

序 言

“学术是根本”是我校的办学理念。围绕这一理念，近年来，我校从制度着手，通过加强学术方面的制度建设，努力营造一个良好的学术发展环境：通过对《湛江师范学院科研项目管理办法》的修订，增加了科研参与学校“一体化全员育人”体系建设的规定，让更多的教师，尤其是青年教师，还有学生都有参加项目研究的机会，极大增强了教师和学生们的学术意识和学术兴趣，通过对《湛江师范学院科研成果奖励办法》的修订，使得奖励在资源有限的情况下能够更公平和合理，以最大限度激发教师们的学术积极性；通过对《湛江师范学院重点学科管理办法》的修订，规范了学校的学术发展方向，让资源得到更有效的配置；而《湛江师范学院举办学术会议管理办法》的出台，则让学校的学术交流更加活跃和开放……。

合理的规章制度和有利的贯彻推行让我校的学术发展日益蓬勃，成效也是显著的，单学术论文一项，2004年到2006年，累计发表学术论文近2500篇，年均论文发表数超过800篇。相对于2004年以前的年均500多篇，在数量上有了一个较大的飞跃。而质量上的提高也是明显的，发表在核心刊物的论文比例从2004年前的25%提高到了35%；三年内被三大索引收录的论文有89篇，年均近30篇，比2004年前的每年不到20篇增加了三分之一。

对学术的重视使我校收获了丰硕的学术成果，基此，我们编印了这部《2004——2006湛江师范学院优秀学术文集》(分社会科学、自然科学版)。这部文集，共收录了我校教师从2004年到2006年发表的部分优秀论文178篇，其中社会科学类论文92篇，自然科学类论文86篇；收录的原则是：社会科学类论文为权威A或权威B级别，自然科学类论文为权威A级别。这本汇编，可以看作是对我校过去三年学术成果的一个总结，同时更是对未来的一个激励，鞭策更多的湛师人如书中作者一样，兢兢业业地从事学术研究，多出成果，出高水平的成果。

编 者

2007年10月

目 录

数 学

A note on the exponential diophantine equation $a^x + b^y = c^z$

(Proc.Japan Acad., 80,Ser. A, 2004; SCI 收录) 乐茂华 (3)

On the diophantine system $x^2 - Dy^2 = 1 - D$ and $x = 2z^2 - 1$

(Math. SCAND.95, 2004;SCI 收录) 乐茂华 (6)

Some identities for Bernoulli and Euler polynomials

(Fibonacci Quart. 42, 2004, No.4; SCI 收录) 吴克俭 (16)

A Conjecture Concerning the Pure Exponential Diophantine Equation $a^x + b^y = c^z$

(Acta Mathematica Sinica, English Series, Vol.21, No.4, 2005; SCI收录)

..... 乐茂华 (21)

广义 Ramanujan-Nagell 方程 $x^2 + D^m = p^n$ 的解数

(数学学报 2005年第1期) 乐茂华 (27)

Pell 方程组 $x^2 - ay^2 = 1$ 和 $z^2 - by^2 = 1$ 的解数 (数学进展 2005 年第 1 期)

..... 乐茂华 (31)

Reinhardt域上和复Hilbert空间单位球上推广的Roper-Suffridge算子

(数学年刊 26A: 5, 2005) 刘小松 (42)

A Representation Theorem for E^* -unitary Categorical Inverse Semigroups

(数学进展 2006 年第 2 期) 陈历敏 (52)

An open problem concerning the Diophantine equation $a^x + b^x = c^z$

(Publ. Math. Debrecen 68/3-4, 2006; SCI 收录) 乐茂华 (57)

A note on the Diophantine equation $x^2 + b^y = c^z$

(Czechoslovak Mathematical Journal,56<131> 2006; SCI 收录) 乐茂华 (70)

An iterative equation on the unit circle

(Acta Mathematica Scientia 2006,26B<3>; SCI 收录) 李晓培 (78)

The generalized Roper-Suffridge extension operator for some biholomorphic mappings

(J. Math. Anal. Appl.324, 2006;SCI 收录) 刘小松 (88)

α 型螺形映照精细的增长、掩盖定理及精细的展开式系数估计

(数学学报 2006 年第 3 期) 刘小松 (99)

关于 α 次的 β 型螺形映照推广的 Roper-Suffridge 算子

(数学年刊 2006. 27 A (6)) 刘小松 (109)

物 理 学

Barriola_Vilenkin 黑洞的统计熵 (物理学报 2004 年第 11 期; SCI 收录) 李固强 (121)

