高等学校试用

英语理工科教材选

第二分册 物理

蔡从政 赵立华 黄 荔 编

戴鸣钟 谢卓杰 柯秉衡 主审

机械工业部部属院校选编

高 等 学 校 试 用

英语理工科教材选

Book Ⅱ

Physics

蔡从政 赵立华 黄 荔 编

戴鸣钟 谢卓杰 柯秉衡 主审

机械工业部部属院校选编

内 容 简 介

本分册是根据教育部制订的“高等工业学校普通物理学教学大纲”的要求，参照中文版普通物理教材的体系而编写的，内容为电磁学部份的前四章。每章末附有适量的习题，并提供答案。对于难度较大的句子，进行了注释，附在各章后面。书末还附有专业词汇表。

高等学校试用

英语理工科教材选

（第二分册 物理）

蔡从政 赵立华 黄 荔 编

戴鸣钟 谢卓杰 柯秉衡 主审

湖南大学教务处 出版发行

湖南大学印刷厂 印制

编 者 的 话

为了提高机械工业部部属院校学生的外语水平，培养学生阅读英语科技书刊的能力，我们选编了这套“英语理工科教材选”。整套“教材选”共分九个分册，内容包括数学、物理、理论力学、材料力学（与理论力学合为一个分册）、电工学、工业电子学、金属工艺学、机械原理、机械另件（与机械原理合为一个分册）、计算机算法语言、管理工程等十一门业务课程。各业务课都选了三章英语原版教材（个别也有选四章），供机械工业部部属院校试用。

在业务课中使用部分外语原版教材，这是我们的一次尝试，也是业务课教材改革、吸收国外先进科学技术的探索。在选材时，我们考虑了我国现行各课程的体系、内容以及学生的外语程度，尽可能选用适合我国实际的外国材料。

本“教材选”的选编工作，是在机械工业部教育局的直接领导下，由部属院校的有关教研室做了大量调查研究后选定的，并进行注释和词汇整理工作，由马泰来、卢思源、李国瑞、柯秉衡、谢卓杰、戴炜华、戴鸣钟等同志（以姓氏笔划为序）组成的审编小组，对选材的文字、注释、词汇作了审校。戴鸣钟教授担任整套“教材选”的总审。

由于时间仓促，选材、注释和编辑必有不尽完善之处，希广大读者提出宝贵意见，以利改进。

1983年4月

CONTENTS

Chapter 7. Electric Field in a Vacuum

7.1 Electric Charge. Coulomb's Law 1
7.2 Electric Field. Field Strength 4
7.3 Gauss's Theorem 10
7.4 Potential 17
7.5 Relation Between Electric Field Strength and Potential 22
7.6 Dipole 26
Problems 31
Notes 39

Chapter 8. Conductors and Dielectrics in an Electric Field

8.1 A Conductor in an External Electric Field 44
8.2 Polarization of Dielectrics 47
8.3 Field Strength in Dielectrics. Electric Displacement Vector 50
8.4 Capacitance. Capacitors 56
8.5 Energy of an Electric Field 60
Problems 64
Notes 70

Chapter 9. Steady Electric Current

9.1 Electric Current 73
9.2 Electromotive Force 76
9.3 Ohm's Law 78
9.4 Kirchhoff's Rules 83
9.5 Power of a Current. Joule-Lenz Law 87
9.6 The Elementary Classical Theory of Metals 88
9.7 Thermoelectricity 91
Problems 94
Notes 98

Chapter 10. Magnetic Field in a Vacuum

10.1 Interaction of Currents. Magnetic Field	101
10.2 The Biot-Savart Law	103
10.3 Ampere's Circuital Law	109
10.4 The Lorentz Force	118
10.5 The Magnetic Force on a Current	123
10.6 Work Done When a Current Moves in a Magnetic Field	130
Problems	132
Notes	141
Answers to Problems	145
Vocabulary	150

CHAPTER 7 ELECTRIC FIELD IN A VACUUM

7.1. Electric Charge. Coulomb's Law

All bodies in nature are capable of becoming electrified, i.e. acquiring an electric charge. The presence of such a charge manifests itself in that a charged body interacts with other charged bodies.① Two kinds of electric charges exist. They are conventionally called positive and negative. Like charges repel each other, and unlike charges attract each other.

