cdot 7) = 1$, $\mathrm{h}(-4 \cdot 11) = 3$ and $\mathrm{h}(-4 \cdot 15) = 2$.

When $t = 3$, we get from (9) that

$$b^{(y-1)/2} = \lambda_1\lambda_2 Y_1(3X_1^2 - bY_1^2). \tag{16}$$

Let $d = \gcd(Y_1, 3X_1^2 - bY_1^2)$. Since $\gcd(X_1, Y_1) = 1$, we have $d = 1$ or 3. Notice that $\gcd(b, c) = 1$ and $\gcd(b, X_1) = 1$ by (10). If $d = 1$ and b is a power of prime, then $b \neq$ a power of 3 and $\gcd(b, 3X_1^2 - bY_1^2) = 1$. Hence, from (16) we get $Y_1 = b^{(y-1)/2}$ and

$$3X_1^2 - b^y = 1, \tag{17}$$

since $b^y \equiv 3 \pmod 4$. Recall that $c \equiv 1 \pmod b$. We get from (10) and (17) that $X_1^2 \equiv 1 \pmod b$ and $3X_1^2 \equiv 1 \pmod b$, respectively. It implies that $3 \equiv 1 \pmod b$, a contradiction. If $d = 3$, then $3 \mid b$, by (16). Since b is not a power of 3, b has at least two distinct prime divisors. Therefore when $d = 1$ and $b \neq a$ power of prime or $d = 3$, by the genus theory of binary quadratic forms (see [4, Section 48]), we have $h(-4b) \equiv 0 \pmod 2$. Further, by (8) and (10), we get $z = 3Z_1$ and $h(-4b) \equiv 0 \pmod{2z/3}$. It follows that

$$h(-4b) \geq \frac{2}{3}b, \tag{18}$$

by (6). Further, by Lemma 3, we obtain from (18) that

$$\frac{2}{3}b < \frac{4\sqrt{b}}{\pi}\log(2e\sqrt{b}), \tag{19}$$

whence we conclude that $b \leq 51$, since $3 \mid b$ for $d = 3$, we have $b \in \{15, 35, 39, 51\}$. But, (18) is impossible, since $h(-4\cdot 15) = 2$, $h(-4\cdot 35) = 2$, $h(-4\cdot 39) = 4$ and $h(-4\cdot 51) = 6$. To sum up, the theorem is proved.

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ON THE DIOPHANTINE SYSTEM $x^2 - Dy^2 = 1 - D$ AND $x = 2z^2 - 1$

MAOHUA LE*

Abstract

Let D be a positive integer such that $D - 1$ is an odd prime power. In this paper we give an elementary method to find all positive integer solutions (x, y, z) of the system of equations $x^2 - Dy^2 = 1 - D$ and $x = 2z^2 - 1$. As a consequence, we determine all solutions of the equations for $D = 6$ and 8.

1. Introduction

Let $\mathbf{Z}$, $\mathbf{N}$ be the sets of all integers and positive integers respectively. Let D be a positive integer with $D > 1$. The determination of all solutions (x, y, z) of the system of equations

$$(1)\quad x^2 - Dy^2 = 1 - D, \quad x = 2z^2 - 1, \qquad x, y, z \in \mathbf{N}, \ \gcd(x, y) = 1$$

is an interesting problem concerning the arithmetic properties of recurrence sequences and the solution of exponential-polynomial equations over real quadratic fields. In 1995 Mignotte and Pethö [9] determined all solution (x, y, z) for $D = 6$. Their proof relied upon deep tools related to linear form in logarithms and reduction technigues. In 1998, Cohn [4] gave an elementary proof of the above mentioned result.

In this paper we give an elementary method to find all solutions of (1) for the general case that $D - 1$ is an odd prime power. We now introduce some needful notations and known results given by Petr [10].

LEMMA 1. *Let D be a nonsquare positive integer, and let $u_1 + v_1\sqrt{D}$ be the fundamental solution of Pell equation*

$$(2)\qquad u^2 - Dv^2 = 1, \qquad u, v \in \mathbf{Z}.$$

Then we have

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(i) *All solutions* (u, v) *of* (2) *can be expressed as*

$$u + v\sqrt{D} = (u_1 + v_1\sqrt{D})^t, \qquad t \in \mathbf{Z}. \tag{3}$$

(ii) *For any positive integer n, let*

$$u_n + v_n\sqrt{D} = (u_1 + v_1\sqrt{D})^n. \tag{4}$$

Then $(u, v) = (u_n, v_n)$ $(n \in \mathbf{N})$ *are all positive integer solutions of* (2).

LEMMA 2. *Let D be an even nonsquare positive integer, and let*

$$D' = \begin{cases} D, \\ \dfrac{D}{4}, \end{cases} \qquad \varepsilon = \begin{cases} 1, & \text{if } v_1 \text{ is even}, \\ 2, & \text{if } v_1 \text{ is odd}. \end{cases} \tag{5}$$

For a fixed D, there exists a unique positive integers pair (D_1, D_2) *such that* $D_1 > 1$, $D_1D_2 = D'$ *and the equation*

$$D_1U^2 - D_2V^2 = 1, \qquad U, V \in \mathbf{N} \tag{6}$$

has solution (U, V).

LEMMA 3. *Let* D_1, D_2 *be positive integers with* $D_1 > 1$. *If* (6) *has solutions* (U, V), *then it has a unique solution* (U_1, V_1) *satisfying* $V_1 \le V$, *where V runs through all solutions* (U, V) *of* (6). *The solution* (U_1, V_1) *is called the least solution of* (6). *Then we have:*

(i) $(U_1\sqrt{D_1} + V_1\sqrt{D_2})^2 = u_1 + v_1\sqrt{D}$.

(ii) *For any odd positive integer m, let*

$$U_m\sqrt{D_1} + V_m\sqrt{D_2} = (U_1\sqrt{D_1} + V_1\sqrt{D_2})^m. \tag{7}$$

Then $(U, V) = (U_m, V_m)$ *for* $m = 1, 3, \ldots$ *are all solutions of* (6).

Under the mentioned notations, using a result of [5], we prove a general result as follows.

THEOREM. *Let D be a positive integer such that* $D-1$ *is an odd prime power. If D is a square, then* $D = 4$ *and* (1) *has only the solution* $(x, y, z) = (1, 1, 1)$. *If D is not a square, then all solutions of* (1) *can be classified into the following five shapes.*

(i) $(x, y, z) = (1, 1, 1)$.

(ii) $(x, y, z) = \left(u_{2n} + Dv_{2n}, u_{2n} + v_{2n}, \sqrt{u_n(u_n + Dv_n)}\right)$.

(iii) $(x, y, z) = \left(-u_{2n} + Dv_{2n}, u_{2n} - v_{2n}, \sqrt{Dv_n(u_n - v_n)}\right)$.