State equations for massless spin fields near the event horizon in Schwarzschild spacetime

(Class. Quantum Grav. 21, 2004; SCI 收录) 黎忠恒 (124)

Matter entropy and entropy of de Sitter universe

(IL NUOVO CIMENTO Vol. 119 B, No. 3, 2004; SCI 收录) 米丽琴 (129)

Anti-de Sitter 时空中黑洞量子熵的发散结构

(物理学报 2004 年第 7 期; SCI 收录) 米丽琴 (135)

A new concept toward industrialization of Cu-III-VI₂ thin film solar cells and some preliminary experiment results

(ACTA METALLURGICA SINICA Vol.18 No.3, 2005) 邵乐喜 (139)

Divergence structure of the statistical entropy of the Dirac field in a plane symmetry black hole geometry (Chinese Physics Vol 14 No 3, 2005; SCI 收录) 李固强 (144)

Kerr-Newman-de Sitter 黑洞的统计熵 (物理学报 2005 年第 7 期; SCI 收录) ... 李固强 (148)

Effect of Spin on Thermodynamical Quantities around Reissner-Nordström Black Holes

(CHIN.PHYS.LETT Vol. 22, No. 6, 2005; SCI 收录) 黎忠恒 (152)

Experimental Study of Effect of Medium Boundary on Light Distribution in Tissue Phantoms	
(中国物理快报 2005年第7期; SCI收录)	许 棠 (156)
Theoretical and experimental study of the intensity distribution in biological tissues	
(中国物理 2005年第9期; SCI收录)	许 棠 (160)
Algebro-Geometric Solution to Two New (2+1)-Dimensional Modified Kadomtsev-Petviashvili Equations	
(中国物理快报 2006 年第 10 期; SCI 收录)	吴勇旗 (168)
高速桥梁瓶颈模型属性的研究	
(物理学报 2006 年第 7 期; SCI 收录)	肖世发 (172)
Logarithmic terms in brick wall model	
(Physics Letters B 643, 2006; SCI收录) ...	黎忠恒 (180)
MODIFICATIONS TO BEKENSTEIN-HAWKING ENTROPY DUE TO ARBITRARY SPIN FIELDS	
(Modern Physics Letters A Vol. 21, No. 23, 2006; SCI收录)	米丽琴 (187)
Thermodynamical quantities around a RNAdS black hole	
(中国物理 2006年第6期; SCI收录)	米丽琴 (194)
Cu₂ZnSnS₄, thin films prepared by sulfurization of ion beam sputtered precursor and their electrical and optical properties	
(RARE METALS Vol. 25, Spec. Issue , 2006; SCI收录)	邵乐喜 (200)
广义不确定关系与黑洞附近的热力学量	
(物理学报 2006年第11期; SCI收录)	刘晓莹 (205)
Effects of Al doping concentration on optical parameters of ZnO:Al thin films by sol-gel technique	
(Physica B 381, 2006; SCI收录)	薛书文 (210)
Effects of annealing and dopant concentration on the optical characteristics of ZnO:Al thin films by sol-gel technique	
(Physica B 382, 2006; SCI收录)	薛书文 (215)
Effects of low doping concentration on interconnected microstructural ZnO:Al thin films prepared by the sol-gel technique	
(Eur. Phys. J. Appl. Phys.35, 2006; SCI收录) ...	薛书文 (219)
Anti-de Sitter时空内柱黑洞的量子熵	
(物理学报 2006年第2期; SCI收录)	李固强 (225)
Black plane's tunneling radiation	
(Europhys. Lett., 75 (2), 2006; SCI收录)	李固强 (229)
Entropy of Spin Fields in Anti-de Sitter Space-time	
(CHINESE JOURNAL OF PHYSICS VOL. 44, No. 4, 2006; SCI收录)	李固强 (233)
Quantum entropy of Dirac field in toroidal black hole	
(Physics A 368, 2006; SCI收录)	李固强 (239)

Black string’s tunnelling radiation (J. Phys. A: Math. Gen. 39,2006;SCI收录)	李固强 (244)
Hawking radiation via tunneling from Kerr-Newman-de Sitter black hole	
(Europhys. Lett., 76 (2), 2006; SCI收录)	李固强 (249)
QUANTUM ENTROPY OF DIRAC FIELDS IN BLACK HOLES	
(Theoretical and Mathematical Physics, 149(1) 2006; SCI收录)	李固强 (254)
Quantum Entropy of Spin Fields in the Schwarzschild-Anti-de Sitter Black Hole with a Global Monopole (Journal of Statistical Physics Vol.125,No3, 2006; SCI 收录)	李固强 (259)

