高等学校试用

英语理工科教材选

第七分册　机械原理与机械零件

赵枢中　曹惟庆（主编）

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机械工业部部属院校选编

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Book Ⅶ

Theory of Machine and Mechanical Elements

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编者的话

为了提高机械工业部部属院校学生的外语水平，培养学生阅读英语科技书刊的能力，我们选编了这套“英语理工科教材选”。整套“教材选”共分九个分册，内容包括数学、物理、理论力学、材料力学（与理论力学合为一个分册）、电工学、工业电子学、金属工艺学、机械原理、机械另件（与机械原理合为一个分册）、计算机算法语言、管理工程等十一门业务课程。各业务课都选了三章英语原版教材（个别也有选四章），供机械工业部部属院校试用。

在业务课中使用部分外语原版教材，这是我们的一次尝试，也是业务课教材改革、汲收国外先进科学技术的探索。在选材时，我们考虑了我国现行各课程的体系、内容以及学生的外语程度，尽可能选用适合我国实际的外国材料。

本“教材选”的选编工作，是在机械工业部教育局的直接领导下，由部属院校的有关教研室做了大量调查研究后选定的，并进行注释和词汇整理工作。由马泰来、卢思源、李国瑞、柯秉衡、谢卓杰、戴炜华、戴鸣钟等同志（以姓氏笔划为序）组成的审编小组，对选材的文字、注释、词汇作了审校。戴鸣钟教授担任整套“教材选”的总审。

由于时间仓促，选材、注释和编辑必有不尽完善之处，希广大读者提出宝贵意见，以利改进。

1983年4月

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1

MECHANISM TERMINOLOGY AND CHARACTERISTICS

1.10 INTRODUCTION

Typically, the design of a new machine begins when there is a perceived need for a mechanical device with certain (often vaguely defined) characteristics. The degree of need may be determined by a formal or informal market analysis and economic study.

To fulfill the identified need, a **conceptual** or **inventive phase** of the design process is required to establish the general nature and type of machine. It may well be that the art of invention is not communicable to any great degree.[1] But all of us, whether endowed with great or little talent for invention, can profit through familiarity with the major features of existing machines. Many of the most important mechanisms in ①
use today will be studied in this book. However, it is highly recommended that the prospective inventor peruse the many existing qualitative descriptions, surveys, and catalogues of existing machines. References to such source materials are given in Sec. **1.55**.

Having arrived at a concept of the general form of the device, one usually makes a preliminary geometric layout, which is subjected to a **kinematic analysis** in order to determine whether the displacements, velocities, and accelerations are suitable for the intended purpose. Closely related to the process of analysis is the process of **kinematic** ②

[1] "At present, questions of this kind can only be solved by that species of intuition which long familiarity with a subject usually confers upon experienced persons, but which they are totally unable to communicate to others. When the mind of a mechanician is occupied with the contrivance of a machine, he must wait until, in the midst of his meditations, some happy combination presents itself to his mind which may answer his purpose."—R. Willis [1870].

synthesis,[2] which is a systematic attempt to compute the major kinematic design vari-
ables (link lengths, fixed angles, pivot locations, etc.). In this book, we shall concen- ③
trate on the problem of *analysis*, as opposed to *direct synthesis*. Sufficiently general ④
and automatic analysis procedures, such as those to be developed in Part 2, make it
practical to reach a satisfactory design by repeated iterations of an "analyze-modify"
procedure. Direct procedures for kinematic synthesis may be found in a number of ⑤
publications cited in Sec. **5.12**.

For low-speed machinery, the kinematic analysis may be followed directly by a **static analysis** of the forces which the links and bearings must withstand. On the basis of these forces, the link cross sections and bearing dimensions may be properly designed by suitable methods of stress analysis and lubrication theory.

For high-speed machinery, or systems with several degrees of freedom, it is frequently necessary to make a complete **dynamic analysis** of the system, because of the pronounced effects of inertia forces. Considerations of statics and dynamics will be taken up in Part 3.

We begin our study of kinematics of machinery by introducing certain fundamental terms and concepts.

