

高等数学

(上册)

习题解答



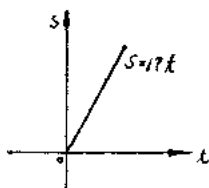
南京邮电学院

一九七五年十二月

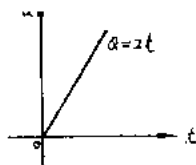
第一章 函 数

习 题 1 P.4

1. 路程速度是变量，加速度是常量。
2. 路程 s 和速度 v 是时间 t 的函数，加速度 a 不是 t 的函数。
3. ① 是 $s = \pi r^2$
 ② 是 $s = \frac{\pi}{4}d^2$
4. $s = 17t$



第 4 题



第 5 题

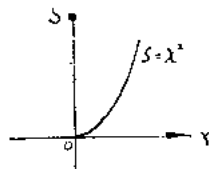
5. $Q = 2t$
6. 是。在一天中不同的时间，气温也不同。
7. 是。
8. ①，②，③是。

习 题 2 P.8

1. ① $s = x^2$ 。②

x	1	2	3	4	...
s	1	4	9	16	...

 ③



第 1 题

2. ①	时间 t (时)	0	4	8	11	17	20	24
	负载 P (瓩)	1500	1200	1500	2230	2300	2150	1500

②不能。

4. $V = 0.95 t + v_0$

5. $l = 1.5 P$

6. $s = 2h + h^2 \quad 0 \leq h \leq H$

7. $V = \frac{1}{9} \pi h^2 \quad [0, 30]$

8. ① $I = \frac{10}{R}$ ② $P = 50 I^2$ ③ $R = \frac{220^2}{P}$, 1210欧。

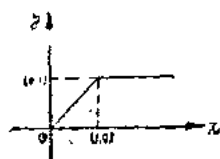
9. $V = 10 + 0.5 t$ (米³) $0 \leq t \leq 180$ (分钟)
 $100 = 10 + 0.5 t$ 解出 $t = 180$ (分钟)

10. $\because R\varphi = R\omega t \quad \therefore s = H - R\omega t \quad 0 \leq t \leq \frac{H}{R\omega}$

11. $V = \pi r^2 h \quad \therefore r = \sqrt{\frac{V}{\pi h}}$

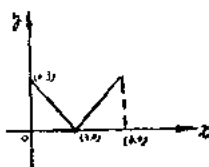
12. $u = \begin{cases} 5t & 0 \leq t \leq 2 \\ 10 & 2 < t \leq 5 \\ 35 - 5t & 5 < t \leq 7 \end{cases}$

13. 是 ①



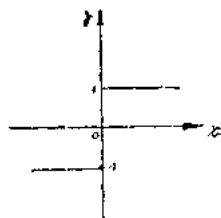
第 13① 题

②



第 13② 题

③



第 13③ 题

14. $0 < x < 1, 1 \leq x \leq 3, -2 < x < \infty, -\infty < x < 0.$

习 题 3 P.11

$$1. \quad y(1)=1, \quad y(0)=-1, \quad y(4)=3, \quad y(a+1)$$

$$= 2\sqrt{a+1}-1$$

$$2. \quad f(0)=-\frac{1}{2}, \quad f(3)=7, \quad f(a)=\frac{2a+1}{a-2},$$

$$f(-a)=\frac{2a-1}{a+2}, \quad f\left(\frac{1}{a}\right)=\frac{a+2}{1-2a},$$

$$f(a+2)=2+\frac{5}{a}, \quad f(x_0+k)=\frac{2(x_0+k)+1}{x_0+k-2}$$

$$3. \quad i(0)=\frac{5\sqrt{2}}{2}, \quad i\left(\frac{1}{8}\right)=5, \quad i\left(\frac{1}{4}\right)=\frac{5\sqrt{2}}{2},$$

$$i\left(\frac{1}{2}\right)=-\frac{5\sqrt{2}}{2}, \quad \left[i\left(\frac{3}{4}\right)\right]^2=\frac{25}{2}.$$