An electric charge is an integral part of certain elementary particles*. The charge of all elementary particles (if it is not absent) is identical in magnitude. It can be called an **elementary charge**. We shall use the symbol e to denote a positive elementary charge.

The elementary particles include, in particular, the electron (carrying the negative charge $-e$), the proton (carrying the positive charge $+e$), and the neutron (carrying no charge). These particles are the bricks which the atoms and molecules of any substance are built of,② therefore all bodies contain electric charges. The particles carrying charges of different signs are usually present in a body in equal numbers and are distributed over it with the same density.③ The algebraic sum of the charges in any elementary volume of the body equals zero in this case, and each such volume (as well as the body as a whole) will be neutral. If in some way or other we create a surplus of particles of one sign in a body (and, correspondingly, a shortage of particles of the opposite sign), the body will be charged. It is also possible, without changing the total number of positive and negative particles, to cause them to be redistributed in a body so that one part of it has a surplus of charges of one sign and the other part a surplus of charges of the opposite sign.④ This can be done by bringing a charged body close to an uncharged metal one.

Since a charge q is formed by a plurality of elementary charges, it is an integral multiple of e:

$$q = \pm Ne. \qquad (7.1)$$

An elementary charge is so small, however, that macroscopic charges may be

* Elementary particles are defined as such microparticles whose internal structure at the present level of development of physics cannot be conceived as a combination of other particles.

considered to have continuously changing magnitudes.

If a physical quantity can take on only definite discrete values, it is said to be quantized. The fact expressed by Eq.(7.1) signifies that an electric charge is quantized.

Electric charges can vanish and appear again. Two elementary charges of opposite signs always appear or vanish simultaneously, however. For example, an electron and a positron (a positive electron) meeting each other annihilate, i.e. transform into neutral gamma-photons. This is attended by vanishing of the charges $-e$ and $+e$. In the course of the process called the birth of a pair, a gamma-photon getting into the field of an atomic nucleus transforms into a pair of particles—an electron and a positron. This process causes the charges $-e$ and $+e$ to appear.

Thus, the total charge of an electrically isolated system* cannot change. This statement forms the law of electric charge conservation.

The law obeyed by the force of interaction of point charges was established experimentally in 1785 by the French physicist Charles A. de Coulomb (1736-1806). A point charge is defined as a charged body whose dimensions may be disregarded in comparison with the distances from this body to other bodies carrying an electric charge.⑤

Using a torsion balance (Fig. 7.1), Coulomb measured the force of interaction of two charged spheres depending on the magnitude of the charges on them and on the distance between them. He proceeded from the fact that when a charged metal sphere was touched by an identical uncharged sphere, the charge would be distributed equally between the two spheres.

As a result of his experiments, Coulomb arrived at the conclusion that the force of interaction between two stationary point charges is proportional to the magnitude of each of them and inversely proportional to the square of the distance between them. The direction of the force coincides with the straight line connecting the charges.

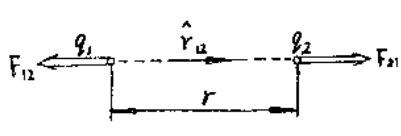

Fig. 7.2

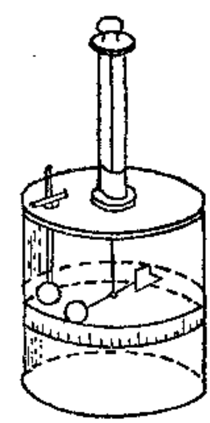

Fig. 7.1

Coulomb's law can be expressed by the formula

$$\mathbf{F}_{12} = -k\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}_{12}. \qquad (7.2)$$

* A system is referred to as electrically isolated if no charged particles can penetrate through the surface confining it.

Here k = proportionality constant assumed to be positive,

q_1 and q_2 = magnitudes of the interacting charges,

r = distance between the charges,

$\hat{r}_{12}$ = unit vector directed from the charge q_1 to q_2,

$\mathbf{F}_{12}$ = force acting on the charge q_1 (Fig. 7.2; the figure corresponds to the case of like charges).