1.20 TERMINOLOGY OF MECHANISM THEORY

The evolution of machines and mechanisms is interwoven with the development
of man's social and economic systems. Thus we find an extensive discussion of ⑥
mechanical devices, and their impact on society, in the work of the noted economic
historian A. B. Usher [1929]. A beautifully succinct historical outline of the science ⑦
of kinematics is given by Hartenberg and Denavit [1964]; a longer, more general,
discussion of machinery, from the time of Watt, is provided by Ferguson [1962]; and
the role of machinery in the history of mechanical engineering is well described by
Burstall [1965]. All writers, on both the historical and technical aspects of mechanisms,
owe a great debt to the penetrating work of Franz Reuleaux [1876]. ⑧

1.21 Definitions of Machine

Until the time of Reuleaux it was usually accepted that there exists a small number
of *simple machines*, which, acting in combination to form so-called *compound ma-
chines*, could produce the most general form of mechanical device. However, as
pointed out by Reuleaux [1876, p. 275], previous writers could not even agree on the
number of simple machines, much less their form. Hero of Alexandria (ca. 50 A.D.) ⑨
had described five *simple machines* or *mechanical powers* (wheel and axle, lever, pulley,
wedge, and screw) used for lifting weights. According to Usher [1929, p. 120], the
Arabic version of Hero's book was called "the book on the raising of heavy weights."
Upon translation into Greek, the title became "Baroulkos" ("the elevator") or
"Mechanics." This last term, says Usher, literally meant "lifting of heavy weights" in
Hero's time. Over the years the word *mechanics* took on a more general meaning, but
most writers on mechanics clung to the idea that *all* machines could be compounded

[2]The determination of design dimensions is often called **number synthesis**, as opposed to **type synthesis**, which denotes the *conceptual* or *inventive phase* of the design cycle.

from the simple machines. The number of such *simple machines* wandered up consid- (10)
erably (in order to include the inclined plane, toothed wheel, cord, etc.) and down to as few as two (since the wheel, lever, and pulley have similar static characteristics, and the wedge, inclined plane, and screw have similar static characteristics). (11)

Although the concept of mechanical powers had some merit when restricted to weight lifting machines, it has no great significance in modern times and has been virtually ignored in contemporary serious works on mechanics. (12)

Most definitions of the term *machine* to be found in current technical literature are variations on the following definition[3] of Reuleaux: "A machine is a combination of resistant bodies so arranged that by their means the mechanical forces of nature can be compelled to do work accompanied by certain determinate motions." It should (13)
also be noted that in current technical usage the term *machine* connotes a device which transmits significant levels of force, as in an automobile engine. When the force transmitted is small and the principal function of the device is to transmit or modify motion, as in a clock, we usually refer to it as a mechanism. A more precise definition (14)
of the term *mechanism* will now be given.

1.22 Kinematic Chains and Mechanisms

The individual rigid bodies which collectively form a machine are said to be **members** or **links**. Links may consist of nonrigid bodies, such as cables or fluid columns, which momentarily serve the same function as rigid bodies and are sometimes referred to as **resistant bodies**. The links are interconnected in pairs at points of contact called **joints**. That part of a link's surface which contacts another link is called a **pair element**. The combination of two such elements constitutes a **kinematic pair**. Note the difference between a *pair* and a *joint*. A joint connecting two links constitutes a **simple pair**. **Double pairs**, **triple pairs**, or **multiple pairs**, in general, occur at joints where three, four, or more links are connected. Simple and double pairs are illustrated at joints C and D, respectively, in Fig. 2(b). A joint which connects $N + 1$ members is said to have **multiplicity** N.

An assemblage of interconnected links is called a **kinematic chain**. When one link of a kinematic chain is held fixed, the chain is said to form a **mechanism**, provided that any of its links may move. The **fixed link** is also called the **ground link** or **frame**. (15)
Different mechanisms may be formed from the same chain by fixing different links; all mechanisms formed from the same chain are said to be **inversions** of one another. When all motion is prohibited by the fixing of one link, the chain is referred to as a **structure**.