$$4. \quad \varphi(1)=2, \quad \varphi(0)=0, \quad \varphi(-2)=-\frac{1}{2},$$

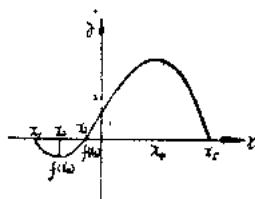
$$\frac{1}{\varphi(a)}=\frac{1}{a2^a}, \quad \varphi(a^2)=a^22^{a^2},$$

$$5. \quad y(0)=1, \quad y(-1)=\sqrt{2}, \quad y(3)=\sqrt{10}, \quad y\left(-\frac{1}{3}\right)=\frac{\sqrt{10}}{3},$$

$$\frac{1}{y(3)}y(x_0)=\frac{1}{\sqrt{10}}\sqrt{x_0^2+1}=\frac{\sqrt{10(x_0^2+1)}}{10},$$

$$y\left(\frac{1}{x_0}\right)=\frac{1}{x_0}\sqrt{1+x_0^2}, \quad \frac{1}{y(x_0)}=\frac{\sqrt{x_0^2+1}}{x_0^2+1}$$

6. ①



② $f(x) > 0, (x_3, x_5)$

$f(x) < 0, (x_1, x_2)$

③ $f(0) = 1.5, f(x_4) = 4.$

第 6 题

7. 证: $f(-a) = -a^3, -f(a) = -(a^3) \therefore f(-a) = -f(a)$

8. 证: $y(-x) = \frac{1}{(-x)^2} = \frac{1}{x^2} = y(x)$

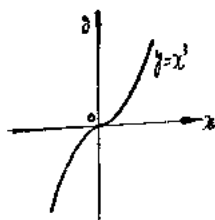
习 题 4 P.23

1. ① $y = x^3$ $y(-x) = -y(x)$ 奇函数。

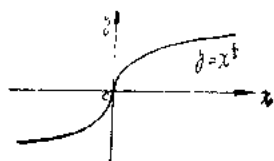
x	0	1	1.5	2	2.5	3	...
y	0	1	33.75	8	15.625	27	...

② $y = \sqrt[3]{x} = x^{\frac{1}{3}}$ 奇函数。

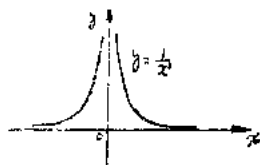
x	0	0.5	1	1.5	2	2.5	3	3.5	4	...
y	0	0.7937	1	1.145	1.26	1.357	1.442	1.518	1.584	...



第 1 ① 题



第 1 ② 题



第 1 ③ 题

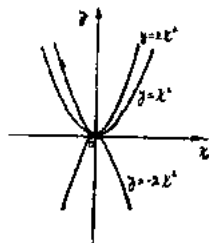
③ $y = \frac{1}{x^2}$ 偶函数。

x	0.5	1	1.5	2	2.5	3	3.5	...
y	4	1	0.44	0.25	0.16	0.11	0.045...	