The force $\mathbf{F}_{21}$ differs from $\mathbf{F}_{12}$ in its sign:

$$\mathbf{F}_{21} = k\frac{q_1 q_2}{r^2}\hat{r}_{12}. \tag{7.3}$$

The magnitude of the interaction force, which is the same for both charges, can be written in the form

$$F = k\frac{|q_1 q_2|}{r^2}. \tag{7.4}$$

Experiments show that the force of interaction between two given charges does not change if other charges are placed near them. Assume that we have the charge q_a and, in addition, N other charges q_1, $q_2, \cdots, q_N$. It can be seen from the above that the resultant force $\mathbf{F}$ with which all the N charges q_i act on q_a is

$$\mathbf{F} = \sum_{i=1}^{N}\mathbf{F}_{a,i}, \tag{7.5}$$

where $\mathbf{F}_{a,i}$ is the force with which the charge q_i acts on q_a in the absence of the other $N-1$ charges.

All experimental facts available lead to the conclusion that Coulomb's law holds for distances from 10^{-15} m to at least several kilometres.⑥ There are grounds to presume that for distances smaller than 10^{-16} m the law stops being correct.⑦ For very great distances, there are no experimental confirmations of Coulomb's law. But there are also no reasons to expect that this law stops being obeyed with very great distances between charges.⑧

The SI unit of charge is the **coulomb** (C). Careful measurements showed that an elementary charge is

$$e = 1.60 \times 10^{-19}\ \text{C}. \tag{7.6}$$

In SI units we can measure q_1, q_2, r, and F in Eq (7.4) in ways that do not depend on Coulomb's law. Numbers with units can be assigned to them. There is no choice about the proportionality constant k: it must have that value which makes the right-hand side of Eq. (7.4) equal to the left-hand side. This value turns out to be

$$k = 9 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2. \tag{7.7}$$

The proportionality constant k is usually written in a more complex way as $1/4\pi\varepsilon_0$. Hence, Eq.(7.4) can be written in the form

$$F = \frac{1}{4\pi\varepsilon_0} \frac{|q_1 q_2|}{r^2}. \qquad (7.8)$$

Certain equations that are derived from Eq. (7.4), but are used more often than it is, will be simpler in form if we do this.⑨

This modified way of writing formulas is called **rationalized.** Systems of units constructed with the use of rationalized formulas are also called **rationalized.** They include the SI system.

The quantity ε_0 is called the **electric constant.**

$$\varepsilon_0 = \frac{1}{4\pi k} = \frac{1}{4\pi \times 9 \times 10^9} = 8.85 \times 10^{-12}\ C^2/N\cdot m^2. \qquad (7.9)$$

7.2. Electric Field. Field Strength

Charges at rest interact through an **electric field.** A charge alters the properties of the space surrounding it—it sets up an electric field in it. This field manifests itself in that an electric charge placed at a point of it experiences the action of a force. Hence, to see whether there is an electric field at a given place, we must place a charged body (in the following we shall say simply a charge for brevity) at it and determine whether or not it experiences the action of an electric force. We can evidently assess the "strength" of the field according to the magnitude of the force exerted on the given charge.

Thus, to detect and study an electric field, we must use a "test" charge. For the force acting on our test charge to characterize the field "at the given point", the test charge must be a point one.

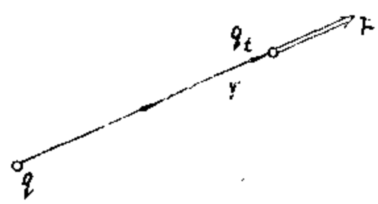

Fig. 7.3

Let us study the field set up by the stationary point charge q with the aid of the point test charge q_t. We place the test charge at a point whose position relative to the charge q is determined by the position vector $\mathbf{r}$ (Fig.7.3). We see that the test charge experiences the force

$$\mathbf{F} = q_t\left(\frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}\hat{\mathbf{r}}\right) \qquad (7.10)$$

(see Eqs. (7.3) and (7.8)). Here $\hat{\mathbf{r}}$ is the unit vector of the position vector $\mathbf{r}$.

A glance at Eq.(7.10) shows that the force acting on our test charge depends not only on the quantities determining the field (on q and $\mathbf{r}$), but also on the magnitude of the test charge q_t.⑩ If we take different test charges q'_t, q''_t, etc., then the forces $\mathbf{F}'$, $\mathbf{F}''$, etc. which they experience at the given point of the field will be different. We can see from Eq. (7.10), however, that the ratio $\mathbf{F}/q_t$ for all the test charges will be the same and depend only on the values of q and r determining the field at the given point. It is therefore natural to adopt this ratio as the quantity characterizing an electric field:

$$\mathbf{E}=\frac{\mathbf{F}}{q_t}. \tag{7.11}$$

This vector quantity is called the **electric field strength (or intensity)** at a given point (i.e. at the point where the test charge q_t experiences the action of the force $\mathbf{F}$).