Most mechanisms consist of **closed chains** wherein each link is connected to at least two other links of the system. Links containing two, three, or four joints are called **binary**, **ternary**, or **quaternary links**. A chain which is not closed, such as a double pendulum, is said to be **open**. An open chain can contain links with only one joint, which are properly called **unitary links**, although they are sometimes referred to by the overworked adjective **singular**. A **closed mechanism** is formed from a closed chain, and an **open mechanism** from an open chain.

---

[3]See p. 35 in Reuleaux [1876]. Note that *24* other definitions, used by earlier writers, are listed on pp. 583–588 of this reference.

Any variable angle or length, associated with a given system, is said to be a **position variable** or **configuration variable**. The minimum number of position variables needed to fully define the configuration of a system is called the **degree of freedom (DOF)** of the system. The DOF of a *mechanism* is often called its **mobility**. Figure 1 shows various closed kinematic chains, each with N **hinged** joints and N links ($N = 3, 4, 5$) including the ground link (1); such systems are called **N-bar chains**.

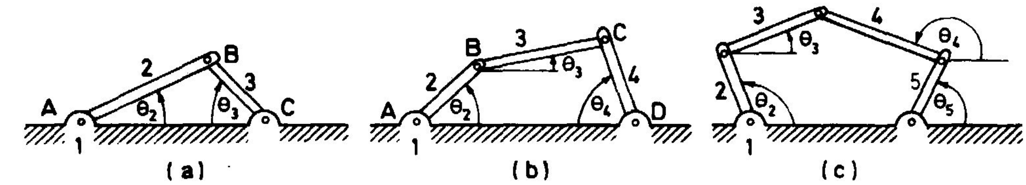

Figure 1.22-1. (a) 3-bar chain or 3 hinged truss (0 D.O.F.); (b) 4-bar mechanism (1 D.O.F.); (c) 5-bar mechanism (2 D.O.F.).

For the three-bar chain of Fig. 1(a) we see that the system is rigid, and therefore it constitutes a *structure* (called a **three-bar truss**). In other words, the angles θ_2 and θ_3 are fixed, and the system has zero DOF. For the four-bar mechanism of Fig. 1(b) we see that if angle θ_2 is specified, the other configuration variables, θ_3 and θ_4, may be calculated[4] (or measured from a drawing); therefore the system has one DOF. Similarly, if we fix angles θ_2 and θ_3 in Fig. 1(c), the remaining angle variables are determined, and the five-bar mechanism is seen to have 2 DOF.

The closed three-bar truss contains the least number of members of any N-bar hinged polygonal chain. Accordingly, it represents the fundamental *building block* for the generation of rigid structures. For example, if we add the two links 4 and 5 to the three-bar truss 1, 2, 3 of Fig. 2(a), the new system is a rigid structure. The process may be repeated by adding bars two at a time (e.g., bars 6 and 7) to form a triangulated rigid structure of just about any shape desired.

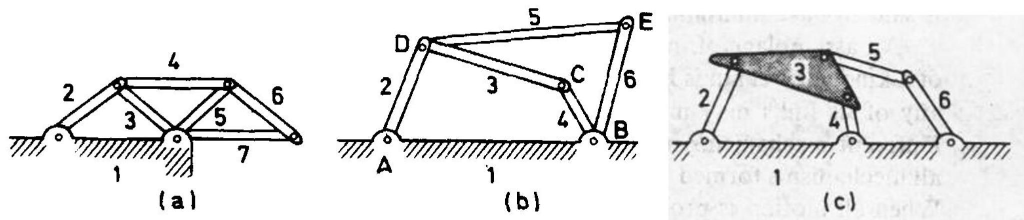

Figure 1.22-2. (a) Rigid truss; (b) A six-bar mechanism with one D.O.F.; (c) A six-link mechanism with one D.O.F.

Similarly, the four-bar mechanism contains the least number of members of any closed N-bar mechanism with at least one DOF. Therefore, mechanisms of great complexity can be built up by adding links, two at a time, to a given four-bar mechanism, as illustrated in Figs. 2(b) and 2(c). A combination of two links pinned together (such as links 5 and 6) is called a **dyad**. We see from Fig. 2 that the addition of a dyad to a mechanism results in a new mechanism without changing the original DOF. By 16

[4]Details of the calculation may be found in Sec. **1.35**.

induction, it may be established that the DOF for mechanisms formed from N links ⑰
and P_s *simple pairs* is

$$F = 3(N - 1) - 2P_s \qquad (1)$$

This result is a special case of more general mobility criteria discussed in greater depth in Sec. **8.20**.