机 械 工 程

可倾瓦推力轴承在变载荷下的瞬态润滑性能研究	
(中国机械工程 2004 年第 15 期; EI 收录)	李 忠 (269)
半环型锥盘滚轮式无级变速器的传动特性研究	
(中国机械工程 2005 年第 6 期; EI 收录)	李 忠 (272)
基于多组件 Agent 的协同设计系统 (中国机械工程 2006 年第 7 期; EI 收录) ...	苏财茂 (278)
半自由场波叠加噪声源识别方法研究 (中国机械工程 2006年第7期;EI收录) ...	潘汉军 (284)

计 算 机 科 学

IPv6 及其在教育教学中的应用 (中国电化教育 2005.9 总第 224 期)	程 志 (291)
--	-----------

作物学

- 土壤机械阻力对玉米根系导水率的影响（水利学报 2004 年第 4 期） 刘晚苟（297）
- 不同诱导处理后水稻悬浮细胞的活性氧变化与有关酶系的关系
（作物学报 2005 年第 1 期） 曾富华（302）
- Synthesis and Antifungal Activities of Alkyl N-(1,2,3-Thiadiazole-4-Carbonyl) Carbamates and
S-Alkyl N-(1,2,3-Thiadiazole-4-Carbonyl) Carbamothioates**
（J. Agric. Food Chem. 2005, 53; SCI 收录） 李再峰（308）
- 水稻稻穗灌浆生长的基因效应全程分析（作物学报 2005 年第 7 期） 左清凡（313）

园艺学

- 基因枪介导的高羊茅基因转化体系的建立（园艺学报 2004 年第 5 期） 马生健（321）
- 高羊茅抗真菌病基因转化的研究（园艺学报 2006 年第 6 期） 马生健（324）

生物学

- 用核糖体 ITS 区序列验证自然杂交种 *Meconopsis × cookei* G. Taylor
（遗传学报 2004 年第 9 期） 袁长春（333）
- 迷卡斗蟋鸣声的声学特征及其生物学意义（昆虫学报 2005 年第 1 期） 陈道海（340）

化学

- 酿酒酵母凋亡细胞的毛细管电泳行为研究（分析化学 2004 年第 9 期） 何金兰（351）
- Preparation of Hyperbranched Polyester Photoresists for Miniaturized Optics**（Journal of Applied

Polymer Science, Vol. 92, 2004;SCI、EI收录)	冯宗财 (355)
纳米磷钼钒杂多酸盐催化苯羟基化合成苯酚 (应用化学 2005年第1期)	石晓波 (360)
纳米 $\text{H}_3\text{CoPMo}_{10}\text{V}_2\text{O}_{40}$ 的室温固相合成及其催化性能 (化学通报 2005 年第 5 期)	石晓波 (365)
Study on Preparation and Properties of $\text{La}_2\text{O}_3/\text{MC}$ Nylon Nanocomposite	
(JOURNAL OF RARE EARTHS Vol.23,No.6,Dec.2005; SCI、EI 收录)	林 轩 (370)
Simultaneous Determination of Cinnamaldehyde, Eugenol and Paeonol in Traditional Chinese Medicinal Preparations by Capillary GC-FID	
(Chem. Pharm. Bull. Vol.54, No.1,2006;SCI 收录)	余炳生 (376)
超支化聚酯微光学光致抗蚀剂 (应用化学 2006 第 7 期)	冯宗财 (379)
固相载体法合成低聚糖 (应用化学 2006 年第 6 期)	冯宗财 (383)
$\text{BaLiF}_3:\text{Ce}^{3+}$ 纳米粒子的制备及其光谱特性 (高等学校化学学报 2006 年第 2 期;SCI 收录)	朱国贤 (387)
稀土掺杂氟化镁钾纳米晶的合成及其光谱特性 (高等学校化学学报 2006 年第 3 期; SCI 收录)	朱国贤 (390)
Effect of oxalic acid on control of postharvest browning of litchi fruit	
(Food Chemistry 96,2006 ; SCI收录)	郑小林 (395)

食 品 科 学

水溶性高聚物萃取分离光度法测定 Sn^{4+} 的研究 (食品科学 2005, Vol.26, No.9)	杜建中 (403)
---	-----------

体 育 学

针灸足三里穴对大负荷训练血液流变学、红细胞形态的干预

(体育科学 2004 年第 1 期) 李 红 (409)

