

# **Cosmological Physics**

**John A. Peacock**

**宇宙物理学**

**CAMBRIDGE**

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# COSMOLOGICAL PHYSICS

J. A. PEACOCK

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## COSMOLOGICAL PHYSICS

This textbook provides a comprehensive introduction to modern cosmology, at a level suitable for advanced undergraduates and graduate students. The essential concepts and key equations used by professional researchers in both theoretical and observational cosmology are derived and explained from first principles.

The first third of the book carefully develops the necessary background in general relativity and quantum fields. The remainder of the volume then provides self-contained accounts of the principal topics in contemporary cosmology, including inflation, topological defects, gravitational lensing, the distance scale, large-scale structure and galaxy formation.

Throughout, the emphasis is on helping students to attain a physical and intuitive understanding of the subject. The book is therefore extensively illustrated, and outline solutions to more than 90 problems are included. All necessary astronomical jargon is clearly explained, ensuring that the book is self-contained for students with a background in undergraduate physics.

**John Peacock** studied physics at the University of Cambridge, obtaining his Ph.D. in 1981. He subsequently worked as a research astronomer at the Royal Observatory, Edinburgh, where he was appointed Head of Research in 1992. In 1998, he moved to become Professor of Cosmology at the University of Edinburgh. His research interests are a mixture of theoretical and observational cosmology: galaxy formation and clustering; high redshift galaxies and quasars; and gravitational lensing. He is married with three children. Time not devoted to science or family is shared between playing classical clarinet and exploring Scottish mountains.

## Preface

This is a textbook on cosmology – a subject that has the modest aim of understanding the entire universe and all its contents. While it can hardly be claimed that this task is complete, it is a fact that recent years have seen astonishing progress towards answering many of the most fundamental questions about the constitution of the universe. The intention of this book is to make these developments accessible to someone who has studied an undergraduate course in physics. I hope that the book will be useful in preparing new Ph.D. students to grapple with the research literature and with more challenging graduate-level texts. I also hope that a good deal of the material will be suitable for use in advanced undergraduate courses.

Cosmology is a demanding subject, not only because of the vast scales with which it deals, but also because of the range of knowledge required on the part of a researcher. The subject draws on just about every branch of physics, which makes it a uniquely stimulating discipline. However, this breadth is undeniably intimidating for the beginner in the subject. As a fresh Ph.D. student, 20 years ago, I was dismayed to discover that even a good undergraduate training had covered only a fraction of the areas of physics that were important in cosmology. Worse still, I learned that cosmologists need a familiarity with astronomy, with all its peculiar historical baggage of arcane terminology. In the past two decades, cosmological knowledge and understanding have advanced almost beyond recognition, and yet undergraduate physics courses have changed only a little. As a result, there is now a yawning gap between the professional literature on cosmology and the knowledge base of a typical physics undergraduate. What I have tried to do in this book is to bridge that gap, by discussing modern cosmology in language that should be familiar to an advanced undergraduate, going back to first principles wherever possible.

The material here is therefore of two kinds: relevant pieces of physics and astronomy that are often not found in undergraduate courses, and applications of these methods to more recent research results. The former category is dominated by general relativity and quantum fields. Relativistic gravitation has always been important in the large-scale issues of cosmology, but the application of modern particle physics to the very early universe is a more recent development. Many excellent texts exist on these subjects, but I wanted to focus on those aspects that are particularly important in cosmology. At times, I have digressed into topics that are strictly ‘unnecessary’, but which were just too interesting to ignore. These are, after all, the crown jewels of twentieth century physics, and I firmly believe that their main features should be a standard part of an undergraduate education in physics. Despite this selective approach, which aims to



concentrate on the essential core of the subject, the book covers a wide range of topics. My original plan was in fact focused more specifically on matters to do with particle physics, the early universe, and structure formation. However, as I wrote, the subject imposed its own logic: it became clear that additional topics simply had to be added in order to tell a consistent story. This tendency for different parts of cosmology to reveal unexpected connections is one of the joys of the subject, and is also a mark of the maturity of the field.