Reuleaux [1876, p. 47] has defined a *mechanism* as a *closed constrained* kinematic
chain with one link fixed. He introduced the term *constrained* to describe a mechanism ⑱
with one DOF. This usage is contrary to the older and still current tradition of mechanics wherein a **constraint** is any condition which reduces the DOF of a system but not necessarily to a single DOF. Henceforth we shall use the expression *constraint* only in the broader sense of mechanics.

Note that our definition of a mechanism includes open chains and those with ⑲
several degrees of freedom; thus, unlike Reuleaux, we would classify a pendulum and
the five-bar linkage of Fig. 1(c) as *mechanisms*. ⑳

1.23 Kinematic Pairs

If all particles of a given system undergo motion in parallel planes, we say that the system experiences **planar motion**. In this book we shall consider only **planar mechanisms**, in which all links undergo planar motion. For planar mechanisms, only four types of joints or kinematics pairs arise; they are called

1. **Hinge** (also called **turning**, **revolute**, or **pinned**) **pairs**
2. **Sliding** (also called **prismatic**) **pairs**
3. **Rolling** (or **gear**) **pairs**
4. **Cam pairs**

All of the joints shown in Figs. **1.22**-1 and **1.22**-2 are hinge pairs.

If the link 4 of the four-bar mechanism shown in Fig. **1.22**-1(b) is very long, it may be desirable to use the more compact, *kinematically equivalent*[5] arrangement shown in Fig. 1(a). When the radius R approaches infinity, the connection between

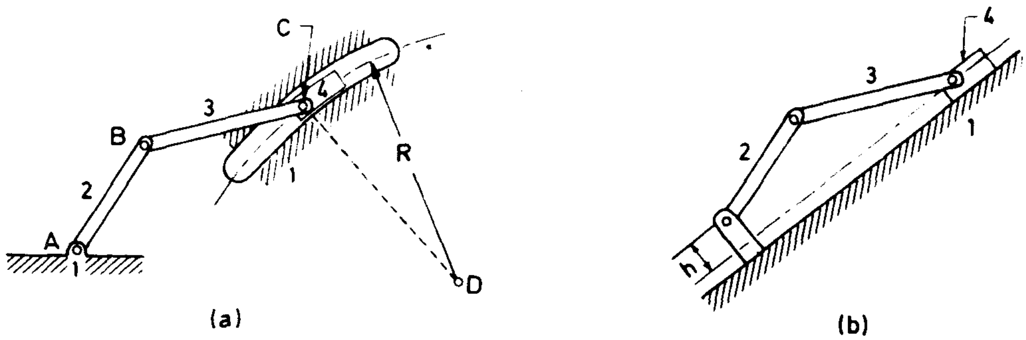

Figure 1.23-1. (a) Kinematic equivalent of 4-bar mechanism; (b) Slider-crank mechanism with offset h.

[5]The concept of *kinematic equivalence* is discussed more fully in Sec. **1.54**.

members 4 and 1 is called a **sliding** or **prismatic pair**. The mechanism shown in Fig. 1(b) is called an **offset slider-crank mechanism**, and the distance h is called the **offset**.

When two friction wheels roll about fixed centers A and B, as shown in Fig. 2, and 21
no slip occurs at points of contact, the kinematic pair is said to be a **rolling pair**. If teeth are cut into the wheels, to provide a driving force independent of friction, the wheels become **gears**, and the equivalent rolling circles are called the **pitch circles** of the gears, which touch at the **pitch point** P; thus the name **gear pair** is equivalent to **rolling pair**.