针灸足三里穴对小鼠运动能力及部分免疫指标的影响

(中国运动医学杂志 2004 年第 1 期) 李 红 (413)

对新时期中学体育与健康课程教学问题的研究

(北京体育大学学报 2004 年第 8 期) 许康成 (417)

中国女排在第 28 届奥运会比赛中进攻技术运用特征的分析

(北京体育大学学报 2004 年第 12 期) 钟 雯 (420)

螺旋藻复方合剂对运动小鼠肝细胞、心肌和骨骼肌细胞超微结构及 HSP₇₀ 表达的影响

(中国运动医学杂志 2005 年第 5 期) 朱梅菊 (422)

螺旋藻复方有效部位配方对慢性运动性疲劳大鼠脑组织基因表达谱的影响

(体育科学 2005 年第 12 期) 朱梅菊 (427)

广东省高校武术运动开展现状及其发展对策 (中国体育科技 2005 年第 4 期) ... 宋亚炳 (431)

竞技散打运动中下潜抱摔的量化分析及训练模型构建

(北京体育大学学报 2005 年第 1 期) 李 刚 (434)

点压肾俞、照海穴对网球运动员定量负荷运动后红细胞免疫与抗氧化功能的影响

(中国运动医学杂志 2005 年第 1 期) 陈筱春 (436)

甲壳寡糖对运动小鼠红细胞畸变及脂质过氧化的保护

(中国体育科技 2005 年第 2 期) 陈筱春 (440)

对运动员赛前心理压力源的调查及调节对策的研究

(北京体育大学学报 2005 年第 11 期) 姚卫宇 (443)

不同强度运动和雌激素联合作用对去卵巢大鼠骨骼生物力学性能的影响

(中国运动医学杂志 2006 年第 2 期) 章晓霜 (445)

不同强度运动和雌激素联合作用对去卵巢大鼠骨骼影响的骨形态计量学研究

(体育科学 2006 年第 8 期) 章晓霜 (450)

大负荷训练后大鼠红细胞形态学的变化及针灸的促恢复作用	
（北京体育大学学报 2006 年第 7 期）	朱梅菊（455）
NR1、NR2A 和 NR2B 在运动性疲劳大鼠海马组织中的表达及配方的调节作用	
（体育科学 2006 年第 6 期）	朱梅菊（458）
甲壳寡糖对力竭运动小鼠红细胞免疫粘附与抗氧化功能的影响	
（北京体育大学学报 2006 年第 11 期）	陈筱春（462）
运动、膳食与脂肪细胞因子（体育科学 2006 年第 4 期）	李世成（464）
补充活性肽对大鼠 1 次离心运动后骨骼肌微细损伤作用的形态学研究	
（中国运动医学杂志 2006 年第 1 期）	李世成（469）

数 学

A note on the exponential diophantine equation $a^x + b^y = c^z$

By Maohua LE

Department of Mathematics, Zhanjing Normal College
29 Cunjin Road, Chikan Zhanjing, Guangdong, P. R. China
(Communicated by Shokichi IYANAGA, M. J. A., April 12, 2004)

Abstract: Let a, b, c be fixed coprime positive integers. In this paper we prove that if $b \equiv 3 \pmod{4}$, $a \equiv -1 \pmod{b^{2l}}$, $a^2 + b^{2l-1} = c$ and c is odd, where l is a positive integer, then the equation $a^x + b^y = c^z$ has only the positive integer solution $(x, y, z) = (2, 2l - 1, 1)$.

Key words: Exponential diophantine equations; primitive divisors of Lucas numbers.

1. Introduction. Let $\mathbb{Z}, \mathbb{N}$ be the sets of all integers and positive integers respectively. Let a, b, c be fixed coprime positive integers. Recently, using the theory of linear forms in logarithms, Terai [7] proved that if b is a prime with $b \equiv 3 \pmod{4}$, $a \equiv -1 \pmod{b^{2l}}$, $a^2 + b^{2l-1} = c$ and c is odd, where $l \in \{1, 2\}$, then the equation

$$(1) \quad a^x + b^y = c^z, \quad x, y, z \in \mathbb{N}$$

has only the solution $(x, y, z) = (2, 2l - 1, 1)$. In this paper, by means of different approach, we shall show that the conditions b is a prime and $l \in \{1, 2\}$ can be eliminated from the above-mentioned result. We prove a general result as follows:

Theorem. Let l be a positive integer. If $b \equiv 3 \pmod{4}$, $a \equiv 1 \pmod{b^{2l}}$, $a^2 + b^{2l-1} = c$ and c is odd, then (1) has only the solution $(x, y, z) = (2, 2l - 1, 1)$.