2. $a > 0$ 开口向上 a 愈大, 函数值变化愈快。

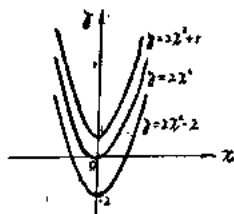
$a < 0$ 开口朝下。

①



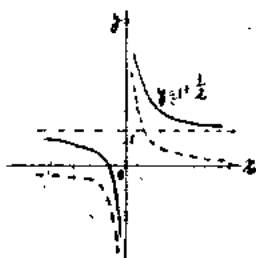
第 2 ① 题

②



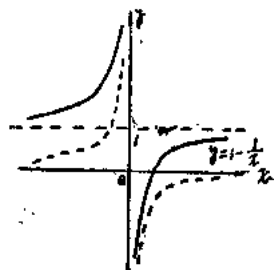
第 2 ② 题

3. ①



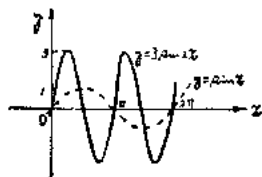
第 3 ① 题

②



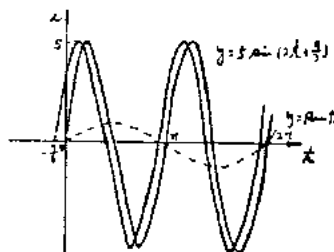
第 3 ② 题

4. ① $y = 3 \sin 2x$



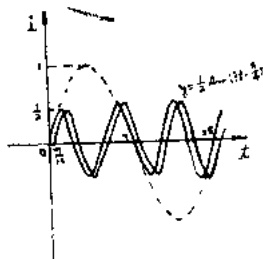
第 4 ① 题

② $u = 5 \sin \left(2t + \frac{\pi}{3} \right)$



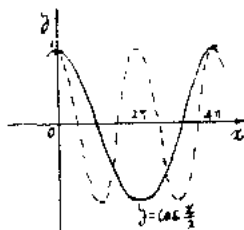
第 4 ② 题

③ $i = \frac{1}{2} \sin \left(3t - \frac{\pi}{4} \right)$



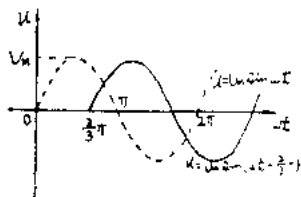
第 4 ③ 题

④ $y = \cos \frac{x}{2}$

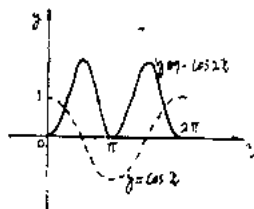


第 4 ④ 题

⑤ $u = U_m \sin \left(\omega t - \frac{2}{3} \pi \right)$ ⑥ $1 - \cos 2x$

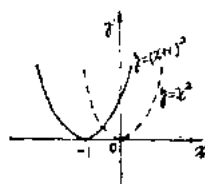


第 4 ⑤ 题



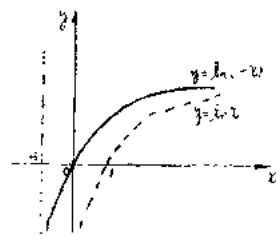
第 4 ⑥ 题

5. ①



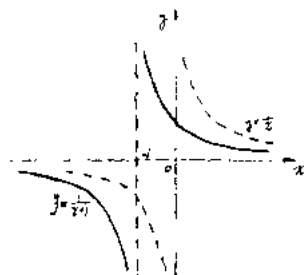
第 5 ① 题

②



第 5 ② 题

③



第 5 ③ 题

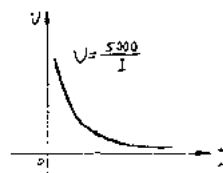
6. ①, ③, ⑤, ⑧, ⑨, ⑩ 偶函数。

②, ④, ⑦ 奇函数。⑥ 非奇非偶。

$$7. U = \frac{5000}{I}$$

$$\begin{aligned} \text{① } P &= 50 \times 80 \\ &= 4000 \text{ (伏安) 安全。} \end{aligned}$$

$$\begin{aligned} \text{② } P &= 20 \times 300 \\ &= 6000 \text{ (伏安) 不安全。} \end{aligned}$$

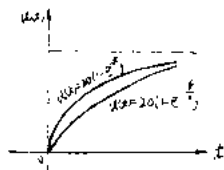


第 7 题

8. ① $u(t) = 20(1 - e^{-t})$ ② $u(t) = 20(1 - e^{-\frac{t}{2}})$

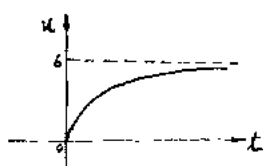
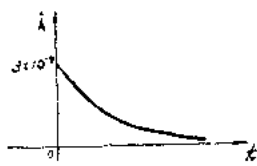
①	t	0	1	2	3	4	5
	$u(t)$	0	12.6	17.2	19	19.6	19.86