It is possible for *noncircular* **pitch curves** to roll together without slipping. When 22
meshing teeth are cut into the pitch curves we speak of the members as **noncircular gears**. Unless indicated otherwise, we shall henceforth assume that the gears we deal with have circular pitch curves. 23

Figure 3 shows two members with curved outlines which touch at a common point C, where sliding action occurs at the so-called **cam pair**. The distinction between the cam pair and the prismatic pair is that the former has a line contact and the latter has surface contact. Reuleaux classified joints with surface contact as **lower pairs** and those with point or line contact as **higher pairs**. Other writers have redefined the terms so that a lower pair refers to the case where the paired members have one DOF relative to each other. For simplicity, we accept Reuleaux's definition whereby hinges and sliding pairs are classified as lower pairs and gear and cam pairs are considered higher pairs.

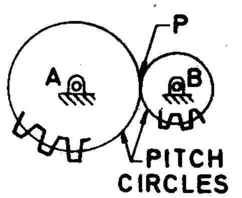

Figure 1.23-2. Friction wheels or gears. No slip occurs at pitch point P, where pitch circles contact.

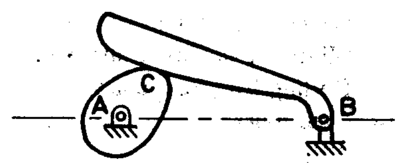

Figure 1.23-3. Cams. Sliding occurs at contact point C.

It has been suggested by Hartenberg and Denavit [1964] that a mechanism which contains only lower pairs be called a **linkage**. This is a useful distinction, which we 24
shall observe. However, the reader should be warned that the term *linkage* has been used widely as a synonym for *mechanism* and in other specialized ways; e.g., by Beggs [1955, p. 403].

Frequently we shall use stylized drawings called **kinematic skeletons**, as in Fig. 4, to represent the essential kinematic features of a mechanism. The following conventions should be noted: The joints are lettered, the links are numbered, and hinges are shown as small circles. Sliding pairs are illustrated in Fig. 4 at M and A. The small triangle on the slider at A indicates that it is welded to link 2.

Binary links are shown as straight lines (e.g., 5) or bent lines (e.g., 9). Ternary and higher-order links may be shown as shaded polygons (e.g., $CFDE$) or by a continuous line (e.g., HG) which carries a hinge (e.g., B) in its interior. When a link (such as 11)

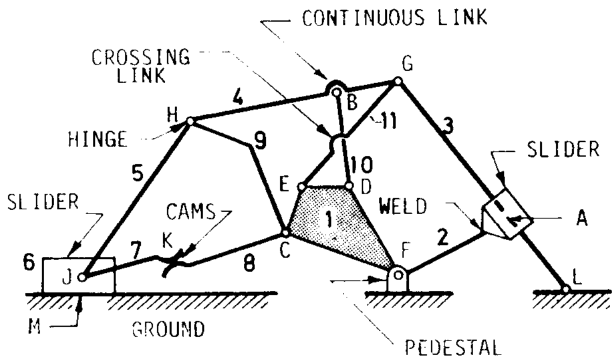

Figure 1.23-4. Conventions used in drawing kinematic skeletons.

crosses over another link (such as 10) a small break is made in the bottom link. Some- 25
times the crossing is emphasized by a small bend in the upper link, over the crossing point. A fixed link is indicated by cross-hatching, and fixed pivots are shown on pedestals (as at F) or embedded in the fixed link (as at L). Cams are indicated by small contacting arcs (as at K).

Additional conventions will be introduced as needed; e.g., in Fig. 3.33-1 for gear trains.

1.24 Problems

Define the following terms:

1. kinematic analysis
2. kinematic synthesis
3. static analysis
4. dynamic analysis
5. resistant body
6. kinematic pair
7. elements of a pair
8. kinematic chain
9. mechanism
10. ground link
11. frame
12. kinematic inversion
13. structure
14. closed mechanism
15. open mechanism
16. binary link
17. ternary link
18. quaternary link
19. unitary link
20. position variable
21. displacement variable
22. degree of freedom
23. mobility
24. simple pair
25. pair multiplicity
26. hinge pair
27. turning pair
28. revolute pair
29. pinned pair
30. sliding pair
31. prismatic pair
32. rolling pair
33. gear pair
34. cam pair

35. kinematic equivalence
36. slider-crank mechanism
37. offset slider-crank mechanism
38. four-bar linkage
39. pitch circle
40. pitch point
41. rolling pair
42. lower pair
43. higher pair
44. linkage
45. dyad
46. *N*-bar chain

