舒萊伯爾大地位置計算公式

中央人民政府人民革命軍事委員會
總參謀部測繪局印
一九五三年九月

大地位置計算

計算者:　　　　　　　　　　　　　　　　　　　　　　　　　　校算者:

已知點:(1)　　　　　　推算點:(3)　　　　　　已知點:(2)

φ_1	38° 05' 47".66	$\alpha_{1\cdot2}$	° ' "	φ_2	38° 07' 52".5189	$\alpha_{2\cdot1}$	° ' "
$\Delta\varphi$	19 24.4684	$\angle 2\cdot1\cdot3$		$\Delta\varphi$	17 19.6095	$\angle 1\cdot2\cdot3$	
φ_3	38 25 12.1284	$\alpha_{1\cdot3}$	336 14 46.151	φ_3	38 25 12.1284	$\alpha_{2\cdot3}$	11 47 20.794
λ_1	114 30 17.70	180°-ε	180 1.442	λ_2	114 14 50.2158	180°-ε	180 – 0.545
$\Delta\lambda$	– 10 51.6646	t	– 6 44.959	$\Delta\lambda$	4 35.8197	t	2 51.400
λ_3	114 19 26.0354	$\alpha_{3\cdot1}$	156 8 2.634	λ_3	114 19 26.0354	$\alpha_{3\cdot2}$	191 50 11.649
						檢驗角∠1.3.2	35 42 9.017

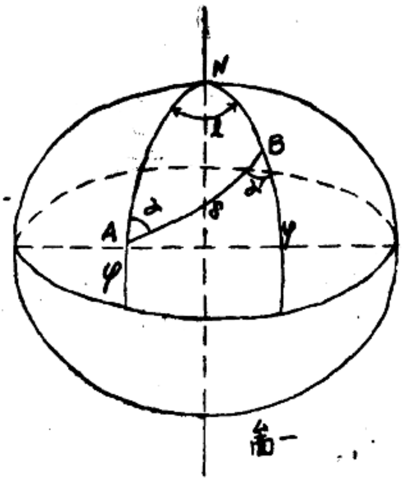
計算次序	項目		項目		計算次序	項目		項目	
1	$lg S_{1\cdot3}$	4.5937 9177			1	$lg S_{2\cdot3}$	4.5152 0130		
2	$lg \cos\alpha_{1\cdot3}$	9.9615 5608	$lg\, u$	4.55 535	2	$lg \cos\alpha_{2\cdot3}$	9.99 7 4110	$lg\, u$	4.50594
3	$lg \sin\alpha_{1\cdot3}$	9.6050 9825 n	$lg(4)_1$	8.52 342	3	$lg \sin\alpha_{2\cdot3}$	9.3102 8960	$lg(4)_2$	8.52355
1+2=4	$lg\, u$	4.5553 4785	$lg(4)_1 u$	3.07 877	1+2=4	$lg\, u$	4.5059 4240	$lg(4)_2 u$	3.02949
5	$lg(1)_1$	8.5109 7862	$(4)_1 u$	1198.9	5	$lg(1)_2$	8.5109 7605	$(4)_2 u$	1070.3
6	$-(4)_1 u$	– 1199			6	$-(4)_2 u$	1070		
7	$(5)_1 v^2$	89	$lg\, v^2$	8.39 778	7	$(5)_2 v^2$	16	$lg\, v^2$	7.65098
8	$(6)_1 u^2$	– 1	$lg(5)_1$	3.55 196	8	$(6)_2 u^2$	– 1	$lg(5)_2$	3.55196
4+5+6+7+8	$lg\, b_0$	3.0663 1536	$lg(5)_1 v^2$	1.94 974	4+5+6+7+8	$lg\, b_0$	3.0169 0790	$lg(5)_2 v^2$	1.20294
	b_0	1164.97166	$(5)_1 v^2$	89.1		b_0	1039.69966	$(5)_2 v^2$	16.0
	b_0	19 24.97166				b_0	17 19.69966		
	$\varphi_0=\varphi_1+b_0$	38 25 12.6317				$\varphi_0=\varphi_2+b_0$	38 25 12 218 6		
1+3=9	$lg\, v$	4.1988 9002 n	$lg\, u^2$	9.11 070	1+3=9	$lg\, v$	3.8254 9090	$lg\, u^2$	9.01 188
10	$lg(2)_0$	8.5091 6193	$lg(5)_1$	3.55 196	10	$lg(2)_0$	8.5091 6194	$lg(5)_2$	3.55 196
11	$-\frac{1}{2}(5)_1 u^2$	– 230	colg 2	9.69 897	11	$-\frac{1}{2}(5)_2 u^2$	– 183	colg 2	9.69 897
9+10+11=12	$lg\, c$	2.7080 4965 n	$lg \frac{1}{2}(5)_1 u^2$	2.36 163	9+10+11=12	$lg\, c$	2.3346 5101	$lg \frac{1}{2}(5)_2 u^2$	2.26 281
13	$lg \sec\varphi_0$	0.1059 7504	$\frac{1}{2}(5)_1 u^2$	229.9	13	$lg \sec\varphi_0$	0.1059 7435	$\frac{1}{2}(5)_2 u^2$	183.2
14	$lg\, tg\, \varphi_0$	9.8993 6284			14	$lg\, tg\, \varphi_0$	9.8993 6106		
12+13=15	$lg\, \lambda$	2.8140 2469 n	$lg\, t^2$	5.21 482	12+13=15	$lg\, \lambda$	2.4406 2536	$lg\, t^2$	4.46802
16	$-2\gamma t^2$	– 56	$lg\, \gamma$	6.23 078	16	$-2\gamma t^2$	– 10	$lg\, \gamma$	6.23078
17	$(9)_0 \lambda^4$	0	$lg\, \gamma t^2$	1.44 560	17	$(9)_0 \lambda^4$	0	$lg\, \gamma t^2$	0.69880
15+16+17	$lg\, \Delta\lambda$	2.8140 2413 n	γt^2	27.9	15+16+17	$lg\, \Delta\lambda$	2.4406 2526	γt^2	5.0
	$\Delta\lambda$	– – 651.66460	$2\gamma t^2$	55.80		$\Delta\lambda$	275.81969	$2\gamma t^2$	10.0
12+14=18	$lg\, t$	2.6074 1249 n			12+14=18	$lg\, t$	2.2340 1207		
19	$-\gamma t^2$	– 28	$lg\, \lambda^2$	5.62 805	19	$-\gamma t^2$	– 5	$lg\, \lambda^2$	4.88 125
20	$-\gamma \lambda^2$	– 72	$lg\, \gamma$	6.23 078	20	$-\gamma \lambda^2$	– 13	$lg\, \gamma$	6.22 078
21	$(7)_0 \lambda^2$	0	$lg\, \gamma \lambda^2$	1.85 883	21	$(7)_0 \lambda^2$	0	$lg\, \gamma \lambda^2$	1.11 203
18+19+20+21	$lg\, t$	2.6074 1149 n	$\gamma \lambda^2$	72.2	18+19+20+21	$lg\, t$	2.2340 1189	$\gamma \lambda^2$	13.2
	t	– 404.9594	$\frac{1}{2}\gamma \lambda^2$	36.1		t	171.4004	$\frac{1}{2}\gamma \lambda^2$	6.6
22	$lg(3)_0$	4.386 338			22	$lg(3)_0$	4.386 338		
12	$lg\, c$	2.708 050 n	$lg\, b$	3.06 632	12	$lg\, c$	2.334 651	$lg\, b$	3.01 671
18	$lg\, t$	2.607 412 n	$lg\, c$	2.70 805 n	18	$lg\, t$	2.234 012	$lg\, c$	2.33 465
22+12+18=23	$lg\, \delta$	9.701 800	colg 2ρ	4.38 454	22+12+18=23	$lg\, \delta$	8.955 001	colg 2ρ	4.38 454
24	$-\gamma t^2$	– 0	$lg\, \varepsilon$	0.15 891 n	24	$-\gamma t^2$	– 0	$lg\, \varepsilon$	9.73 610
25	$-\frac{1}{2}\gamma \lambda^2$	– 0	ε	– 1.4418	25	$-\frac{1}{2}\gamma \lambda^2$	– 0	ε	0.5446
26	$(8)_0 \lambda^2$	0	–ε	1.4418	26	$(8)_0 \lambda^2$	0	–ε	0.5446
23+24+25+26	$lg\, d$	9.701 800			23+24+25+26	$lg\, d$	8.955 001		
	d	0.50327				d	0.09016		
	$\Delta\varphi=b_0-d$	14' 24".46839				$\Delta\varphi=b_0-d$	17' 19".00750		