②	t	0	1	2	3	4	5
	$u(t)$	0	7.8	12.6	15.8	17.2	18.4



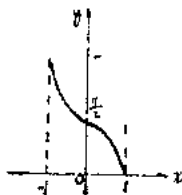
第 8 题

9. $i = 3 \times 10^{-4} e^{-\frac{t}{2}}$ $u(t) = 6(1 - e^{-\frac{t}{2}})$



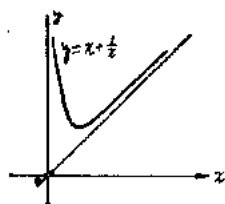
第 9 题

10. $y = \arccos x$



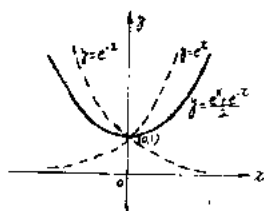
第 10 题

11. ①



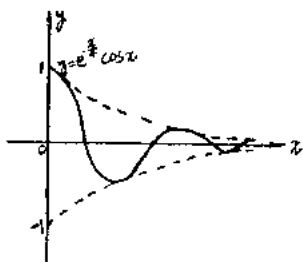
第 11① 题

②

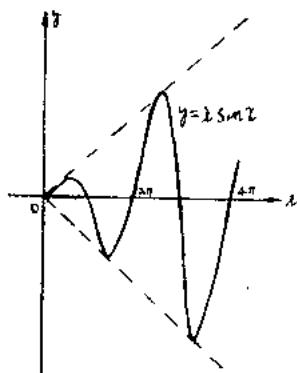


第 11② 题

12. ① $y = e^{-\frac{x}{4}} \cos x$ ($x \geq 0$) ② $y = x \sin x$ ($x \geq 0$)



第 12① 题

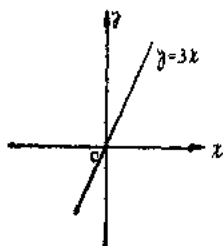


第 12② 题

习 题 5 P.28

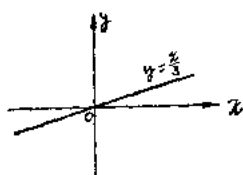
1 ①, ②, ③, ④ 是线性函数其余都不是。

2. ①



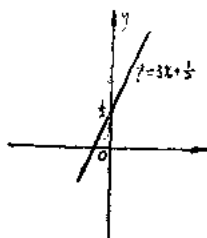
第 2 ① 题

②



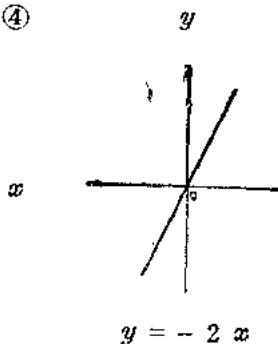
第 2 ② 题

③



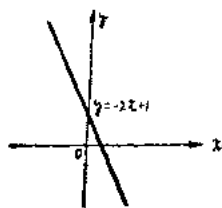
第 2 ③ 题

④



第 2 ④ 题

⑤ 自变量的系数就是直线的斜率。



第 2 ⑤ 题

3. 是。在 t 秒内通过导线横截面的电量。

$$4. \begin{cases} 1.00012 = 10a + b \\ 1.00060 = 50a + b, \end{cases} \quad a = 1.2 \times 10^{-5}, \quad b = 1$$

a 是温度系数, b 是在 0° 时的轴的长度。

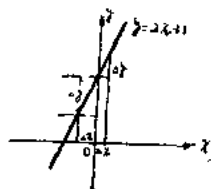
$$5. \textcircled{1} \frac{\Delta y}{\Delta x} = \frac{y(x + \Delta x) - y(x)}{\Delta x} = 2 \quad \textcircled{2} \frac{\Delta y}{\Delta x} = 2$$

