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三氏微積分題解

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陳明鑑編

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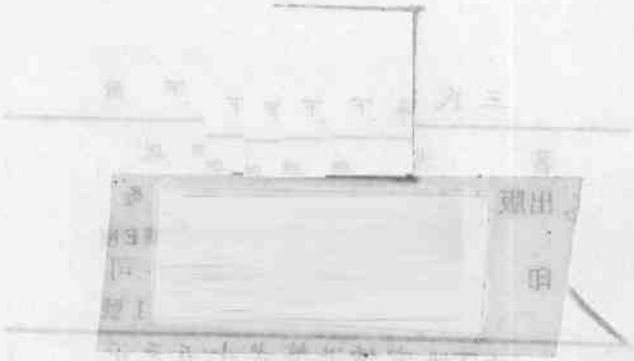

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三氏微積分題解 下 册

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出版及發行：文　源　書　局
澳門大馬路348號二樓E座
印　　刷：偉聯印刷公司
澳門草堆街151號

上下册 定價港幣式十五元正

第十六章 標準基本形式之積分；應用

習題解答 P-259

作出下列積分式：

1. $\int \frac{dx}{2+3x}$

〔解〕設 $v=2+3x \quad dv=3dx$.

$$\int \frac{dx}{2+3x}=\frac{1}{3}\int \frac{3dx}{2+3x}=\frac{1}{3}\ln(2+3x)+C.$$

2. $\int \frac{3dx}{2x-3}$

〔解〕設 $v=2x-3$; $dv=2dx$

$$\int \frac{3dx}{2x-3}=\frac{3}{2}\int \frac{2dx}{2x-3}\cdot\frac{3}{2}\ln(2x-3)+C$$

3. $\int \frac{adx}{b-cx}$

〔解〕設 $v=b-cx$, $dv=-cdx$.

$$\int \frac{adx}{b-cx}=-\frac{a}{c}\int \frac{-cdx}{b-cx}=-\frac{a}{c}\ln(b-cx)+C$$

4. $\int \frac{tdt}{a+bt^2}$

〔解〕設 $v=a+bt^2$, $dv=2btdt$

$$\int \frac{tdt}{a+bt^2}=\frac{1}{2b}\int \frac{2btdt}{a+bt^2}=\frac{1}{2b}\ln(a+bt^2)+C.$$

5. $\int \frac{(2x+3)dx}{x^2+3x}$

〔解〕設 $v=x^2+3x$; $dv=(2x+3)dx$

$$\int \frac{(2x+3)dx}{x^2+3x} = \ln(x^2+3x)+C$$

6. $\int \frac{y^2dy}{3+y^3}$

〔解〕設 $v=3+y^3$；$dv=3y^2dy$

$$\int \frac{y^2dy}{3+y^3} = \frac{1}{3}\int \frac{3y^2dy}{3+y^3} = \frac{1}{3}\ln(3+y^3)+C$$

7. $\int \frac{e^xdx}{ae^x+b}$

〔解〕設 $v=ae^x+b$；$dv=ae^xdx$

$$\int \frac{e^xdx}{ae^x+b} = \frac{1}{a}\int \frac{ae^xdx}{ae^x+b} = \frac{1}{a}\ln(ae^x+b)+C$$

8. $\int \frac{\cos\theta d\theta}{\sin\theta+2}$

〔解〕設 $v=\sin\theta+2$；$dv=\cos\theta d\theta$

$$\int \frac{\cos\theta d\theta}{\sin\theta+2} = \ln(\sin\theta+2)+C$$

9. $\int \frac{\sin\theta d\theta}{3-2\cos\theta}$

〔解〕設 $v=3-2\cos\theta$；$dv=2\sin\theta d\theta$

$$\int \frac{\sin\theta d\theta}{3-2\cos\theta} = \frac{1}{2}\int \frac{2\sin\theta d\theta}{3-2\cos\theta} = \frac{1}{2}\ln(3-2\cos\theta)+C$$

10. $\int \frac{\sec^2\phi d\phi}{\sqrt{3\tan\phi+1}}$

〔解〕設 $v=3\tan\phi+1$；$dv=3\sec^2\phi d\phi$

$$\int \frac{\sec^2\phi\, d\phi}{\sqrt{3\tan\phi+1}} = \frac{1}{3}\int \frac{3\sec^2\phi\, d\phi}{\sqrt{3\tan\phi+1}} = \frac{2}{3}\sqrt{3\tan\phi+1}+C$$

11. $\int \frac{e^{2t}dt}{\sqrt{e^{2t}+4}}$

〔解〕設 $v = e^{2t}+4$；$dv = 2e^{2t}dt$.