附記　1. 經緯度算至小數點四位，方位角算至小數點三位
　　　2 本計算用紙使用於一等三角計算

舒萊伯爾大地位置計算公式

盧福康　編

1． 由一已知大地位置的三角點，在參考橢圓體上推算另一三角點的大地位置，設此兩三角點間大地綫長爲已知，已知點至所求點的方位角亦已知，則可用大地綫 S 之冪級數以計算之。如圖一，N 爲參考橢圓體之北極，A 爲已知點其經緯度爲 λ，φ，A 至 B 之



圖一

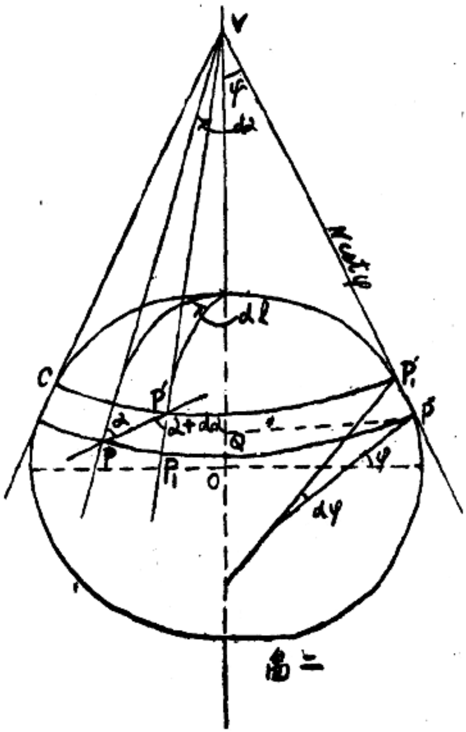
方位角爲 α，B 點爲所求點，設其經緯度爲 λ' φ' B 至 A 之反方位角爲 $180+\alpha'$，又如圖二．設有極靠近的兩點 P．P'，其間距離爲 ds，過 P 及 P' 點作兩平行圈由 P 點作子午圈切綫和地軸相交於 V，