$$\textcircled{3} \frac{\Delta y}{\Delta x} = 2$$

$$\textcircled{4} \frac{\Delta y}{\Delta x} = 2$$

① ②

$\frac{\Delta y}{\Delta x}$ 与 x 无关, 与 Δx 也无关。



第 5 题

6. ① $\frac{\Delta y}{\Delta x} = \frac{y(x + \Delta x) - y(x)}{\Delta x} = -\frac{1}{2}$, ② $\frac{\Delta y}{\Delta x} = -\frac{1}{2}$,

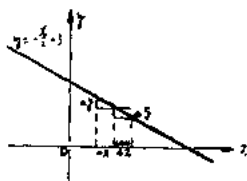
③ $\frac{\Delta y}{\Delta x} = -\frac{1}{2}$,

④ $\frac{\Delta y}{\Delta x} = -\frac{1}{2}$

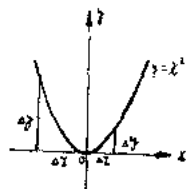
与 x 和 Δx 都无关。

③ ④

① ②



第 6 题



第 7 题

7. $\Delta y = (x + \Delta x)^2 - x^2 = 2x\Delta x + (\Delta x)^2$, $\frac{\Delta y}{\Delta x} = 2x + \Delta x$

① $\frac{\Delta y}{\Delta x} = 0.1$,

② $\frac{\Delta y}{\Delta x} = -0.2$,

③ $\frac{\Delta y}{\Delta x} = 2 + 0.1 = 2.1$,

④ $\frac{\Delta y}{\Delta x} = 2 - 0.2 = 1.8$

与 x 和 Δx 都有关。

$$8. \textcircled{1} \Delta y = f(x + \Delta x) - f(x) = 3 - 2(x + \Delta x)^2 - 3 + 2x^2 \\ = -2\Delta x(2x + \Delta x)$$

$$\therefore \Delta y = -2 \times 0.1(2 + 0.1) = -0.42$$

$$\textcircled{2} \Delta y = -2x\Delta x(2x_0 + \Delta x) = -4x_0\Delta x - 2(\Delta x)^2$$

$$9. \textcircled{1} \Delta y = \sin 2\left(\frac{\pi}{6} + \frac{7}{12}\pi\right) - \sin 2\frac{\pi}{6}$$

$$= \sin \frac{3}{2}\pi - \sin \frac{\pi}{3} = -1.866$$

$$\textcircled{2} \Delta y = \sin 2(x + \Delta x) - \sin 2x = 2\cos(2x + \Delta x)\sin \Delta x$$

习 题 6 P.38

$$2. n \rightarrow \infty \text{ 时 } \frac{1000}{n} \rightarrow 0, \frac{1}{n-10000} \rightarrow 0, \frac{1}{10000-n} \rightarrow 0$$

4. 否, 没有指明 x 变化趋势。

$$5. \textcircled{1} +\infty, -\infty \quad \textcircled{2} +\infty, 0 \quad \textcircled{3} -\infty, +\infty$$

$$\textcircled{4} -\frac{\pi}{2}, \frac{\pi}{2} \quad \textcircled{5} -\infty, +\infty$$

$$6. \textcircled{1} 1 \quad \textcircled{2} -1$$

$$\textcircled{3} \lim_{x \rightarrow \infty} \left(e^{-x} - \frac{1}{x} \right) = \lim_{x \rightarrow \infty} e^{-x} - \lim_{x \rightarrow \infty} \frac{1}{x} = 0 - 0 = 0$$

$$\textcircled{4} -2 \quad \textcircled{5} 1 \quad \textcircled{6} 0 \quad \textcircled{7} -\frac{1}{x^2} \quad \textcircled{8} \frac{22}{9}$$

$$7. 0, 2, \infty$$

$$8. \textcircled{1} -\frac{1}{3} \quad \textcircled{2} 0$$

$$\textcircled{3} \lim_{n \rightarrow \infty} \frac{n(n-1)(2n-1)}{6n^2} = \lim_{n \rightarrow \infty} \frac{1}{6} \left(1 - \frac{1}{n} \right) \left(2 - \frac{1}{n} \right) = \frac{1}{3}$$