According to Eq. (7.11), the electric field strength numerically equals the force acting on a unit point charge at the given point of the field. The direction of the vector $\mathbf{E}$ coincides with that of the force acting on a positive charge.

It must be noted that Eq. (7.11) also holds when the test charge is negative ($q_t<0$). In this case, the vectors $\mathbf{E}$ and $\mathbf{F}$ have opposite directions.

We have arrived at the concept of electric field strength when studying the field of a stationary point charge. Definition (7.11), however, also covers the case of a field set up by any collection of stationary charges. But here the following clarification is needed. The arrangement of the charges setting up the field being studied may change under the action of the test charge.① This will happen, for example, when the charges producing the field are on a conductor and can freely move within its limits. Therefore, to avoid appreciable alterations in the field being studied, a sufficiently small test charge must be taken.

It follows from Eqs. (7.11) and (7.10) that the field strength of a point charge varies directly with the magnitude of the charge q and inversely with the square of the distance r from the charge to the given point of the field:

$$\mathbf{E}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}. \tag{7.12}$$

The vector $\mathbf{E}$ is directed along the radial straight line passing through the charge and the given point of the field, from the charge if the latter is positive and toward the charge if it is negative.

The unit of electric field strength is the strength at a point where unit force (1 N) acts on unit charge (1 C). The SI unit of electric field strength is called the newton per coulomb (N/C) [or the volt per metre (V/m), see Eq. (7.54)].

According to Eq. (7.11), the force exerted on a test charge is

$$\mathbf{F}=q_t\mathbf{E}.$$

It is obvious that any point charge q^* at a point of a field with the strength $\mathbf{E}$ will

---

* In Eq. (7.12), q stands for the charge setting up the field. In Eq. (7.13), q stands for the charge experiencing the force F at a point of strength E.

experience the force

$$\mathbf{F}=q\mathbf{E}. \qquad (7.13)$$

If the charge q is positive, the direction of the force coincides with that of the vector $\mathbf{E}$. If q is negative, the vectors $\mathbf{F}$ and $\mathbf{E}$ are directed oppositely.

We mentioned in the preceding section that the force with which a system of charges acts on a charge not belonging to the system equals the vector sum of the forces which each of the charges of the system exerts separately on the given charge [see Eq. (7.5)].⑬ Hence it follows that the field strength of a system of charges equals the vector sum of the field strengths that would be produced by each of the charges of the system separately:

$$\mathbf{E}=\sum\mathbf{E}_i. \qquad (7.14)$$

This statement is called the **principle of electric field superposition**.

If the charge distribution is a continuous one, the field it sets up at any point p can be computed by dividing the charge into infinitesimal elements dq. The field $d\mathbf{E}$ due to each element at the point in question is then calculated, treating the elements as point charges. The magnitude of $d\mathbf{E}$ (see Eq.(7.12)) is given by

$$dE=\frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}, \qquad (7.15)$$

where r is the distance from the charge element dq to the point p. The resultant field at p is then found from the superposition principle by adding (that is, integrating) the field contributions due to all the charge elements, or

$$\mathbf{E}=\int d\mathbf{E}. \qquad (7.16)$$

The integration, like the sum in Eq. (7.14), is a vector operation; in Example 7.1 we will see how such an integral is handled in a simple case.

Example 7.1

Ring of charge. Fig. 7.4 shows a ring of charge q and radius a. Calculate $\mathbf{E}$ for points on the axis of the ring a distance x from its center.

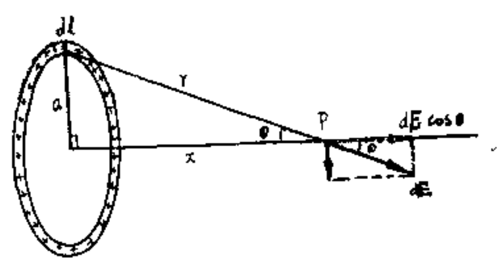

Fig. 7.4

Consider a differential element of the ring of length dl, located at the top of the ring in Fig. 7.4. It contains an element of charge given by

$$dq=q\frac{dl}{2\pi a},$$

where $2\pi a$ is the circumference of the ring. This element sets up a differential electric field $d\mathbf{E}$ at point p.