PVO 角等於 P 點的緯度 φ。由圖易知 $P''Q$ 等於 $N\cos\varphi$， N 爲 P 點卯酉圈曲率半徑，$VP=N\cot\varphi$， P 和 P' 的經差 dl 由圖可以很容易地推出：

$P_1P'=P''P_1'=Md\varphi$，M 爲子午圈曲率半徑。$PP_1=N\cos\varphi dl$

$P_1P'=ds\ \cos\alpha$， $P\ P_1=ds\ \sin\alpha$

又：
$$d\alpha=\frac{P\ P_1}{P_1\ V}=\frac{N\cos\varphi dl}{N\cot\varphi}=\sin\varphi dl$$



圖二

根據以上的叙述可得以下的關係式:

$$ds\cos\alpha = M\,d\varphi \qquad (1)$$

$$ds\sin\alpha = N\cos\varphi\,dl \qquad (2)$$

$$d\alpha = \sin\varphi\,dl \qquad (3)$$

子午圈曲率半徑算式:

$$M = \frac{a^2 b^2}{\left(a^2\cos^2\varphi + b^2\sin^2\varphi\right)^{\frac{3}{2}}} \qquad (4)$$

又:

$$N = \frac{a^2}{\left(a^2\cos^2\varphi + b^2\sin^2\varphi\right)^{\frac{1}{2}}} \qquad (5)$$

爲簡便計，令 $\frac{a^2}{b} = c$ 則得:

$$a^2\cos^2\varphi + b^2\sin^2\varphi = \frac{c^2\,(1+e'^2\cos^2\varphi)}{(1+e'^2)^2}$$

又設 $V^2 = 1 + e'^2\cos^2\varphi$ 則得:

$$M = \frac{c}{V^3} \qquad N = \frac{c}{V}$$

將 M，N，的簡寫式代入（1）（2）（3） 得

$$\frac{d\varphi}{ds} = \frac{1}{c}V^3\cos\alpha \qquad (6)$$

$$\frac{dl}{ds} = \frac{1}{c}\,V\frac{\sin\alpha}{\cos\varphi} \qquad (7)$$

$$\frac{d\alpha}{ds} = \frac{1}{c}V\sin\alpha\tan\varphi \qquad (8)$$

上面已叙述過 λ，φ，α 都可以寫成 s 的函數式如:

$$\lambda' = f\,(s) \qquad \varphi' = \Psi\,(s) \qquad \alpha = \zeta\,(s)$$

並且在 $s=o$ 時（λ'）（φ'）（α'）是已知點A的相應值 λ，φ，α，亦即

$$\lambda = f\,(o) \qquad \varphi = \Psi\,(o) \qquad \alpha = \zeta\,(o)$$

今將上述的函數用*Maclaurin*級數展開，則有:

$$\lambda'-\lambda=f^{\mathrm{I}}(o)\,s+f^{\mathrm{II}}(o)\frac{s^2}{2}+f^{\mathrm{III}}(o)\frac{s^3}{3!}$$

$$+f^{\mathrm{IV}}(o)\frac{s^4}{4!}+f^{\mathrm{V}}(o)\frac{s^5}{5!} \qquad (9)$$

$$\varphi'-\varphi=\Psi^{\mathrm{I}}(o)\,s+\Psi^{\mathrm{II}}(o)\frac{s^2}{2}+\Psi^{\mathrm{III}}(o)\frac{s^3}{3!}$$

$$+\Psi^{\mathrm{IV}}(o)\frac{s^4}{4!}+\Psi^{\mathrm{V}}(o)\frac{s^5}{5!} \qquad (10)$$

$$\alpha'-\alpha=\zeta^{\mathrm{I}}(o)\,s+\zeta^{\mathrm{II}}(o)\frac{s^2}{2}+\zeta^{\mathrm{III}}(o)\frac{s^3}{3!}$$

$$+\zeta^{\mathrm{IV}}(o)\frac{s^4}{4!}+\zeta^{\mathrm{V}}(o)\frac{s^5}{5!} \qquad (11)$$

以上函式的各項微分係數可由（6）（7）（8）式導出:

$$f'(s)=\frac{df(s)}{ds}=\frac{dl}{ds}=\frac{1}{c}V\sin\alpha\sec\varphi \qquad (12)$$

$$\Psi'(s)=\frac{d\Psi(s)}{ds}=\frac{d\varphi}{ds}=\frac{1}{c}V^2\cos\alpha \qquad (13)$$

$$\zeta'(s)=\frac{d\zeta(s)}{ds}=\frac{d\alpha}{ds}=\frac{1}{c}V\sin\alpha\tan\varphi \qquad (14)$$