④ 5 ⑤ ω ⑥ 2

⑦ $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x} = \lim_{x \rightarrow 0} 2 \cdot \frac{\sin 2x}{2x} \cdot \frac{3x}{3 \sin 3x} = \frac{2}{3}$

⑧ $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2} = \lim_{x \rightarrow 0} \frac{2 \sin^2 x}{x^2} = 2$

⑨ 0 ⑩ 0

9. ① $x \rightarrow 0$ 和 $x \rightarrow \infty$ 时 $y \rightarrow 0$ ② $x \rightarrow \pm 1$ 时 $y \rightarrow \infty$

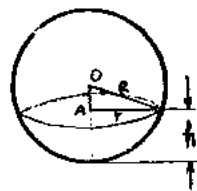
10. ① $t \rightarrow \infty$ $u \rightarrow 12$ ② $t \rightarrow 0$ $u \rightarrow 0$

习 题 7 P.44

1. $Q = Cu$

2. $P = i^2 R = RI_m^2 \sin^2 \omega t$

3. $S(\alpha) = \frac{1}{2} OB \cdot AB$
 $= \frac{1}{2} \alpha y = \frac{1}{2} x^2$



第 4 题

4. $A(h) = \pi r^2 = \pi [R^2 - (R - h)^2]$
 $= \pi h(2R - h) \quad [0, 2R]$

5. ① $W = kbh^2 = \frac{1}{6} b(30^3 - b^3) \quad (0 \leq b \leq 30)$

② $W(20) = \frac{1}{6} \times 20(900 - 400) = \frac{10000}{6}$

6. 设华氏 G 温标为 y , 摄氏温标为 x 。

设 $y = ax + b$, 由 $x = 0$ $y = 32$ 约 $b = 32$

$x = 100$, $y = 212$ $a = 1.8$

$\therefore y = 1.8x + 32$

7. 设方程为 $x^2 = -2P(y - 4)$

由 $x = 0$, $y = 0$ 约 $-2P = -25$

∴ 方程为 $x^2 = -25(y - 4)$ 即 $y = 4 - \frac{x^2}{25}$

$$h_3 = 4 - \frac{\left(-\frac{20}{8}\right)^2}{25} = 3.75$$

$$h_2 = 4 - \frac{\left(-\frac{20}{4}\right)^2}{25} = 3$$

$$h_1 = 4 - \frac{\left(-3 \times \frac{20}{8}\right)^2}{25} = 1.75$$

8. $u(0) = 3$ $u(t) = u(1) - 3e^{-2t} = \frac{3}{e^2}$

$$u(t-1) = 3e^{-2(t-1)} \quad u(t+\Delta t) - u(t) = 3e^{-2t}(e^{-2\Delta t} - 1)$$

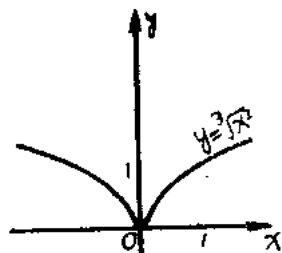
9. $f(0) = 0$ $f(a-1) = \ln a$ $\frac{1}{f(1)} = \frac{1}{\ln 2}$

$$2f(3) - f(1) = 2\ln 4 - \ln 2 = 3\ln 2 \quad f(a^2) = \ln(a^2 + 1)$$

$$f(x + \Delta x) - f(x) = \ln(1 + x + \Delta x) - f(1 + x)$$

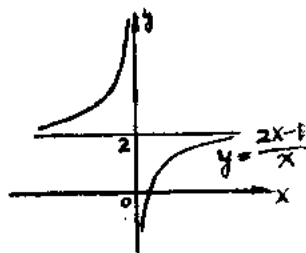
$$= \ln\left(1 + \frac{\Delta x}{x + \Delta x}\right)$$