Partly because of the variety of material treated here, my emphasis throughout has been on making the exposition as simple as possible. Nevertheless, I wanted the final result to be useful at a professional level, so I have tried to concentrate on explaining the techniques and formulae that are used in practice in the research literature. In places, I was unable to reduce the full treatment to an argument of tolerable length and complexity, and I then settled for an approximate analysis. In such cases, I have outlined the steps needed to obtain the exact answer and contrasted the approximate analysis with the full result. Wherever possible, I have tried to motivate the more detailed calculations with physical and order-of-magnitude arguments. This is particularly important for the novice, who will be much more willing to accept lengthy and complicated reasoning if it is clear where the argument is going. Because of this desire to expose the logic of the subject, as well as the need to keep the total length manageable, the treatment is not always simple in the sense of giving every last step of a calculation. Readers will have to be prepared to put in some work at various stages to prove to themselves that equation *C* really does result from combining equations *A* and *B*. Usually this sort of manipulation is straightforward, and it can be skipped on a first reading. Where the derivation is a little more challenging, I have left it as a problem, indicated by [problem] in the text. Other problems are collected at the end of each chapter, and include some basic exercises to test comprehension of the material, as well as more discursive and advanced points that would have disrupted the main presentation. There is a set of solutions to all the problems, although these are often quite schematic, since the best way to develop real understanding is to work through the calculations.

A good deal of the book is based on lectures given to final-year undergraduates and early postgraduates, and I have tried to keep the same style in the written version. The advantage of lectures is that they steer a clear course through a subject, whereas textbooks can make a subject that is in essence quite simple seem impenetrable by burdening it with too complete a treatment. What I have attempted to do is to cover the basics thoroughly, so that a student can strike out with confidence towards more specialized topics. I have also tried to make the presentation as locally self-contained as possible, even at the cost of a little duplication. I do not expect that many readers will be masochistic enough to want to read the book in order from cover to cover, and I have tried to ensure that individual chapters and even sections make sense in isolation. In this way, I hope the book can be useful for a variety of courses. It is a very distant ideal to expect that this material in its entirety could be taught to undergraduates. However, some topics are inevitably more straightforward than others, and are suitable for more specialized undergraduate courses.

Many people deserve thanks for having helped this project reach its present stage. First, Malcolm Longair for the initial opportunity to try out my ideas on what could sensibly be lectured to undergraduates. Second, generations of patient students, many of whom at least pretended to enjoy the experience. Third, Malcolm Longair again for the initial suggestion that I should write up my notes – even though he probably realized

just how much work this would involve. Fourth, the production of this tome would have been unthinkable without the marvellous electronic tools that are now widely available. The creators of  $\text{\TeX}$ , PostScript, xfig and pgplot all have my humble thanks. Fifth, many scientific colleagues have had a great influence on the contents, both by helpful comments on drafts and particularly through the innumerable conversations and arguments that have formed my understanding of cosmology; I shall not attempt a list, but I owe each one a debt.

Thanks are due especially to all those who generously contributed data for the figures. I have produced many illustrations that involve recent research results, not in some futile attempt to be up to date, but to emphasize that cosmology is above all a subject built on foundations of observational data. Many of the latest results have clarified long-contentious issues, and this reduction of uncertainty has been an unintended benefit of the delay in completing the book. In recent months, I have often felt like a small child in a sweetshop as astronomers all round the world have sent me the most mouthwatering new data. Those who contributed in this way, or gave permission for their results to be used, are as follows. Figure 4.5, T. Muxlow; figure 4.6, W. Sutherland; figure 4.7, R. Ellis; figure 5.1, R. Jimenez; figure 5.2, E. Feigelson; figure 5.3, N. Tanvir; figure 5.4, A. Riess and S. Perlmutter; figure 9.2, M. Smith; figure 12.1, T. Ressel and M. Turner; figure 12.2, C. Steidel and M. Pettini; figure 12.4, T. van Albada; figure 12.5, R. Kolb; figure 13.2, A. Dressler; figure 13.4, S. Driver; figure 13.6, W. Couch; figure 13.7, N. Metcalfe and K. Glazebrook; figure 14.1, P. Hewett and C. Foltz; figure 14.2, C. Carilli; figure 14.6, Y. Dabrowski; figure 15.1, J. Huchra; figure 15.5, J. Colberg; figure 16.9, V. Icke; figure 16.11, M. Davis; figure 18.2, M. White. Figure 13.1 was made using data from the Digitized Sky Survey, which was produced at the Space Telescope Science Institute under US Government grant NAG W-2166. The DSS images are based on photographic data obtained using the Oschin Schmidt Telescope on Palomar Mountain and the UK Schmidt Telescope. I also thank Jörg Colberg, Hugh Couchman, George Efstathiou, Carlos Frenk, Adrian Jenkins, Alistair Nelson, Frazer Pearce, Peter Thomas and Simon White, my colleagues in the Virgo Consortium, for permission to reproduce the cover image. The Virgo Consortium uses computers based at the Computing Centre of the Max-Planck Society in Garching and at the Edinburgh Parallel Computing Centre.

