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大學代數

UNDERGRADUATE ALGEBRA

(英文本)

徐氏基金會出版

Translated from the original French text *Cours d'Algèbre*, first published by Hermann in 1963.

本書原著者乃法國巴黎大學數學系主任教授，於 1963 年以法文出版。書名 COURS D'ALGÈBRE，各國均有譯本。英文譯本係根據 1966 年之法文訂正版於 1969 年在美國出版。中文譯本由徐氏基金會出版。當代世界各大學多採用為一二年級課本或作教師與學生之參考用書。

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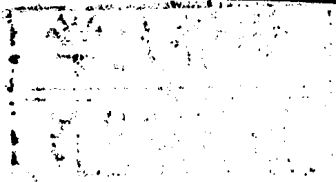
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Preface

This book is based on lecture-courses given by the author at the University of Paris. Although designed to meet the needs of French undergraduates, it covers the beginning algebra course in an American University and the average Honors mathematics course in a British University. The book contains a thorough treatment of linear algebra and can therefore be used as an algebra textbook by the student throughout his undergraduate career.

The topics covered in this book are those which are universally considered to be essential for the future mathematician or physicist: sets and functions; groups, rings, fields, complex numbers; vector spaces, linear mappings, matrices; finite-dimensional vector spaces, systems of linear equations, determinants, Cramer's formulae; polynomials, rational fractions, algebraic equations; reduction of matrices. This choice of subject-matter reflects the evolution of mathematics in the last half-century, and we have thought it proper that this evolution should also be reflected by the use of a style which hitherto has been reserved for treatises addressed to professional mathematicians.

Many people—in particular the majority of those whose attitude to mathematics is purely *utilitarian*—are of the opinion that, in textbooks designed to be read by beginners, it is useless or even dangerous to put great emphasis on rigour, to prove every statement, to introduce notions of great generality or to use carefully and strictly defined terminology. If this view were correct it would imply that, contrary to all professional mathematicians and to common sense, the worst-written textbooks would be those most easily understood by the beginner. The professional latinists succeed in deciphering the truncated and incomplete inscriptions which are continually being dug up from the subsoil of Italy: this is their expertise: but no professor of Latin has yet had the idea of using such texts for teaching the language to beginners. Instead he prefers to rely on a well-written grammar. It is just the same in mathematics: when it is a question of interpreting correctly the sense of an obscurely phrased definition, of filling in the gaps in an incomplete proof, or of uncovering the real reasons why a certain theorem is true, it is unreasonable to expect the novice to display the same flair as the professional.

It should also be remarked that the progress of mathematics in this century has brought with it the possibility of a substantial renovation of mathematical teaching. New concepts have emerged which by their generality and simplicity can considerably enlarge the range of application of a traditional piece of mathematical reasoning, and new proofs have been discovered which bring within the reach of

undergraduate students results which used to be considered too difficult for them. Moreover, the concern for rigour, which has always been characteristic of the great mathematicians of the theory of numbers, has in the last thirty years spread through all branches of mathematics and is now filtering down (with varying success) to the authors of textbooks, to the point that some of them are in this respect in advance of the body of professional mathematicians . . . This renovation, and the exaggerations which sometimes go with it, brings protests from some users of mathematics who are irritated that they have difficulty in understanding their children's textbooks, and the reproach is sometimes heard that mathematicians exaggerate the importance of their own contributions instead of directing the attention of their students to more concrete problems. Undoubtedly there is a basis of truth here; but from this point of view what is to be said of the space research specialists who see nothing remarkable in expending vast sums of money to send a rocket probe to Venus whilst hundreds of millions of their fellow human beings are on the edge of starvation? At least mathematics has the advantage of being cheap.

At the risk of arousing in certain quarters the feelings of horror and consternation so marvellously represented by Paolo Uccello in his *Desecration of the Host*, we feel obliged—for the question arises more and more often—to record our disagreement with the large number of public personalities at the present time who demand of scientists in general and mathematicians in particular that they should devote their energies to producing the legions of technologists whose existence is, it appears, urgently indispensable to our survival. Things being as they are, it seems to us that in the scientifically and technologically over-developed “great” nations in which we live, the first duty of the mathematician—and of many others—is to produce what is *not* demanded of him, namely men who are capable of thinking for themselves, of unmasking false arguments and ambiguous phrases, and to whom the dissemination of truth is infinitely more important than, for example, world-wide three-dimensional colour T.V.: free men, and not robots ruled by technocrats. It is sad but true that the best way of producing such men does not consist in teaching them mathematics and physical science; for these are branches of knowledge which ignore the very existence of human problems, and it is a disturbing thought that our most highly civilized societies accord them the first place. But even in the teaching of mathematics it is at least possible to attempt to impart a taste for freedom and reason, and to accustom the young to being treated as human beings endowed with the faculty of reason.

To return to the beginners in mathematics, to whom this book is addressed: we have therefore sought to speak to them in the language of professional mathematicians, by defining all technical terms unambiguously once and for all, by stating all theorems explicitly, and by proving them all completely, with a few exceptions imposed by the need to keep within reasonable limits (*).

(*) Almost all of the theorems which are not proved are in §§ 0 to 5; it is clearly out of the question, in an elementary textbook, to give an account of the theory of sets and formal logic without assuming many “obvious” results. The purpose of § 0, on logical reasoning, is not only to explain to the beginner which types of argument are “legitimate” and which are not (anyone who has marked examination papers will be convinced of the necessity of this), but also to show that the “philosophy of mathematics” does not necessarily reduce to a verbalism devoid of structure.