茲再由上式推求二次微分:

$$\frac{d^2l}{ds^2}=\frac{1}{c}\,\frac{dV}{ds}\sin\alpha\sec\varphi+\frac{V}{c}\sec\varphi\cos\alpha\frac{d\alpha}{ds}$$

$$+\frac{1}{c}V\sin\alpha\sec\varphi\tan\varphi\frac{d\varphi}{ds}$$

因:
$$\frac{dV}{ds}=\frac{dV}{d\varphi}\cdot\frac{d\varphi}{ds}=-\frac{\eta^2}{V}\tan\varphi\cdot\frac{1}{c}V^2\cos\alpha$$

$$=-\frac{\eta^2V^2}{c}\cos\alpha\tan\varphi$$

故

$$\frac{d^2 l}{ds^2}=-\frac{1}{c^2}\eta^2 V^2 \sin\alpha\cos\alpha\sec\varphi\tan\varphi$$

$$+\frac{V^2}{c^2}\sin\alpha\cos\alpha\sec\varphi\tan\varphi$$

$$+\frac{V^4}{c^2}\sin\alpha\cos\alpha\sec\varphi\tan\varphi$$

$$=\frac{V^2}{c^2}\sin\alpha\cos\alpha\sec\varphi\tan\varphi\,(-\eta^2+1+V^2)$$

$$=\frac{2V^2 t}{c^2\cos\varphi}\sin\alpha\cos\alpha \tag{15}$$

上式中 $t=\tan\varphi$。

同理可得:

$$\frac{d^3 l}{ds^3}=\frac{4Vt}{c^2\cos\varphi}\sin\alpha\cos\alpha\frac{dV}{ds}+\frac{2V^2 t}{c^2\cos\varphi}\cos^2\alpha\frac{d\alpha}{ds}$$

$$-\frac{2V^2 t}{c^2\cos\varphi}\sin^2\alpha\frac{d\alpha}{ds}+\frac{2V^2 t^2}{c^2\cos\varphi}\sin\alpha\cos\alpha\frac{d\varphi}{ds}$$

$$+\frac{2V^2}{c^2\cos\varphi}\sin\alpha\cos\alpha\sec^2\varphi\frac{d\varphi}{ds}$$

$$=-\frac{4\eta^2 V^3 t^2}{c^3\cos\varphi}\sin\alpha\cos^2\alpha+\frac{2V^3 t^2}{c^3\cos\varphi}\sin\alpha\cos^2\alpha$$

$$-\frac{2V^3 t^2}{c^3\cos\varphi}\sin^3\alpha+\frac{2V^5 t^2}{c^3\cos\varphi}\sin\alpha\cos^2\alpha$$

$$+\frac{2V^5}{c^3\cos^3\varphi}\sin\alpha\cos^2\alpha$$

$$=\frac{2V^3}{c^3\cos\varphi}\left\{\left(-2\eta^2 t^2+t^2+V^2 t^2\right.\right.$$

$$\left.\left.+\frac{V^2}{\cos^2\varphi}\right)\sin\alpha\cos^2\alpha-t^2\sin^3\alpha\right\}$$

$$=\frac{2v^3}{c^3\cos\varphi}\Big\{\Big(-2\eta^2t^2+t^2+V^2t^2+V^2\ (1+t^2)\Big)\sin\alpha\cos^2\alpha$$

$$-t^2\sin^3\alpha\Big\}$$

$$=\frac{2V^3}{c^3\cos\varphi}\Big\{\ (1+3t^2+\eta^2)\sin\alpha\cos^2\alpha-t^2\sin^3\alpha\Big\}\quad(16)$$

再由(13)式求$\frac{d\varphi}{ds}$的二次微分:

$$\frac{d^2\varphi}{ds^2}=\frac{3}{c}V^2\cos\alpha\frac{dV}{ds}+\frac{1}{c}V^3(-\sin\alpha)\frac{d\alpha}{ds}$$

$$=\frac{3V^4}{c^2}\eta^2\cos^2\alpha\, t-\frac{V^4}{c^2}\sin^2\alpha\, t$$

$$=-\frac{V^4}{c^2}(t\sin^2\alpha+3\eta^2t\cos^2\alpha)\quad(17)$$

再由(17)式求$\frac{d\varphi}{ds}$的三次微分:

$$\frac{d^3\varphi}{ds^3}=-\frac{4V^3}{c^2}t\sin^2\alpha\ \frac{dV}{ds}-\frac{V^4}{c^2}\sin^2\alpha\sec^2\varphi\ \frac{d\varphi}{ds}$$

$$-\frac{V^4}{c^2}t.2\sin\alpha\cos\alpha\ \frac{d\alpha}{ds}-\frac{6V^4}{c^2}\eta\, t\cos^2\alpha\frac{d\eta}{ds}$$

$$-\frac{3V^4}{c^2}\ \eta^2\sec^2\varphi\cos^2\ \frac{d\varphi}{ds}+\frac{6V^4}{c^2}\eta^2t\cos\alpha\ \sin\alpha\ \frac{d\alpha}{ds}$$

$$-\frac{12V^3}{c^2}\eta^2t\cos^2\alpha\ \frac{dV}{ds}$$

因爲　　$\eta^2=e'^2\cos^2\varphi$　故　$2\eta\, d\eta=-2e'^2\cos\varphi\sin\varphi\, d\varphi$

即:　　$\eta\frac{d\eta}{d\varphi}=-e'^2\cos\varphi\sin\varphi=-e'^2\cos^2\varphi\tan\varphi=-\eta^2t$

$$\eta\frac{d\eta}{ds}=\eta\frac{d\eta}{d\varphi}\cdot\frac{d\varphi}{ds}=-\eta^2t\cdot\frac{V^3}{c}\cos\alpha$$