To finish, some apologies. My professional colleagues will doubtless disagree in places with my opinions and emphases. I have tried to make it clear when I am giving a personal view, and have balanced my own simplifications of the issues with frequent references to the literature, so that readers can explore the details of the arguments for themselves. However, the literature is so vast that inevitably many outstanding papers have been omitted. I apologize to those whose work I could have cited, but did not. This is not intended to be a comprehensive review, nor does it try to give a full history of the development of ideas. In places, I have cited classic papers that everyone should read, but in the main papers are cited where I happen to know that they contain further discussion of relevant points. One consequence of this is that the number of references to my own research output exaggerates any contribution I may have made to cosmology.

It is an unfortunate fact that a book of this length can never hope to be entirely free of errors and misprints. I shall be grateful to have any corrections brought to my attention; a list of these will be maintained at <http://www.roe.ac.uk/japwww>

Lastly, I now appreciate why so many authors acknowledge their families: the



conflicting demands for any spare time inevitably make it difficult for families and books to co-exist peacefully. I hope my loved ones will forgive me for the times I neglected them, and I thank the publishers for their patience.

John Peacock

### Note to the fifth reprint

In this printing, a number of misprints have been corrected. These will inevitably not be the last, and I thank those readers who are kind enough to keep reporting them. I have also taken the opportunity of bringing the contents up to date in a few places, specifically new results on the supernova Hubble diagram (figure 5.4) and CMB anisotropies (figure 18.2).

Given the pace of cosmological research, I am surprised, but pleased, to see that the basic framework described in the original text survives without the need for revolutionary change. Nevertheless, some very significant developments have occurred since the first printing. Here is a personal list of recent highlights:

(1) Results on atmospheric neutrinos show that the  $\mu$  neutrino oscillates, probably to a  $\tau$  neutrino. If so, the  $\tau$  neutrino mass is 0.06 eV, and hot dark matter is unimportant (hep-ex/9912007). The SNO experiment has proved that oscillations are also the explanation for the solar neutrino problem (see e.g. hep-ph/0204314).

(2) The supernova Hubble diagram now argues very strongly for vacuum energy, and an accelerating expansion (see the new figure 5.4 and astro-ph/0005229).

(3) Small-scale CMB data show a clear set of acoustic peaks in the power spectrum (see the new figure 18.2). The combination of large-scale structure and CMB data (e.g. astro-ph/0109152 and astro-ph/0206256) requires a flat CDM model with  $\Omega_m = 0.31 \pm 0.06$  and  $h = 0.67 \pm 0.05$ . No tensor anisotropies are required, and the scalar spectrum shows no significant tilt:  $n = 0.96 \pm 0.04$ . The CMB has also been shown to be linearly polarized, at the expected level (astro-ph/0209478).

(4) Given that the  $\Lambda$ CDM model works so well on large scales, it is essential to understand the discrepancy with the form of galaxy correlations. Galaxy-formation models are now starting to pinpoint the origin of the required scale-dependent bias (astro-ph/9910488).

(5) Nevertheless, worries about the basic CDM paradigm persist, and are most severe on small scales, where the structure of galaxy-scale dark matter haloes appears not to match observations (astro-ph/9901240 and astro-ph/9907411). The nature of dark matter continues to be perhaps the greatest uncertainty in cosmology.

(6) Quasars have now been detected out to  $z = 6.28$ . This object shows evidence for an intergalactic medium that is much less strongly ionized than at slightly lower redshift (astro-ph/0108097). We may be seeing close to the era of general reionization.

(7) The debate on initial conditions continues. Models with extra dimensions (the ‘brane world’) have suggested alternatives to inflation (hep-th/0111030). A small cosmological constant presents a fine-tuning problem, and ‘quintessence’ models attempt to solve this by using scalar-field dynamics (astro-ph/9901388). There is an increasingly good case, however, that a small non-zero vacuum density may need to be explained anthropically (hep-th/0106083).

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PART 1 GRAVITATION AND RELATIVITY

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# 1 Essentials of general relativity

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## 1.1 The concepts of general relativity

**SPECIAL RELATIVITY** To understand the issues involved in general relativity, it is helpful to begin with a brief summary of the way space and time are treated in special relativity. The latter theory is an elaboration of the intuitive point of view that the properties of empty space should be the same throughout the universe. This is just a generalization of everyday experience: the world in our vicinity looks much the same whether we are stationary or in motion (leaving aside the inertial forces experienced by accelerated observers, to which we will return shortly).