$$\int \frac{e^{2t}dt}{\sqrt{e^{2t}+4}} = \frac{1}{2}\int \frac{2e^{2t}dt}{\sqrt{e^{2t}+4}} = \sqrt{e^{2t}+4}+C$$

12. $\int \sqrt{2\sin\theta+5}\cos\theta\, d\theta$

〔解〕設 $v = 2\sin\theta+5$；$dv = 2\cos\theta\, d\theta$

$$\int \sqrt{2\sin\theta+5}\cos\theta\, d\theta = \frac{1}{2}\int \sqrt{2\sin\theta+5}\, 2\cos\theta\, d\theta$$

$$= \frac{1}{2}\cdot\frac{2}{3}(2\sin\theta+5)^{\frac{3}{2}}+C = \frac{1}{3}(2\sin\theta+5)^{\frac{3}{2}}+C$$

13. $\int_0^1 \frac{x\,dx}{x^2+1}$

〔解〕設 $v = x^2+1$；$dv = 2x\,dx$

$$\int_0^1 \frac{x\,dx}{x^2+1} = \frac{1}{2}\int_0^1 \frac{2x\,dx}{x^2+1} = \frac{1}{2}\left[\ln(x^2+1)\right]_0^1$$

$$= \frac{1}{2}\ln 2 = 0.347$$

14. $\int_3^5 \frac{dx}{x-1}$

〔解〕設 $v = x-1$；$dv = dx$

$$\int_3^5 \frac{dx}{x-1} = \left[\ln(x-1)\right]_3^5 = \ln 4 - \ln 2 = 1.385 - 0.694$$

$$= 0.691.$$

15. $\int_1^2 \frac{(x+1)\,dx}{x^2+2x}$

〔解〕 設 $v=x^2+2x$； $dv=(2x+2)dx$

$$\int_1^2 \frac{(x+1)dx}{x^2+2x}=\frac{1}{2}\int_1^2 \frac{(2x+2)dx}{x^2+2x}=\frac{1}{2}\left[\ln(x^2+2x)\right]_1^2$$

$$=\frac{1}{2}[\ln 8-\ln 3]=\frac{1}{2}(2.08-1.1)$$

$$=0.49.$$

16. $\int_0^{\frac{\pi}{4}} \frac{\cos 2\theta d\theta}{3-\sin 2\theta}$

〔解〕 設 $v=3-\sin 2\theta$； $dv=-2\cos 2\theta d\theta$.

$$\int_0^{\frac{\pi}{4}} \frac{\cos 2\theta d\theta}{3-\sin 2\theta}=\frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{-2\cos 2\theta d\theta}{3-\sin 2\theta}=-\frac{1}{2}\left[\ln 3-\sin\theta\right]_0^{\frac{\pi}{4}}$$

$$=\frac{1}{2}[\ln 3-\ln 2]=\frac{1}{2}\ln\frac{3}{2}=\frac{1}{2}[1.09-0.692]$$

$$=0.203.$$

17. $\int_0^{\frac{\pi}{2}} \frac{\sin\phi d\phi}{2\cos\phi+5}$

〔解〕 設 $v=2\cos\phi+5$， $dv=-2\sin\phi d\phi$.

$$\int_0^{\frac{\pi}{2}} \frac{\sin\phi d\phi}{2\cos\phi+5}=-\frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{-2\sin\phi d\phi}{2\cos\phi+5}=-\frac{1}{2}\left[\ln(2\cos\phi+5)\right]_0^{\frac{\pi}{2}}$$

$$=\frac{1}{2}[\ln 7-\ln 5]=\frac{1}{2}[1.944-1.61]=0.167$$

18. $\int_0^{\frac{\pi}{4}} \frac{\sec^2 t dt}{2-\tan t}$.

〔解〕 設 $v=2-\tan t$； $dv=-\sec^2 t$.