故有:

$$\frac{d^3\varphi}{ds^3}=\frac{4V^5\eta^2}{c^3}t^2\sin^2\alpha\cos\alpha-\frac{V^7}{c^3}\sin^2\alpha\sec\varphi\cos\alpha$$

$$-\frac{2V^5}{c^3}t^2\sin^2\alpha\cos\alpha+\frac{6V^7}{c^3}\eta^2t^2\cos^3\alpha$$

$$-\frac{3V^7}{c^3}\eta^2\cos^3\alpha\sec^2\varphi+\frac{6V^5}{c^3}\eta^2t^2\cos\alpha\sin^2\alpha$$

$$+\frac{12V^5}{c^3}\eta^4t^2\cos^3\alpha.$$

$$=-\frac{V^5\cos\alpha}{c^3}\Big\{\Big(-6V^2\eta^2t^2+3V^2\eta^2(1+t^2)-12\eta^4t^2\Big)\cos^2\alpha$$

$$+\Big(-4\eta^2t^2+V^2(1+t^2)+2t^2-6\eta^2t^2\Big)\sin^2\alpha\Big\}$$

$$=-\frac{V^5\cos\alpha}{c^3}\Big\{(3\eta^2-3\eta^2t^2+3\eta^4-15\eta^4t^2)\cos^2\alpha$$

$$+(1+t^2+\eta^2-9\eta^2t^2)\sin^2\alpha\Big\} \tag{18}$$

同理由(14)式可求$\frac{d\alpha}{ds}$的二次及三次微分:

$$\frac{d^2\alpha}{ds^2}=\frac{1}{c}\sin\alpha\, t\frac{dV}{ds}+\frac{V}{c}t\cos\alpha\frac{d\alpha}{ds}+\frac{V}{c}\sin\alpha\sec^2\varphi\frac{d\varphi}{ds}$$

$$=-\frac{\eta^2V^4}{c^2}t^2\sin\alpha\cos\alpha+\frac{V^2}{c^2}t^2\sin\alpha\cos\alpha$$

$$+\frac{V^4}{c^2}\sin\alpha\cos\alpha\sec^2\varphi$$

$$=\frac{^2V}{c^2}\sin\alpha\cos\alpha\Big(-\eta^2t^2+t^2+v^2(1+t^2)\Big)$$

$$=\frac{V^2}{c^2}\sin\alpha\cos\alpha(1+2t^2+\eta^2) \tag{19}$$

$$\frac{d^3\alpha}{ds^3}=\frac{2V}{c^2}\sin\alpha\cos\alpha\frac{dV}{ds}+\frac{V^2}{c^2}\cos^2\alpha\;\frac{d\alpha}{ds}-\frac{V^2}{c^2}\sin^2\alpha\;\frac{d\alpha}{ds}$$

$$+\frac{4Vt^2}{c^2}\sin\alpha\cos\alpha\frac{dV}{ds}+\frac{4V^2t}{c^2}\sin\alpha\cos\alpha\sec^2\varphi\frac{d\varphi}{ds}$$

$$+\frac{2V^2t^2}{c^2}\cos^2\alpha\frac{d\alpha}{ds}-\frac{2V^2t^2}{c^2}\sin^2\alpha\;\frac{d\alpha}{ds}$$

$$+\frac{V^2}{c^2}\sin\alpha\cos\alpha\frac{d\eta^2}{ds}+\frac{2V\eta^2}{c^2}\sin\alpha\cos\alpha\frac{dV}{ds}$$

$$+\frac{\eta^2V^2}{c^2}\cos^2\alpha\frac{d\alpha}{ds}-\frac{\eta^2V^2}{c^2}\sin^2\alpha\;\frac{d\alpha}{ds}$$

$$=-\frac{2V^3}{c^3}\eta^2 t\sin\alpha\cos^2\alpha+\frac{V^3}{c^3}t\sin\alpha\cos^2\alpha-\frac{V^3}{c^3}t\sin^3\alpha$$

$$-\frac{4\eta^2V^3t^3}{c^3}\sin\alpha\cos^2\alpha+\frac{4V^3t}{c^3}\sin\alpha\cos^2\alpha\sec^2\varphi$$

$$+\frac{2V^3t^3}{c^3}\sin\alpha\cos^2\alpha-\frac{2V^3\eta^2}{c^3}t\sin\alpha\cos^2\alpha$$

$$-\frac{2V^3}{c^3}t^3\sin^3\alpha-\frac{2V^3\eta^4}{c^3}t\sin\alpha\cos^2\alpha$$

$$+\frac{\eta^2V^3}{c^3}t\sin\alpha\cos^2\alpha-\frac{\eta^2V^3}{c^3}t\sin^3\alpha$$

$$=\frac{V^3}{c^3}\Big\{t\sin\alpha\cos^2\alpha\Big(-2\eta^2+1-4\eta^2t^2+4(1+\eta^2)$$

$$(1+t^2)+2t^2-2V^2\eta^2-2\eta^4+\eta^2\Big)$$

$$-t\sin^3\alpha(1+2t^2+\eta^2)$$

$$=\frac{V^3}{c^3}\Big\{t\sin\alpha\cos^2\alpha\,(5+6t^2+\eta^2-4\eta^4)$$

$$-t\sin^3\alpha(1+2t^2+\eta^2)\qquad(20)$$

採用上述同樣的方法可求出四次，五次的微分，並且設

$$\xi = \frac{V}{c} s \sin \alpha = \frac{s}{N} \sin \alpha = t \qquad (21)$$

$$\zeta = \frac{V}{c} s \cos \alpha = \frac{s}{N} \cos \alpha = u \qquad (22)$$

以V^2除（10）式，以 $\cos\varphi$ 乘（9）式，現在寫出各項的主要部份如次：

$$\frac{d\varphi}{ds} \quad \frac{s}{V^2} = +\zeta$$

$$\frac{d^2\varphi}{ds^2} \quad \frac{s^2}{V^2} = -\xi^2 t - \zeta^2(3\eta^2 t)$$