The resultant field $\mathbf{E}$ at p is found by integrating the effects of all the elements that make up the ring. From symmetry this resultant field must lie along the ring axis. Thus only the component of $d\mathbf{E}$ parallel to this axis contributes to the final result. The component perpendicular to the axis is canceled out by an equal but opposite component established by the charge element on the opposite side of the ring.

Thus the general vector integral (Eq. 7.16)

$$\mathbf{E}=\int d\mathbf{E}$$

becomes a scalar integral $$E=\int dE\cos\theta.$$

The quantity dE follows from Eq. (7.15), or

$$dE=\frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}=\frac{1}{4\pi\varepsilon_0}\left(\frac{q\,dl}{2\pi a}\right)\frac{1}{a^2+x^2}.$$

From Fig. 7.4 we have $$\cos\theta=\frac{x}{\sqrt{a^2+x^2}}.$$

Noting that, for a given point p, x has the same value for all charge elements and is not a variable and that l is the variable of integration, we obtain

$$E=\int dE\cos\theta=\int\frac{1}{4\pi\varepsilon_0}\frac{q\,dl}{(2\pi a)(a^2+x^2)}\frac{x}{\sqrt{a^2+x^2}}$$
$$=\frac{1}{4\pi\varepsilon_0}\frac{qx}{(2\pi a)(a^2+x^2)^{3/2}}\int dl.$$

The integral is simply the circumference of the ring($=2\pi a$), so that

$$E=\frac{1}{4\pi\varepsilon_0}\frac{qx}{(a^2+x^2)^{3/2}}.$$

Does this expression for E reduce to an expected result for $x=0$? For $x\gg a$ we can neglect a in the denominator of this equation, yielding

$$E\cong\frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}.$$

This is an expected result [compare Eq. (7.12)] because at great enough distances the ring behaves like a point charge q.

Example 7.2

Infinite line of charge. Fig.7.5 shows a section of an infinite line of charge whose linear charge density (that is, the charge per unit length, measured in C/m) has the constant value λ. Calculate the field $\mathbf{E}$ a distance y from the line.

The magnitude of the field contribution dE due to charge element $dq(=\lambda\,dx)$ is given, using Eq.(7.15), by

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{y^2+x^2}.$$

The vector $d\mathbf{E}$, as Fig. 7.5 shows, has the components

$$dE_x = -dE \sin\theta,$$
$$dE_y = dE \cos\theta.$$

The minus sign in front of dE_x indicates that $d\mathbf{E}_x$ points in the negative x direction. The x and y components of the resultant vector $\mathbf{E}$ at point p are given by

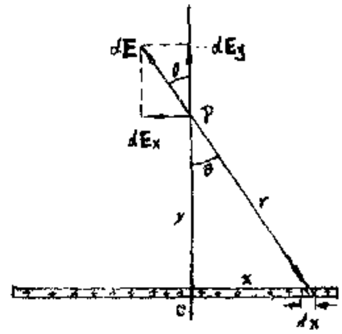

Fig. 7.5

$$E_x = \int dE_x = -\int_{x=-\infty}^{x=+\infty} \sin\theta\, dE \quad \text{and} \quad E_y = \int dE_y = \int_{x=-\infty}^{x=+\infty} \cos\theta\, dE.$$

E_x must be zero because every charge element on the right has a corresponding element on the left such that their field contributions in the x direction cancel. Thus $\mathbf{E}$ points entirely in the y direction. Because the contributions to E_y from the right- and left-hand halves of the rod are equal, we can write

$$E = E_y = 2\int_{x=0}^{x=+\infty} \cos\theta\, dE.$$

Note that we have changed the lower limit of integration and have introduced a compensating factor of two.