2. Preliminaries.

Lemma 1 ([2, 3]). The equation $X^2 + 3^{2m+1} = Y^n$, $X, Y, m, n \in \mathbb{Z}$, $X > 0$, $Y > 0$, $\gcd(X, Y) = 1$, $m \geq 0$, $n > 1$ has only the solution $(X, Y, m, n) = (10, 7, 2, 3)$ with n an odd prime.

Let D be a positive integer, and let $h(-4D)$ denote the class number of positive binary quadratic forms of discriminant $-4D$.

Lemma 2. Let k be an odd integer with $\gcd(D, k) = 1$. If $D > 3$, then every solution (X, Y, Z) of the equation

$$X^2 + DY^2 = k^Z, \quad X, Y, Z \in \mathbb{Z}, \\ \gcd(X, Y) = 1, \quad Z > 0$$

can be expressed as

$$Z = Z_1 t, \quad t \in \mathbb{N}, \\ X + Y\sqrt{-D} = \lambda_1(X_1 + \lambda_2 Y_1 \sqrt{-D})^t, \\ \lambda_1 \lambda_2 \in \{1, -1\},$$

where X_1, Y_1, Z_1 are positive integers satisfying

$$X_1^2 + DY_1^2 = k^{Z_1}, \quad \gcd(X_1, Y_1) = 1, \\ h(-4D) \equiv 0 \pmod{Z_1}.$$

Proof. This lemma is the special case of [6, Theorems 1 and 2] for $D_1 = 1$ and $D_2 < 3$. $\square$

Lemma 3 ([5, Theorems 12.10.1 and 12.14.3]). For any positive integer D , we have

$$h(-4D) < \frac{4\sqrt{D}}{\pi} \log(2e\sqrt{D}).$$

Let α, β be algebraic integers. If $\alpha + \beta$ and $\alpha\beta$ are nonzero coprime integers and α/β is not a root of unity, then (α, β) is called a Lucas pair. Further, let $A = \alpha + \beta$ and $C = \alpha\beta$. Then we have

$$\alpha = \frac{1}{2}(A + \lambda\sqrt{B}), \quad \beta = \frac{1}{2}(A - \lambda\sqrt{B}), \quad \lambda \in \{1, -1\},$$

where $B = A^2 - 4C$. We call (A, B) the parameters of the Lucas pair (α, β) . Two Lucas pairs (α_1, β_1) and (α_2, β_2) are equivalent if $\alpha_1/\alpha_2 = \beta_1/\beta_2 = \pm 1$. Given a Lucas pair (α, β) , one defines the corresponding sequence of Lucas numbers by

$$L_s(\alpha, \beta) = \frac{\alpha^s - \beta^s}{\alpha - \beta}, \quad s = 0, 1, 2, \dots$$

For equivalent Lucas pairs (α_1, β_1) and (α_2, β_2) , we have $L_s(\alpha_1, \beta_1) = \pm L_s(\alpha_2, \beta_2)$ for any $s \geq 0$. A prime p is called a primitive divisor of $L_s(\alpha, \beta)$ ($s > 1$) if

$$p \mid L_s(\alpha, \beta) \quad \text{and} \quad p \nmid BL_1(\alpha, \beta) \cdots L_{s-1}(\alpha, \beta).$$

A Lucas pair (α, β) such that $L_s(\alpha, \beta)$ has no primitive divisors will be called a s -defective Lucas pair. Further, a positive integer s is called totally non-defective if no Lucas pair is s -defective.

Lemma 4 ([8]). *Let s satisfy $4 < s \leq 30$ and $s \neq 6$. Then, up to equivalence, all parameters of s -defective Lucas pairs are given as follows:*

- (i) $s = 5$, $(A, B) = (1, 5), (1, -7), (2, -40), (1, -11), (1, -15), (12, -76), (12, -1364)$.
- (ii) $s = 7$, $(A, B) = (1, -7), (1, -19)$.
- (iii) $s = 8$, $(A, B) = (2, -24), (1, -7)$.
- (iv) $s = 10$, $(A, B) = (2, -8), (5, -3), (5, -47)$.
- (v) $s = 12$, $(A, B) = (1, 5), (1, -7), (1, -11), (2, -56), (1, -15), (1, -19)$.
- (vi) $s \in \{13, 18, 30\}$, $(A, B) = (1, -7)$.