$$\frac{d^3\varphi}{ds^3} \quad \frac{s^3}{V^2} = -\xi^2 \zeta (1+3t^2+\eta^2-9\eta^2 t^2)$$
$$-3\zeta^3 \eta^2(1+t^2+\eta^2-5\eta^2 t^2)$$

$$\frac{d^4\varphi}{ds^4} \quad \frac{s^4}{V^2} = +\xi^4 t (1+3t^2+\eta^2-9\eta^2 t^2) - 2\xi^2\zeta^2 t (4+6t^2$$
$$-13\eta^2-9\eta^2 t^2-17\eta^4+45\eta^4 t^2)$$
$$+\zeta^4 t\eta^2(12+63\eta^2-45\eta^2 t^2+57\eta^4-105\eta^4 t^2)$$

$$\frac{d^5\varphi}{ds^5} \quad \frac{s^5 c}{v^2} = +\xi^4 \zeta (1+30t^2+45t^4)$$
$$-2\xi^2\zeta^3(4+30t^2+30t^4)$$

$$\frac{dl}{ds} s \cos\varphi = +\xi$$

$$\frac{d^2 l}{ds^2} s^2 \cos\varphi = +2\xi\zeta t$$

$$\frac{d^3 l}{ds^3} s^3 \cos\varphi = +2\xi\zeta^2(1+3t^2+\eta^2) - 2\xi^3 t^2$$

$$\frac{d^4 l}{ds^4} s^4 \cos\varphi = +8\xi\zeta^3 t (2+3t^2+\eta^2-\eta^4)$$
$$-8\xi^3\zeta t(1+3t^2+\eta^2)$$

$$\frac{d^5 l}{ds^5} s^5 \cos\varphi = +8\xi\zeta^4(2+15t^2+15t^4)$$

$$-8\,\xi^3\zeta^2(1+20t^2+30t^4)+8\,\xi^5 t^2(1+3t^2)$$

$$\frac{d\alpha}{ds}s=\xi t$$

$$\frac{d^2\alpha}{ds^2}s^2=\xi\zeta(1+2t^2+\eta^2)$$

$$\frac{d^3\alpha}{ds^3}s^3=\xi\zeta^2 t\,(5+6t^2+\eta^2-4\eta^2)-\xi^2 t\,(1+2t^2+\eta^2)$$

$$\frac{d^4\alpha}{ds^4}s^4=\xi\zeta^3(5+28t^2+24t^4+6\eta^2+8\eta^2 t^2+3\eta^4+4\eta^4 t^2$$
$$-4\eta^6+24\eta^6 t^2)-\xi^2\zeta(1+20t^2+24t^4+2\eta^2+8\eta^2 t^2$$
$$+\eta^4-12\eta^4 t^2)$$

$$\frac{d^5\alpha}{ds^5}s^5=\xi\zeta^4 t(61+180t^2+120t^4)-\xi^3\zeta^2 t\,(58+280t^2$$
$$+240t^4)+\xi^5 t\,(1+20t^2+24t^4)$$

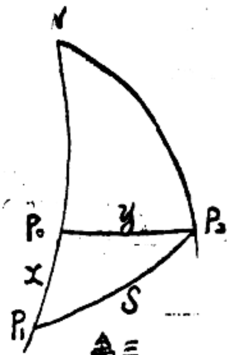
將以上各值代入(9)(10)(11)，並以　$l=\lambda'-\lambda$

$$\left.\begin{aligned}
l\cos\varphi=\xi&+\xi\zeta t-\frac{1}{3}\xi^3 t^2+\frac{1}{3}\xi\zeta^2(1+3t^2+\eta^2)\\
&-\frac{1}{3}\xi^3\zeta t(1+3t^2+\eta^2)\\
&+\frac{1}{3}\xi\zeta^3 t\,(2+3t+\eta^2)\\
&+\frac{1}{15}\xi^5 t^2(1+3t^2)\\
&+\frac{1}{15}\xi\zeta^4(2+15t^2+15t^4)\\
&-\frac{1}{15}\xi^3\zeta^2(1+20t^2+30t^4)
\end{aligned}\right\}\quad(23)$$

$$\left.\begin{aligned}\frac{\varphi^1-\varphi}{V^2} = \zeta - \frac{1}{2}\xi^2 t - \frac{3}{2}\zeta^2\eta^2 t - \frac{1}{6}\xi^2\zeta\ (1+3t^2 \\ +\eta^2-9\eta^2 t^2) - \frac{1}{2}\zeta^3\eta^2\ (1-t^2) \\ +\frac{1}{24}\xi^4 t\ (1+3t^2+\eta^2-9\eta^2t^2) \\ -\frac{1}{12}\xi^2\zeta^2 t(4+6t^2-12\eta^2-9\eta^2t^2)+\frac{1}{2}\zeta^4\eta^2 t \\ +\frac{1}{120}\xi^4\zeta\ (1+30t^2+45t^4) - \frac{1}{30}\xi^2\zeta^3\ (2 \\ +15t^2+15t^4)\end{aligned}\right\} \quad (24)$$