$$\int_0^{\frac{\pi}{4}} \frac{\sec^2 t dt}{2-\tan t}=-\int_0^{\frac{\pi}{4}} \frac{-\sec^2 t dt}{2-\tan t}=-\left[\ln(2-\tan t)\right]_0^{\frac{\pi}{4}}$$

$$=\ln 2=0.692.$$

19. $\int_0^{\frac{\pi}{4}} \frac{\sec^2 x\tan x dx}{\sqrt{2+\sec^2 x}}$

〔解〕設 $v=2+\sec^2 x$；$dv=2\sec^2 x\tan x\,dx$

$$\int_0^{\frac{\pi}{4}}\frac{\sec^2 x\tan x\,dx}{\sqrt{2+\sec^2 x}}=\frac{1}{2}\int_0^{\frac{\pi}{4}}\frac{2\sec^2 x\tan x\,dx}{\sqrt{2+\sec^2 x}}$$

$$=\left[\sqrt{2+\sec^2 x}\right]_0^{\frac{\pi}{4}}=2-\sqrt{3}=0.27$$

20 $\int_0^{\frac{\pi}{2}}\sqrt{5-\sin\theta}\cdot\cos\theta\,d\theta$

〔解〕設 $v=5-\sin\theta$；$dv=-\cos\theta\,d\theta$

$$\int_0^{\frac{\pi}{2}}\sqrt{5-\sin\theta}\cos\theta\,d\theta=-\int_0^{\frac{\pi}{2}}\sqrt{5-\sin\theta}\,(-\cos\theta)\,d\theta$$

$$=-\frac{2}{3}\left[(5-\sin\theta)^{\frac{3}{2}}\right]_0^{\frac{\pi}{2}}=\frac{2}{3}(5\sqrt{5}-8)$$

21 $\int_0^{\pi}\frac{\sin\frac{1}{2}\phi\,d\phi}{\sqrt{4+\cos\frac{1}{2}\phi}}$

〔解〕設 $v=4+\cos\frac{1}{2}\phi$；$dv=-\frac{1}{2}\sin\frac{1}{2}\phi\,d\phi$

$$\int_0^{\pi}\frac{\sin\frac{1}{2}\phi\,d\phi}{\sqrt{4+\cos\frac{1}{2}\phi}}=-2\int_0^{\pi}\frac{-\frac{1}{2}\sin\frac{1}{2}\phi\,d\phi}{\sqrt{4+\cos\frac{1}{2}\phi}}=-4\left[(4+\cos\tfrac{1}{2}\phi)^{\frac{1}{2}}\right]_0^{\pi}$$

$$=4(\sqrt{5}-2)$$

22 $\int\frac{(x^2+2)\,dx}{x+1}$

〔解〕$\int\frac{(x^2+2)\,dx}{x+1}=\int(x-1+\frac{3}{x+1})\,dx$

$$=\tfrac{1}{2}x^2-x+3\ln(x+1)+C$$

23 $\int\frac{(2x+3)\,dx}{x+2}$

〔解〕$\int\frac{(2x+3)\,dx}{x+2}=\int(2-\frac{1}{x+2})\,dx=2x-\ln(x+2)+C$

24. $\int \frac{(2x+7)\,dx}{x+3}$

〔解〕$\int \frac{(2x+7)\,dx}{x+3} = \int (2+\frac{1}{x+3})\,dx = 2x + \ln(x+3)+C$

25. $\int \frac{(x^3-3x)\,dx}{x^2+1}$

〔解〕$\int \frac{(x^3-3x)\,dx}{x^2+1} = \int (x+\frac{2x}{x^2+1})\,dx$

$= \frac{1}{2}x^2 + \int \frac{2x\,dx}{x^2+1} = \frac{1}{2}x^2 + \ln(x^2+1)+C$

26. $\int \frac{(x^2+2x+2)\,dx}{x^2+2}$

〔解〕$\int \frac{(x^2+2x+2)\,dx}{x^2+2} = \int (1+\frac{2x}{x^2+2})\,dx$

$= x + \ln(x^2+2)+C$

27. $\int \frac{(2x^2+4x+1)\,dx}{x^2+x}$

〔解〕$\int \frac{(2x^2+4x+1)\,dx}{x^2+x} = \int (2+\frac{2x+1}{x^2+x})\,dx = 2x+\ln(x^2+x)+C$

28 $\int_0^1 \frac{(x^2+x+2)\,dx}{x+1}$

〔解〕$\int_0^1 \frac{(x^2+x+2)\,dx}{x+1} = \int_0^1 (x+\frac{2}{x+1})\,dx$

$= \left[\frac{1}{2}x + 2\ln(x+1)\right]_0^1 = \frac{1}{2} + 2\ln 2 = 1.886$

29 $\int_0^1 \frac{(x^2+3x+3)\,dx}{x+2}$

【解】$\int_0^1 \frac{(x^2+3x+3)\,dx}{x+2} = \int_0^1 (x+1+\frac{1}{x+2})\,dx$

$= \left[\frac{1}{2}x^2+x+\ln(x+2)\right]_0^1 = \frac{1}{2}+1+\ln 3-\ln 2$

$= \frac{3}{2}+\ln\frac{3}{2} = 1.5+0.405 = 1.905$

30. $\int_1^2 \frac{(4x-1)\,dx}{2x-1}$

【解】$\int_1^2 \frac{(4x-1)\,dx}{2x-1} = \int_1^2 (2+\frac{1}{2x-1})\,dx = \left[2x+\frac{1}{2}\ln(2x-1)\right]_1^2$