1.30 KINEMATIC ANALYSIS OF THE FOUR-BAR MECHANISM

We have seen that the four-bar mechanism is the simplest closed kinematic chain of hinged links with a single degree of freedom (after fixing one link) and that more complex mechanisms can be built up by using one four-bar mechanism to drive one or more others. Because of this property, and because of the wide variety of motions[6] which can be generated directly by four-bar mechanisms, they are often found at the heart of machines and subsystems such as punch presses, film transports, quick returns, analog computers, and function generators. The study of the four-bar linkage (26) is well justified not only because of its many direct applications but also because most of the basic problems encountered in more general linkages show up in a simpler and more understandable way in the four-bar linkage.

1.31 Summary of Basic Types

There are many schemes to classify the various types of four-bar mechanisms. We shall follow a modified version[7] of a classification scheme introduced by Grashof [1883], but first it will be necessary to introduce some basic nomenclature.

In a four-bar chain, we shall refer to the *line segment between hinges* on a given link as a **bar**[8] and shall let

$$s = \text{length of shortest bar}$$

$$l = \text{length of longest bar}$$

$$p, q = \text{lengths of intermediate bars}$$

We shall also use $s, l, p,$ and q as the names of the corresponding bars or links. When a link of the chain is fixed it is called the **frame**, or **fixed link**. The opposite link is called the **coupler link**, and the links which are hinged to the frame are called **side links**. We shall frequently use the following symbols (e.g., as in Fig. **1.32**-1):

$$c = \text{length of coupler bar}$$

$$f = \text{length of fixed bar}$$

$$a, b = \text{length of side bars}$$

A link which is free to rotate through 360° with respect to a second link will be

[6]To appreciate the variety of curves traced out by a point on the coupler bar of a four-bar linkage one should peruse the 7300 **coupler curves** which occupy 730 large pages (11 × 17 in.) of the book by Hrones and Nelson [1951]. An interesting historical account of the application of coupler curves is given by Nolle [1974a, 1974b, 1975].

[7]See Appendix A.

[8]Note: We distinguish between a *bar*, which is a line segment, and a *link*, which is an extended plane.

said to **revolve** relative to the second link. A side link which revolves relative to the frame is called a **crank**. Any link which does not revolve is called a **rocker**.

If it is possible for all four bars to become simultaneously aligned, such a state is called a **change point** and the linkage is said to be a **change-point mechanism**.

Grashof's theorem states that *a four-bar mechanism has at least one revolving link if*

$$s + l \leq p + q \tag{1}$$

and all three links will rock if

$$s + l > p + q \tag{2}$$

Inequality (1) is **Grashof's criterion**, and mechanisms which satisfy it are called **Grashof mechanisms**. Mechanisms which violate Grashof's criterion are called **non-Grashofian**.

It may be shown (see Appendix **A**) that all four-bar mechanisms fall into one of the five categories listed in Table 1 and briefly described below:

TABLE 1.31-1 *Classification of Four-Bar Mechanisms*

Case	$l + s \gtreqless p + q$	*Shortest Bar*	*Type*
1	<	Frame	Double-crank
2	<	Side	Rocker-crank
3	<	Coupler	Double rocker
4	=	Any	Change point
5	>	Any	Triple-rocker