$$\left.\begin{aligned}\alpha' - \alpha = \xi t + \frac{1}{2}\xi\zeta\ (1+2t^2+\eta^2) \\ -\frac{1}{6}\xi^3 t\ (1+2t^2+\eta^2) \\ +\frac{1}{6}\xi\zeta^2 t\ (5+6t^2+\eta^2-4\eta^4) \\ -\frac{1}{24}\xi^3\zeta\ (1+20t^2+24t^4+2\eta^2+8\eta^2t^2) \\ +\frac{1}{24}\xi\zeta^3\ (5+28t^2+24t^4+6\eta^2+8\eta^2t^2) \\ +\frac{1}{120}\xi^5 t\ (1+20t^2+24t^4) \\ -\frac{1}{120}\xi^3\zeta^2 t\ (58+280t^2+240t^4) \\ +\frac{1}{120}\xi\zeta^4 t\ (61+180t^2+120t^4)\end{aligned}\right\} \quad (25)$$

(23) (24) (25) 三式爲緯差$\Delta\varphi$，經差 l，子午線收斂$\Delta\alpha$



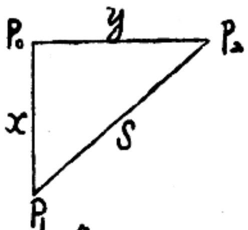
圖三

以大地綫長 S 及方位角 α 之函數，用級數表之公式，在大地測量上用途頗多。

2. *Schreibel* 推演關於大地位置計算之公式如下:

設有如圖三之 p_1及p_2兩點，p_1爲已知點，p_2爲所求點$p_1p_2=S$，S的數值不超過150 公里，設pN子午圈上選取一點p_0，使$\angle p_1p_0p_2$爲直角，p_0p_2 是 p_0，p_2 間的大地綫，p_1p_2 的方位角是α，這三角形的球面角超爲

$$\epsilon = \frac{s^2 \sin\alpha\cos\alpha}{2MN}\rho''$$



圖四

依據 *Legendre* 的理論，球面三角形 $p_1p_0p_2$ 可用圖 4 的平面三角形來計算邊長，這三角形的三個角爲

$$p_1\text{角} = \alpha - \frac{1}{3}\epsilon \quad p_0\text{角} = 90° - \frac{1}{3}\epsilon$$

$$p_2\text{角} = 90 - \left(\alpha - \frac{2}{3}\epsilon\right)$$

故可得:

$$x = S\frac{\sin\left[90° - \left(\alpha - \frac{2}{3}\epsilon\right)\right]}{\sin\left(90° - \frac{1}{3}\epsilon\right)}$$

$$y = S\frac{\sin\left(\alpha - \frac{\epsilon}{3}\right)}{\sin\left(90° - \frac{\epsilon}{3}\right)}$$

或

$$x = S\frac{\cos\left(\alpha - \frac{2}{3}\epsilon\right)}{\cos\frac{1}{3}\epsilon} \quad y = S\frac{\sin\left(\alpha - \frac{\epsilon}{3}\right)}{\cos\frac{\epsilon}{3}}$$

因爲 ϵ 是很小數值，$\cos\frac{1}{3}\epsilon \doteqdot 1$ 故可得：

$$x = s\cos\alpha + s\sin\alpha\,\frac{2\epsilon}{3}$$

$$y = s\sin\alpha - s\cos\alpha\,\frac{\epsilon}{3} \tag{26}$$

再就圖三先由 p_1 點推算 p_0 的緯度，利用公式（24），可知在這情況時（24）式中的 $\alpha=0$，因 p_1p_0 同在一個子午圈上。x 相當於 S，用 $\alpha=0$ $S=x$ 代入（24）式，各項的 $\xi=0$ 故有：

$$\frac{\varphi_0-\varphi_1}{V_1^2} = \frac{x}{N_1} - \frac{2}{3}\,\frac{x^2}{N_1^2}\eta^2 t_1 - \frac{1}{2}\,\frac{x^3}{N_1^3}\eta^2\ (1-t_1^2) \tag{27}$$

再假設 $s\sin\alpha = V$ $\quad s\cos\alpha = u$ （28）

將（26）（28）的關係代入（27）並將 x^2x^3 之値含 ϵ 項略去不計。

$$\frac{\varphi_0-\varphi_1}{V_1^2} = \frac{1}{N_1}(u + v\frac{2\epsilon}{3} - \frac{2}{3}\,\frac{\eta^2}{a_1^2}t_1(u+v\frac{2}{3}\epsilon)^2 - \frac{1}{2}\,\frac{\eta^2}{N_1^3}\ (1-t_1^2)\ (u+v\frac{2}{3}\epsilon)^3$$

$$\doteqdot \frac{1}{N_1}\ (u + v\frac{uv}{3M_1N_1}) - \frac{2}{3}\,\frac{u^2}{N_1^2}t_1\eta^2 - \frac{1}{3}\,\frac{u^3}{N_1^3}\eta^2\ (1-t_1^2)$$

$$\varphi_0-\varphi_1 = u\frac{V^2}{N_1}\left\{1 + \frac{1}{3}\,\frac{v^2}{M_1N_1} - \frac{2}{3}\,\frac{u}{N_1}\eta^2 t_1 - \frac{1}{3}\,\frac{u^2}{N_1^2}\eta^2\ (1-t_1^2)\right\}$$