Substituting the expression for dE into this equation gives

$$E = \frac{\lambda}{2\pi\varepsilon_0}\int_{x=0}^{x=\infty} \cos\theta \frac{dx}{y^2+x^2}.$$

From Fig. 7.5, we see that the quantities θ and x are not independent. We must eliminate one of them, say x. The relation between x and θ is (see figure)

$$x = y \tan\theta.$$

Differentiating, we obtain $dx = y \sec^2\theta\, d\theta.$

Substituting these two expressions leads finally to

$$E = \frac{\lambda}{2\pi\varepsilon_0 y}\int_{\theta=0}^{\theta=\pi/2} \cos\theta\, d\theta.$$

You should check this step carefully, noting that the limits must now be on θ and not on x. For example, as $x \to +\infty$, $\theta \to \pi/2$, as Fig. 7.5 shows. This equation integrates readily to

$$E = \frac{\lambda}{2\pi\varepsilon_0 y}\left(\sin\theta\right)\Big|_0^{\pi/2} = \frac{\lambda}{2\pi\varepsilon_0 y}.$$

You may wonder about the usefulness of solving a problem involving an infinite rod of charge when any actual rod must have a finite length. However, for points close enough to finite rods and not near their ends, the equation that we have just derived yields results that are so close to the correct values that the difference can be ignored in many practical situations.⑲ It is usually unnecessary to solve exactly every geometry encountered in practical problems. Indeed, if idealizations or approximations are not made, the vast majority of significant problems of all kinds in physics and engineering cannot be solved at all.

An electric field can be described by indicating the magnitude and direction of the vector **E** for each of its points. The combination of these vectors forms the field of the electric field strength vector (compare with the field of the velocity vector). The velocity vector field can be represented very illustratively with the aid of flow lines. Similarly, an electric field can be described with the aid of strength lines, which we shall call for short **E** lines or field lines. These lines are drawn so that a tangent to them at every point coincides with the direction of the vector **E**. The density of the lines is selected so that their number passing through a unit area at right angles to the lines equals the numerical value of the vector **E**. Hence, the pattern of field lines permits us to assess the direction and magnitude of the vector **E** at various points of space (Fig. 7.6).

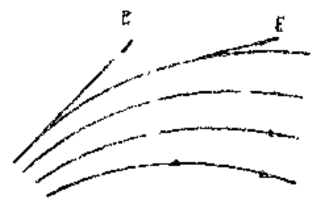

Fig. 7.6

The **E** lines of a point charge field are a collection of radial straight lines directed away from the charge if it is positive and toward it if it is negative (Fig.7.7). One end of each line is at the charge, and the other extends to infinity. Indeed, the total number of lines intersecting a spherical surface of arbitrary radius r will equal the product of the density of the lines and the surface area of the sphere $4\pi r^2$. We have assumed that the density of the lines numerically equals $E=(1/4\pi\varepsilon_0)(q/r^2)$. Hence, the number of lines is $(1/4\pi\varepsilon_0)(q/r^2)\,4\pi r^2=q/\varepsilon_0$. This result signifies that the number of lines at any distance from a charge will be the same. It thus follows that the lines do not begin and do not terminate anywhere except

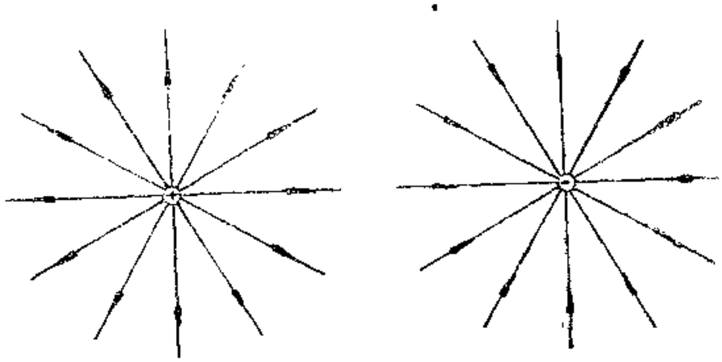

Fig. 7.7

for the charge. Beginning at the charge, they extend to infinity (the charge is positive), or arriving from infinity, they terminate at the charge (the latter is negative). This property of the **E** lines is common for all electrostatic fields, i.e. fields set up by any system of stationary charges: the field lines can begin or terminate only at charges or extend to infinity.

7.3. Gauss's Theorem

Before we discuss Gauss's theorem we must develop a new concept, that of the flux of a vector field.