1. In a **double-crank**, also known as a **drag-link** mechanism, both side links revolve. See Fig. 1(a).

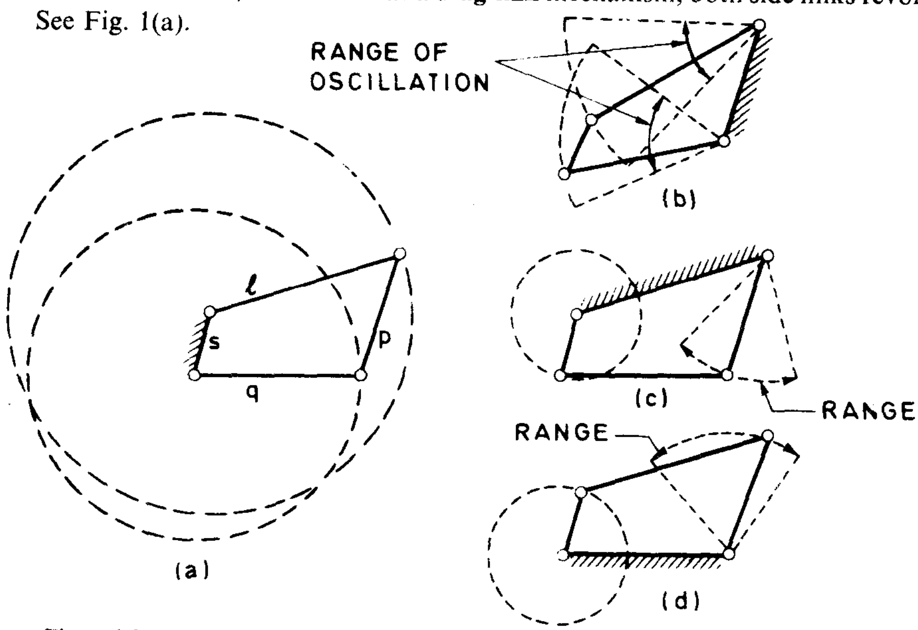

Figure 1.31-1. All possible inversions of a Grashof chain ($s + l < p + q$): (a) Drag-link; (b) Double-rocker; (c) Rocker-crank; (d) Rocker-crank.

2. In a **rocker-crank** the shorter side link revolves and the other one rocks (i.e., oscillates). See Figs. 1(c) and (d).
3. In this inversion, both side links rock and the coupler revolves. This *revolving-coupler double rocker* may be designated more simply as a **double-rocker**, provided we note the distinction between it and case 5. See Fig. 1(b).
4. A **change-point mechanism** is a limiting case which separates Grashof and non-Grashofian mechanisms. Its special characteristics will be discussed in Sec. **1.33**.
5. In a **triple-rocker** all three links rock; Grashof called this mechanism a "double-rocker"—the same name he used for case 3, despite the important differences between the two cases.

Figure 1 shows all possible inversions of a Grashof chain. For a non-Grashofian chain all inversions are triple-rockers.

For an interesting discussion of practical mechanisms (e.g., lawn sprinklers, automobile-hood-raising linkages, forklift mechanisms, etc.) which utilize the various types of four-bar linkages, see Hall [1966, pp. 1–11].

1.32 Dead-Point Configurations

When a side link, such as a in Fig. 1, becomes aligned with the coupler c, it can only be compressed or extended by the coupler. In this configuration, a torque applied to the other side link b cannot induce rotation in link a, which is therefore said to be at a **dead point** (sometimes called a **toggle point**). 28

If a is a crank, we see, from Fig. 1, that it can become aligned with c in full *extension*[9] along the line AD_1C_1 or in *flexion* with AD_2 folded over D_2C_2. We shall denote the angle ABC by ϕ and BAD by θ and shall use subscripts 1 or 2 to denote the extended or flexed state of links a and c. In the extended state, point C cannot move clockwise along the circular arc CC_1 without stretching or compressing the theoretically rigid line AC_1. Therefore, link b cannot move into the "forbidden zone" below BC_1, and ϕ_1 must be at one of its two extreme positions; in other words, **link b is at an extremum**. A second extremum of link b occurs with $\phi = \phi_2$. Note that the ***extreme positions of a side link occur simultaneously with the dead points of the opposite link.***

Although b is temporarily immovable at its extremum points, a can effectively serve as the *driving* link with finite angular velocity ω_a. If ω_b denotes the angular velocity of link b, we can conclude that at the dead points of link a, $\omega_a/\omega_b = \infty$. In the neighborhood of the dead point, the velocity ratio ω_a/ω_b can be huge; i.e., a mechanism can function as a *velocity* (or *displacement*) *magnifier* near its dead points.