因 $\frac{c}{V_1}=N_1$ $\frac{C}{V_1^3}=M_1$ 故 $\frac{N_1}{M_1}=V_1^2$ 將這關係導入上式，並以

² 代替 N_1^2，得：

$$\varphi_0-\varphi_1=\frac{u}{M_1}\left\{1+\frac{1}{3}\frac{v^2}{M_1N_1}-\frac{3}{2}\frac{u}{M_1V_1^2}\eta^2 t_1\right.$$

$$\left.-\frac{1}{2}\frac{u^2}{a^2}\eta^2\ (1-t_1^2)\right\} \qquad (29)$$

又因 $\eta_1^2=e'^2\cos^2\varphi_1 \qquad 1+e'^2=\frac{1}{1-e^2} \qquad e'^2=\frac{e^2}{1-e^2}$

$V_1^2\ (1-e^2)\ =W_1^2$

故：

$$\frac{\eta_1^2}{V_1^2}=\frac{e^2\cos^2\varphi_1}{(1-e^2)\ V_1^2}=\frac{e^2\ \cos^2\varphi_1}{W_1^2}$$

將上式代入（29）式，並將 u^2 項內的 η^2 代以 $e^2\ \cos^2\varphi$

$$\varphi_0-\varphi_1=\frac{u}{M_1}\left\{1+\frac{1}{3}\frac{v^2}{M_1N_1}-\frac{3}{2}\frac{u}{M_1\ W_1^2}\ e^2\cos^2\varphi_1\tan\varphi_1\right.$$

$$\left.-\frac{1}{2}\frac{u^2}{a^2}e^2\ \cos^2\varphi_1\ (1-\ \tan^2\varphi_1)\right\}$$

$$=\frac{u}{M_1}\left\{\ 1+\frac{1}{3}\frac{v^2}{M_1N_1}-\frac{3}{4}\frac{u}{M_1W_1^2}\ e^2\sin 2\varphi_1\right.$$

$$\left.-\frac{1}{2}\frac{u^2}{a^2}e^2\ \cos 2\varphi_1\right\} \qquad (30)$$

$\varphi_0-\varphi_1$若以秒爲單位：

$$\varphi_0-\varphi_1=\frac{u\,\rho''}{M_1}\left\{1+\ (\frac{1}{3}\frac{v^2}{M_1N_1}-\frac{3}{4}\frac{u}{M_1W_1^2}e^2\sin 2\varphi_1\right.$$

$$\left.-\frac{1}{2}\frac{u^2}{a^2}\ e^2\cos 2\ \varphi_1)\ \right\}$$

以對數計算可用下式：

$$\log(\varphi_0-\varphi_1)=\log\frac{u\rho''}{M_1}+\frac{\mu\ v^2}{3M_1N_1}-\frac{3\ u\mu}{4M_1W_1^2}e^2\sin 2\varphi_1$$

$$-\frac{u^2\mu}{2a^2}e\cos 2\varphi_1 \tag{31}$$

再命

$$\varphi_0-\varphi_1=b,\ \frac{\rho''}{M_1}=(1)_1,(1)_1u=\beta,\frac{\mu}{3M_1N_1}=\frac{\mu}{3\gamma^2}=(5)_1$$

$$\frac{3\mu e^2\sin 2\varphi_1}{4W_1^2M_1}=(4)_1,\ -\frac{\mu e^2\cos 2\varphi_1}{2a^2}=(6)_1$$

故(31)式可寫爲:

$$\left.\begin{aligned}\log b&=\log\beta-(4)_1u+(5)_1u^2+(6)_1u^2\\ \varphi_0&=\varphi_1+b\end{aligned}\right\} \tag{32}$$

由(32)式算出 φ_0 後。再用類似辦法推出 $\varphi_2-\varphi_0$。仍依據上述的(24)式，p_0p_2 的方位角爲90°，y 相當於 S，以 $\alpha=90°$，$S=y$ 代入(24)式得：$\zeta=0$ 故得

$$\frac{\varphi_2-\varphi_0}{V_0^2}=-\frac{1}{2}\frac{y^2}{N_0^2}\rho''t_0+\frac{1}{24}\frac{y^4}{N_0^4}\rho''t_0(1+3t_0^2+\eta_0^2-9\eta_0^2t_0^2)$$

設　$c=\frac{y}{N_0}\rho''$　代入上式得:

$$\frac{\varphi_2-\varphi_0}{V_0^2}=-\frac{1}{2}\frac{c^2}{\rho}t_0+\frac{1}{24}\frac{c^4}{\rho^3}t_0(1+3t_0^2+\eta_0^2-9\eta_0^2t_0^2) \tag{33}$$

再以 $e^2\cos^2\varphi$ 代替 η^2，則 $1+3t_0^2+\eta_0^2-9\eta_0^2t_0^2$ 可寫成下式

$$1+3t_0^2+e^2-e^2\sin^2\varphi_0-9e^2\sin^2\varphi_0+3e^2t_0^2-3e^2t_0^2$$

若 e^4 項略去不計，上式亦可寫成下列形式

$$(1+3t_0^2-10e^2\sin^2\varphi_0-3e^2t_0^2)\quad(1+e^2)$$

或　$$(1+3t_0^2-10e^2\sin^2\varphi_0-3e^2t_0^2)\frac{1}{1-e^2}$$

將上列値代入（33）式，並將 $\varphi_2-\varphi_0$ 改爲 $\varphi_0-\varphi_2$