10. ①



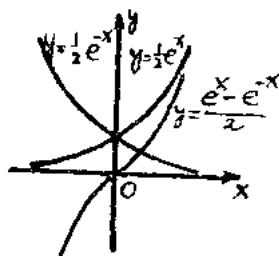
第 10① 题

②



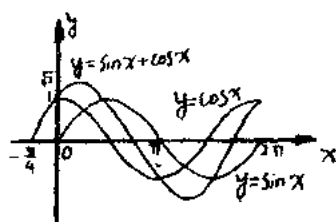
第 10② 题

③



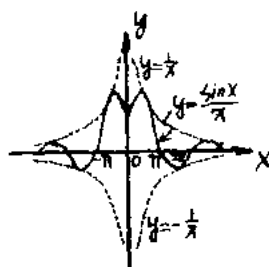
第 10③ 题

④



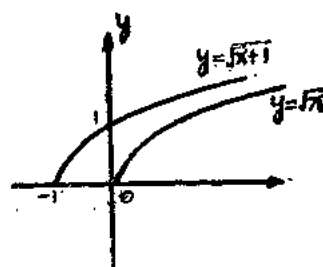
第 10④ 题

⑤



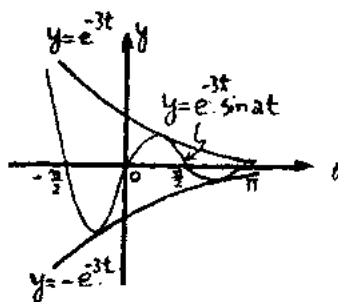
第 10⑤ 题

⑥



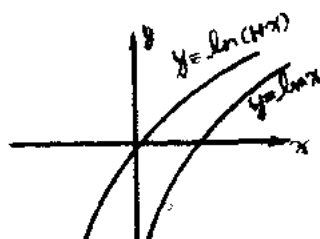
第 10⑥ 题

⑦



第 10⑦ 题

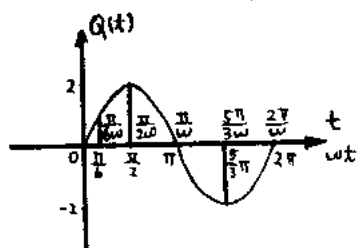
⑧



第 10⑧ 题

11.

①



第 11 题

$$\textcircled{2} \quad Q(0) = 0$$

$$Q\left(\frac{\pi}{6\omega}\right) = 2 \sin \frac{\pi}{6} = 1$$

$$Q\left(\frac{\pi}{2\omega}\right) = 2 \sin \frac{\pi}{2} = 2$$

$$Q\left(\frac{5\pi}{3\omega}\right) = 2 \sin \frac{5\pi}{3} = -\sqrt{3}$$

$$\begin{aligned} \textcircled{3} \quad Q(t + \Delta t) - Q(t) &= 2 \sin \omega(t + \Delta t) - 2 \sin \omega t \\ &= 4 \cos\left(\omega t + \frac{\omega \Delta t}{2}\right) \sin \frac{\omega \Delta t}{2} \end{aligned}$$

$$12. \quad \frac{\Delta s}{\Delta t} = g t + \frac{1}{2} g \Delta t$$

$$\textcircled{1} \quad \Delta t \rightarrow \infty \quad \frac{\Delta s}{\Delta t} \rightarrow \infty \quad t \text{ 固定, } \Delta t \text{ 增大, 平均速度增大.}$$

$$\textcircled{2} \quad t \rightarrow \infty \quad \frac{\Delta s}{\Delta t} \rightarrow \infty \quad \Delta t \text{ 固定, 平均速度随时间而增大.}$$