Lemma 5 ([1]). *If $s > 30$, then s is totally non-defective.*

3. Proof of theorem. Let (x, y, z) be a solution of (1) with $(x, y, z) \neq (2, 2l-1, 1)$. Since $a \equiv -1 \pmod{b}$ and $c \equiv a^2 \equiv 1 \pmod{b}$, we see from (1) that x must be even. Since $b \equiv 3 \pmod{4}$ and c is odd, we see from $a^2 + b^{2l-1} = c$ that a is even and $c \equiv 3 \pmod{4}$. Hence, by (1), we get $y \equiv z \pmod{2}$. Further, since $c \equiv 3 \pmod{4}$, we conclude that $y \equiv z \equiv 1 \pmod{2}$ by (1). It implies that y and z are both odd. Hence, by Lemma 1, we may assume that b is not a power of 3.

Since $a \equiv -1 \pmod{b^{2l}}$ and $a^2 + b^{2l-1} = c$, we have $c \equiv 1 + b^{2l-1} \pmod{b^{2l}}$. Hence, by (1), we get $1 + b^y \equiv 1 \pmod{b^{2l-1}}$ and $y \geq 2l-1$. If $y = 2l-1$, then from (1) we get

$$(2) \quad 1 + b^{2l-1} \equiv (1 + b^{2l-1})^z \pmod{b^{2l}},$$

whence we obtain

$$(3) \quad z - 1 \equiv 0 \pmod{b}.$$

Further, since $y = 2l-1$ and $(x, y, z) \neq (2, 2l-1, 1)$, we have $z > 1$. Therefore, by (3), we get

$$(4) \quad z - 1 \geq b.$$

If $y > 2l-1$, then from (1) we get

$$(5) \quad 1 \equiv (1 + b^{2l-1})^z \pmod{b^{2l}}.$$

It implies that $z \equiv 0 \pmod{b}$ and

$$(6) \quad z \geq b.$$

Therefore, by (4), (6) holds for any case.

Since $b > 3$ and y is odd, we find from (1) that $(X, Y, Z) = (a^{x/2}, b^{(y-1)/2}z)$ is a solution of the equation

$$(7) \quad X^2 + bY^2 = c^Z, \quad X, Y, Z \in \mathbb{Z}, \\ \gcd(X, Y) = 1, \quad Z > 0.$$

Since c is odd, by Lemma 2, we obtain

$$(8) \quad z = Z_1 t, \quad t \in \mathbb{N},$$

$$(9) \quad a^{x/2} + b^{(y-1)/2} \sqrt{-b} = \lambda_1 (X_1 + \lambda_2 Y_1 \sqrt{-b})^t, \\ \lambda_1 \lambda_2 \in \{1, -1\},$$

where X_1, Y_1, Z_1 are positive integers satisfying

$$(10) \quad X_1^2 + bY_1^2 = c^{Z_1}, \quad \gcd(X_1, Y_1) = 1, \\ h(-4b) \equiv 0 \pmod{Z_1}.$$

Moreover, since z is odd, we see from (8) that t must be odd.

Let

$$(11) \quad \alpha = X_1 + Y_1 \sqrt{-b}, \quad \beta = X_1 - Y_1 \sqrt{-b}.$$

By (10) and (11), we have

$$(12) \quad \alpha + \beta = 2X_1, \quad \alpha\beta = c^{Z_1}, \\ \frac{\alpha}{\beta} = \frac{1}{c^{Z_1}} ((X_1^2 - bY_1^2) + 2X_1Y_1\sqrt{-b}).$$

Since $\gcd(X_1, Y_1) = \gcd(b, c) = 1$, we observe from (12) that $\alpha + \beta$ and $\alpha\beta$ are nonzero coprime integers and α/β is not a root of unity. Hence, (α, β) is a Lucas pair with parameters $(2X_1, -4bY_1^2)$. Further, let $L_s(\alpha, \beta)$ ($s = 0, 1, 2, \dots$) denote the corresponding Lucas numbers. By (9) and (11), we get

$$(13) \quad b^{(y-1)/2} = Y_1 |L_t(\alpha, \beta)|.$$

We find from (13) that the Lucas number $L_t(\alpha, \beta)$ has no primitive divisors. Therefore, by Lemma 5, we get $t \leq 30$. Further, it is easy to remove all cases in Lemma 4 and conclude that $t \leq 4$. So we have $t \in \{1, 3\}$.