To find the four critical angles ($\phi_1, \theta_1, \phi_2, \theta_2$), consider triangle ABC and introduce the notation

$$e \equiv AC, \qquad \phi^* = \angle ABC, \qquad \theta^* = \angle BAC \tag{1}$$

[9]The words **extension** and **flexion** are used in their anatomical sense. For example, a fully outstretched arm is in extension, but a complete bend of the elbow joint puts it into a state of flexion.

where $0 \leq \phi^*$ and $\theta^* \leq \pi$. From the law of cosines we find that

$$\phi^* = \arccos\left(\frac{f^2 + b^2 - e^2}{2fb}\right) \quad (2)$$

$$\theta^* = \arccos\left(\frac{f^2 + e^2 - b^2}{2fe}\right) \quad (3)$$

For the rocker-crank shown in Fig. 1, it is apparent that in state 1 (extension of a and c)

$$e \equiv AC_1 = c + a, \qquad \phi_1 = \phi^*, \qquad \theta_1 = \theta^* \quad (4)$$

whereas for state 2 (flexion of a and c)

$$e \equiv AC_2 = c - a, \qquad \phi_2 = \phi^*, \qquad \theta_2 = \theta^* + \pi \quad (5a)$$

For the case of a revolving coupler mechanism (Fig. 2), Eqs. (1)–(4) remain valid but Eqs. (5a) should be replaced by

$$e \equiv e_2 = a - c, \qquad \phi_2 = \phi^*, \qquad \theta_2 = \theta^* \quad (5b)$$

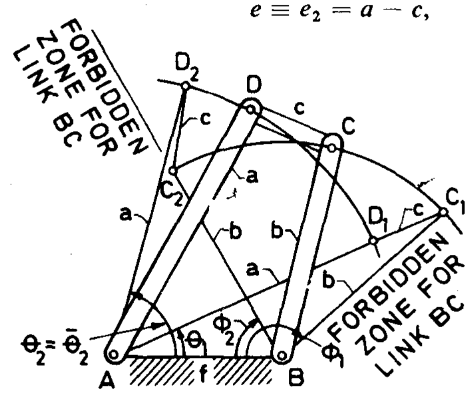

Figure 1.32-1. Dead points of a rocker-crank $\phi_{max} = \phi_1$, $\phi_{min} = \phi_2$.

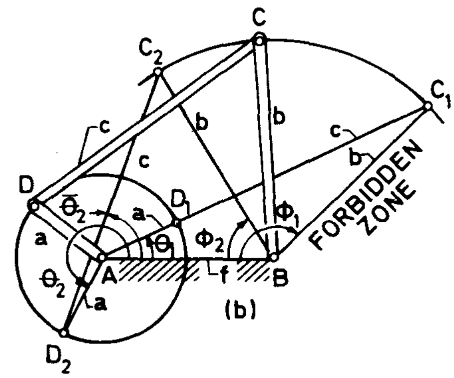

Figure 1.32-2. Dead points of link b of a revolving coupler.

To find the extremum points of rocker-link a of Fig. 2 it is only necessary to interchange a and b and to interchange symbols ϕ and θ everywhere in Eqs. (1)–(4) and in Eq. (5b). Then subscript 1 (2) represents the state where b and c are aligned in extension (flexion).

Since double-crank mechanisms cannot (by definition) have any extremum points, Eqs. (1)–(5) cover all possible Grashof mechanisms.[10] For non-Grashofian linkages (triple-rockers), the extremum angles (ϕ_1, ϕ_2) of the follower bar b and the associated dead points (θ_1, θ_2) of the input bar a are given in Table 1. The required angles ϕ^* and θ^* are

TABLE 1.32-1 *Extremum Angles (ϕ) and Associated Dead Points (θ) for Triple-Rockers*

Case	Longest Bar Is	e	ϕ_1	θ_1	ϕ_2	θ_2
1	Frame, f	$a + c$	ϕ^*	θ^*	$-\phi^*$	$-\theta^*$
2	Coupler, c	$c - a$	$2\pi - \phi^*$	$\pi - \theta^*$	ϕ^*	$\pi + \theta^*$
3	Follower, b	$a + c$	ϕ^*	θ^*	$-\phi^*$	$2\pi - \theta^*$
4	Input, a	$a - c$	$2\pi - \phi^*$	$-\theta^*$	ϕ^*	θ^*

[10]Some minor reinterpretation of symbols may be necessary for linkages that are assembled in modes opposite to those shown in Figs. 1 and 2. See the discussion of mode of assembly in Sec. 1.35.