The immediate consequence of this assumption is that any process that depends only on the properties of empty space must appear the same to all observers: the velocity of light or gravitational radiation should be a constant. The development of special relativity can of course proceed from the experimental constancy of  $c$ , as revealed by the Michelson-Morley experiment, but it is worth noting that Einstein considered the result of this experiment to be inevitable on intuitive grounds (see Pais 1982 for a detailed account of the conceptual development of relativity). Despite the mathematical complexity that can result, general relativity is at heart a highly intuitive theory; the way in which our everyday experience can be generalized to deduce the large-scale structure of the universe is one of the most magical parts of physics. The most important concepts of the theory can be dealt with without requiring much mathematical sophistication, and we begin with these physical fundamentals.

**4-VECTORS** From the constancy of  $c$ , it is simple to show that the only possible linear transformation relating the coordinates measured by different observers is the Lorentz transformation:

$$\begin{aligned} dx' &= \gamma \left( dx - \frac{v}{c} c dt \right) \\ c dt' &= \gamma \left( c dt - \frac{v}{c} dx \right). \end{aligned} \tag{1.1}$$

Note that this is written in a form that makes it explicit that  $x$  and  $ct$  are treated in the same way. To reflect this interchangeability of space and time, and the absence of any preferred frame, we say that special relativity requires all true physical relations to be written in terms of **4-vectors**. An equation valid for one observer will then apply to all

others because the quantities on either side of the equation will transform in the same way. We ensure that this is so by constructing physical 4-vectors out of the fundamental interval

$$dx^\mu = (c dt, dx, dy, dz) \quad \mu = 0, 1, 2, 3, \quad (1.2)$$

by manipulations with relativistic invariants such as rest mass  $m$  and proper time  $d\tau$ , where

$$(c d\tau)^2 = (c dt)^2 - (dx^2 + dy^2 + dz^2). \quad (1.3)$$

Thus, defining the 4-momentum  $P^\mu = m dx^\mu/d\tau$  allows an immediate relativistic generalization of conservation of mass and momentum, since the equation  $\Delta P^\mu = 0$  reduces to these laws for an observer who sees a set of slowly moving particles. This is a very powerful principle, as it allows us to reject 'obviously wrong' physical laws at sight. For example, Newton's second law  $\mathbf{F} = m d\mathbf{u}/dt$  is not a relation between the spatial components of two 4-vectors. The obvious way to define 4-force is  $F^\mu = dP^\mu/d\tau$ , but where does the 3-force  $\mathbf{F}$  sit in  $F^\mu$ ? Force will still be defined as rate of change of momentum,  $\mathbf{F} = d\mathbf{P}/dt$ ; the required components of  $F^\mu$  are  $\gamma(\dot{\mathbf{E}}, \mathbf{F})$ , and the correct relativistic force-acceleration relation is

$$\mathbf{F} = m \frac{d}{dt}(\gamma \mathbf{u}). \quad (1.4)$$

Note again that the symbol  $m$  denotes the **rest mass** of the particle, which is one of the invariant scalar quantities of special relativity. The whole ethos of special relativity is that, in the frame in which a particle is at rest, its intrinsic properties such as mass are always the same, independently of how fast it is moving. The general way in which quantities are calculated in relativity is to evaluate them in the rest frame where things are simple, and then to transform out into the lab frame.

**GENERAL RELATIVITY** Nothing that has been said so far seems to depend on whether or not observers move at constant velocity. We have in fact already dealt with the main principle of general relativity, which states that the only valid physical laws are those that equate two quantities that transform in the same way under any arbitrary change of coordinates.

Before getting too pleased with ourselves, we should ask how we are going to construct general analogues of 4-vectors. Consider how the components of  $dx^\mu$  transform under the adoption of a new set of coordinates  $x'^\mu$ , which are functions of  $x^\nu$ :

$$dx'^\mu = \frac{\partial x'^\mu}{\partial x^\nu} dx^\nu. \quad (1.5)$$

This apparently trivial equation (which assumes, as usual, the summation convention on repeated indices) may be divided by  $d\tau$  on either side to obtain a similar transformation law for 4-velocity,  $U^\mu$ ; so  $U^\mu$  is a general 4-vector. Things unfortunately go wrong at the next level, when we try to differentiate this new equation to form the 4-acceleration  $A^\mu = dU^\mu/d\tau$ :

$$A'^\mu = \frac{\partial x'^\mu}{\partial x^\nu} A^\nu + \frac{\partial^2 x'^\mu}{\partial \tau \partial x^\nu} U^\nu. \quad (1.6)$$