When $t = 1$, we get from (8) and (10) that $z = Z_1$ and $h(-4b) \equiv 0 \pmod{z}$. It implies that $h(-4b) \geq z$. Further, by (6),

$$(14) \quad h(-4b) \geq b.$$

By Lemma 3, we see from (14) that

$$(15) \quad b < \frac{4\sqrt{b}}{\pi} \log(2e\sqrt{b}),$$

whence we conclude that $b < 19$. Recall that $b \equiv 3 \pmod{4}$ and b is not a power of 3. We have $b \in \{7, 11, 15\}$. But, (14) is impossible, since $h(-4 \cdot 7) = 1$, $h(-4 \cdot 11) = 3$ and $h(-4 \cdot 15) = 2$.

When $t = 3$, we get from (9) that

$$(16) \quad b^{(y-1)/2} = \lambda_1 \lambda_2 Y_1 (3X_1^2 - bY_1^2).$$

Let $d = \gcd(Y_1, 3X_1^2 - bY_1^2)$. Since $\gcd(X_1, Y_1) = 1$, we have $d = 1$ or 3 . Notice that $\gcd(b, c) = 1$ and $\gcd(b, X_1) = 1$ by (10). If $d = 1$ and b is a power of prime, then $b \neq$ a power of 3 and $\gcd(b, 3X_1^2 - bY_1^2) = 1$. Hence, from (16) we get $Y_1 = b^{(y-1)/2}$ and

$$(17) \quad 3X_1^2 - b^y = 1,$$

since $b^y \equiv 3 \pmod{4}$. Recall that $c \equiv 1 \pmod{b}$. We get from (10) and (17) that $X_1^2 \equiv 1 \pmod{b}$ and $3X_1^2 \equiv 1 \pmod{b}$, respectively. It implies that $3 \equiv 1 \pmod{b}$, a contradiction. If $d = 3$, then $3 \mid b$, by (16). Since b is not a power of 3 , b has at least two distinct prime divisors. Therefore when $d = 1$ and $b \neq$ a power of prime or $d = 3$, by the genus theory of binary quadratic forms (see [4, Section 48]), we have $h(-4b) \equiv 0 \pmod{2}$. Further, by (8) and (10), we get $z = 3Z_1$ and $h(-4b) \equiv 0 \pmod{2z/3}$. It follows that

$$(18) \quad h(-4b) \geq \frac{2}{3}b,$$

by (6). Further, by Lemma 3, we obtain from (18) that

$$(19) \quad \frac{2}{3}b < \frac{4\sqrt{b}}{\pi} \log(2e\sqrt{b}),$$

whence we conclude that $b \leq 51$, since $3 \mid b$ for $d = 3$, we have $b \in \{15, 35, 39, 51\}$. But, (18) is impossible, since $h(-4 \cdot 15) = 2$, $h(-4 \cdot 35) = 2$, $h(-4 \cdot 39) = 4$ and $h(-4 \cdot 51) = 6$. To sum up, the theorem is proved.

Acknowledgements. This paper was supported by the National Natural Science Foundation of China (No. 10271104), the Guangdong Provincial Natural Science Foundation (No. 011781) and the Natural Science Foundation of the Education Department of Guangdong Province (No. 0161). The author would like to thank the referees for their valuable suggestions.

References

- [1] Bilu, Y., Hanrot, G., and Voutier, P. M.: Existence of primitive divisors of Lucas and Lehmer numbers. With an appendix by M. Mignotte. *J. Reine Angew. Math.*, **539**, 75–122 (2001).
- [2] Brown, E.: Diophantine equations of the form $x^2 + D = y^n$. *J. Reine Angew. Math.*, **274/275**, 385–389 (1975).
- [3] Brown, E.: Diophantine equations of the form $ax^2 + Db^2 = y^p$. *J. Reine Angew. Math.*, **291**, 118–127 (1977).
- [4] Hecke, E.: *Vorlesungen über die Theorie der algebraischen Zahlen*. Akademische Verlagsgesellschaft, Leipzig (1923).
- [5] Hua, L.-K.: *Introduction to Number Theory*. Springer Verlag, Berlin (1982).
- [6] Le, M.-H.: Some exponential diophantine equations I. The equation $D_1x^2 - D_2y^2 = \lambda k^2$. *J. Number Theory*, **55**, 209–221 (1995).
- [7] Terai, N.: On the exponential diophantine equation $a^x + l^y = c^z$. *Proc. Japan Acad.*, **77A**, 151–154 (2001).
- [8] Voutier, P. M.: Primitive divisors of Lucas and Lehmer sequences. *Math. Comp.*, **64**, 869–888 (1995).