Assume that the flow of a liquid is characterized by the field of the velocity vector. The volume of liquid flowing in unit time through an imaginary surface S is called the flux of the liquid through this surface. To find the flux, let us divide the surface into elementary sections of the size ΔS. It can be seen from Fig. 7.8 that during the time Δt a volume of liquid equal to

$$\Delta V = (\Delta S \cos \alpha)\, v\, \Delta t$$

will pass through section ΔS. Dividing this volume by the time Δt, we shall find the flux through surface ΔS:

$$\Delta \Phi = \frac{\Delta V}{\Delta t} = \Delta S v \cos \alpha.$$

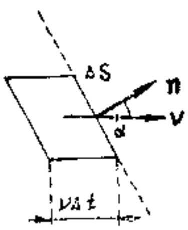

Fig. 7.8

Passing over to differentials, we find that

$$d\Phi = (v \cos \alpha)\, dS = v_n\, dS. \qquad (7.17)$$

We can introduce the vector $d\mathbf{S}$ whose magnitude equals that of area dS, while its direction coincides with the direction of a normal $\mathbf{n}$ to the area:

$$d\mathbf{S} = dS\, \mathbf{n}.$$

Since the direction of the vector $\mathbf{n}$ is chosen arbitrarily (it can be directed to either side of the area), then $d\mathbf{S}$ is not a true vector, but is a pseudo vector. The angle α in Eq. (7.17) is the angle between the vectors $\mathbf{v}$ and $d\mathbf{S}$. Hence, this equation can be written in the form

$$d\Phi = \mathbf{v} \cdot d\mathbf{S}. \qquad (7.18)$$

By summating the fluxes through all the elementary areas into which we have divided surface S, we get the flux of the liquid through S:

$$\Phi_v = \int_S \mathbf{v} \cdot d\mathbf{S} = \int_S v_n\, dS. \qquad (7.19)$$

A similar expression written for an arbitrary vector field **a**, i.e. the quantity

$$\Phi_a = \int_S \mathbf{a} \cdot d\mathbf{S} = \int_S a_n\, dS \qquad (7.20)$$

is called the flux of the vector a through surface S. In accordance with this definition, the flux of a liquid can be called the flux of the vector v through the relevant surface (see Eq. (7.19)).

The flux of a vector is an algebraic quantity. Its sign depends on the choice of the direction of a normal to the elementary areas into which surface S is divided in calculating the flux. Reversal of the direction of the normal changes the sign of a_n and, therefore, the sign of the quantity (7.20).

We can give an illustrative geometrical interpretation of the vector flux. For this purpose, we shall represent a vector field by a system of lines a constructed so that the density of the lines at every point is numerically equal to the magnitude of the vector a at the same point of the field (compare with the rule for constructing the lines of the vector E set out at the end of the preceding section). let us find the number ΔN of intersections of the field lines with the imagimary area ΔS. A glance at Fig. 7.9 shows that this number equals the density of the lines (i.e. a) multiplied by $\Delta S_{\perp} = \Delta S \cos \alpha$:

$$\Delta N (=) a \, \Delta S \cos \alpha = a_n \, \Delta S.$$

We are speaking only about the numerical equality between ΔN and $a_n \, \Delta S$. This is why the equality sign is confined in parentheses. According to Eq. (7.20), the expression $a_n \, \Delta S$ is $\Delta \Phi_a$—the flux of the vector a through area ΔS. Thus,

$$\Delta N (=) \Delta \Phi_a. \qquad (7.21)$$

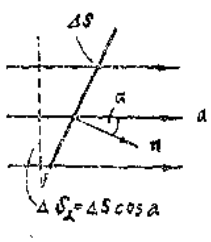

Fig. 7.9

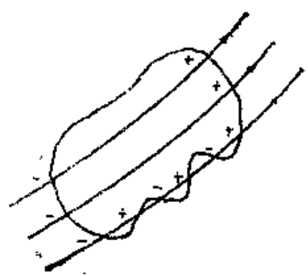

Fig. 7.10

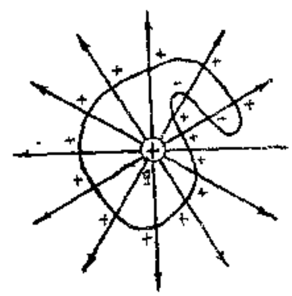

Fig. 7.11

For the sign of ΔN to coincide with that of $\Delta \Phi_a$, we must consider those intersections to be positive for which the angle α between the positive direction of a field line and a normal to the area is acute.⑬ The intersection should be considered negative if the angle α is obtuse.

An outward normal is considered to be positive for a closed surface (Fig.7.10). Therefore, the intersections corresponding to outward protrusion of the lines (in this case the angle α is acute) must be taken with the plus sign, and the ones appearing when the lines enter the surface (in this case the angle α is obtuse) must be taken with the minus sign.