ON THE DIOPHANTINE SYSTEM $x^2 - Dy^2 = 1 - D$ AND $x = 2z^2 - 1$

MAOHUA LE*

Abstract

Let D be a positive integer such that $D - 1$ is an odd prime power. In this paper we give an elementary method to find all positive integer solutions (x, y, z) of the system of equations $x^2 - Dy^2 = 1 - D$ and $x = 2z^2 - 1$. As a consequence, we determine all solutions of the equations for $D = 6$ and 8 .

1. Introduction

Let $\mathbb{Z}$, $\mathbb{N}$ be the sets of all integers and positive integers respectively. Let D be a positive integer with $D > 1$. The determination of all solutions (x, y, z) of the system of equations

$$(1) \quad x^2 - Dy^2 = 1 - D, \quad x = 2z^2 - 1, \quad x, y, z \in \mathbb{N}, \quad \gcd(x, y) = 1$$

is an interesting problem concerning the arithmetic properties of recurrence sequences and the solution of exponential-polynomial equations over real quadratic fields. In 1995 Mignotte and Pethő [9] determined all solution (x, y, z) for $D = 6$. Their proof relied upon deep tools related to linear form in logarithms and reduction techniques. In 1998, Cohn [4] gave an elementary proof of the above mentioned result.

In this paper we give an elementary method to find all solutions of (1) for the general case that $D - 1$ is an odd prime power. We now introduce some needful notations and known results given by Petr [10].

LEMMA 1. *Let D be a nonsquare positive integer, and let $u_1 + v_1\sqrt{D}$ be the fundamental solution of Pell equation*

$$(2) \quad u^2 - Dv^2 = 1, \quad u, v \in \mathbb{Z}.$$

Then we have

* Supported by the National Natural Science Foundation of China (No. 10271104), the Guangdong Provincial Natural Science Foundation (No. 011781) and the Natural Science Foundation of the Education Department of Guangdong Province (No. 0161).

Received September 22, 2003.

(i) All solutions (u, v) of (2) can be expressed as

$$(3) \quad u + v\sqrt{D} = (u_1 + v_1\sqrt{D})^t, \quad t \in \mathbb{Z}.$$

(ii) For any positive integer n , let

$$(4) \quad u_n + v_n\sqrt{D} = (u_1 + v_1\sqrt{D})^n.$$

Then $(u, v) = (u_n, v_n)$ ($n \in \mathbb{N}$) are all positive integer solutions of (2).

LEMMA 2. Let D be an even nonsquare positive integer, and let

$$(5) \quad D' = \begin{cases} D, \\ \frac{D}{4}, \end{cases} \quad \varepsilon = \begin{cases} 1, & \text{if } v_1 \text{ is even,} \\ 2, & \text{if } v_1 \text{ is odd.} \end{cases}$$

For a fixed D , there exists a unique positive integers pair (D_1, D_2) such that $D_1 > 1$, $D_1 D_2 = D'$ and the equation

$$(6) \quad D_1 U^2 - D_2 V^2 = 1, \quad U, V \in \mathbb{N}$$

has solution (U, V) .

LEMMA 3. Let D_1, D_2 be positive integers with $D_1 > 1$. If (6) has solutions (U, V) , then it has a unique solution (U_1, V_1) satisfying $V_1 \leq V$, where V runs through all solutions (U, V) of (6). The solution (U_1, V_1) is called the least solution of (6). Then we have:

$$(i) \quad (U_1\sqrt{D_1} + V_1\sqrt{D_2})^2 = u_1 + v_1\sqrt{D}.$$

(ii) For any odd positive integer m , let

$$(7) \quad U_m\sqrt{D_1} + V_m\sqrt{D_2} = (U_1\sqrt{D_1} + V_1\sqrt{D_2})^m.$$

Then $(U, V) = (U_m, V_m)$ for $m = 1, 3, \dots$ are all solutions of (6).

Under the mentioned notations, using a result of [5], we prove a general result as follows.

THEOREM. Let D be a positive integer such that $D-1$ is an odd prime power. If D is a square, then $D = 4$ and (1) has only the solution $(x, y, z) = (1, 1, 1)$. If D is not a square, then all solutions of (1) can be classified into the following five shapes.

$$(i) \quad (x, y, z) = (1, 1, 1).$$

$$(ii) \quad (x, y, z) = (u_{2n} + Dv_{2n}, u_{2n} + v_{2n}, \sqrt{u_n(u_n + Dv_n)}).$$

$$(iii) \quad (x, y, z) = (-u_{2n} + Dv_{2n}, u_{2n} - v_{2n}, \sqrt{Dv_n(u_n - v_n)}).$$