$= 2+\frac{1}{2}\ln 3 = 2.549$

31. $\int \frac{(ae^x+b)\,dx}{ae^x-b}$

【解】$\int \frac{(ae^x+b)\,dx}{ae^x-b} = \int (\frac{2ae^x}{ae^x-b}-1)\,dx = 2\ln(ae^x-b)-x+C$

32. $\int \frac{(7e^x+8)\,dx}{e^x+2}$

【解】$\int \frac{(7e^x+8)\,dx}{e^x+2} = \int (\frac{3e^x}{e^x+2}-4)\,dx = 3\ln(e^x+2)-4x+C$

33. $\int \frac{(5e^{2x}-2)\,dx}{e^{2x}-1}$

【解】$\int \frac{(5e^{2x}-2)\,dx}{e^{2x}-1} = \int (\frac{3e^{2x}}{e^{2x}-1}+2)\,dx$

$= 3\ln(e^x-1)+2x+C$

34. $\int \frac{(\sin t+4)\cos t\,dt}{\sin t+2}$

〔解〕 $\int \frac{(\sin t+4)\cos t\,dt}{\sin t+2} = \int \frac{(\sin t\cos t+4\cos t)\,dt}{\sin t+2}$

$= \int (\cos t + \frac{2\cos t}{\sin t+2})\,dt = \sin t + 2\ln(\sin t+2)+C$

35. $\int \frac{(\cos^2\theta+\cos\theta-1)\sin\theta\,d\theta}{\cos\theta-1}$

〔解〕 $\int \frac{(\cos^2\theta+\cos\theta-1)\sin\theta\,d\theta}{\cos\theta-1} = \int (\cos\theta\sin\theta+2\sin\theta-\frac{3\sin\theta}{\cos\theta-1})(d\theta)$

$= \cos^2\theta - 2\cos\theta + 3\ln(\cos\theta-1)+C$

36. $\int \frac{\tan^2\phi\sec^2\phi\,d\phi}{\tan\phi+1}$

〔解〕 $\int \frac{\tan^2\phi\sec^2\phi\,d\phi}{\tan\phi+1} = \int (\tan\phi\sec^2\phi+\sec^2\phi-\frac{\sec^2\phi}{\tan\phi+1})d\phi$

$= \tan^2\phi + \tan\phi - \ln(\tan\phi+1)+C$

習題解答 p-261

作出下列積分式：

1. $\int 6e^{3x}dx$

〔解〕 設 $v=3x$；$dv=3dx$

$\int 6e^{3x}dx = 2\int e^{3x}\cdot 3dx = 2e^{3x}+C$

2. $\int 10^{2x}dx$

〔解〕 設 $v=2x$；$dv=2dx$

$\int 10^{2x}dx = \frac{1}{2}\int 10^{2x}\cdot 2dx = \frac{1}{2}\cdot\frac{10^{2x}}{\ln 10}+C$

3. $\int \frac{dx}{e^x}$

〔解〕 $\int \frac{dx}{e^x} = \int e^{-x}dx$，設 $v=-x$；$dv=-dx$

$\therefore \int \frac{dx}{e^x} = -\int e^{-x}(-dx) = -e^{-x}+C = -\frac{1}{e^x}+C$

4. $\int \frac{dx}{5^x}$

〔解〕 設 $v=-x$；$dv=-dx$

$\int \frac{dx}{5^x} = -\int 5^{-x}(-dx) = -\frac{5^{-x}}{\ln 5}+C = -\frac{1}{\ln 5 \cdot 5^x}+C$

5. $\int \frac{e^{\sqrt{x}}dx}{\sqrt{x}}$

〔解〕 設 $v=\sqrt{x}$，$dv=\frac{1}{2}\frac{dx}{\sqrt{x}}$

$\int \frac{e^{\sqrt{x}}dx}{\sqrt{x}} = 2\int e^{\sqrt{x}}(\frac{1}{2}\frac{dx}{\sqrt{x}}) = 2e^{\sqrt{x}}+C$

6. $\int xe^{x^2}dx$

〔解〕 設 $v=x^2$，$dv=2xdx$

$\int xe^{x^2}dx = \frac{1}{2}\int e^{x^2}(2xdx) = \frac{1}{2}e^{x^2}+C$

7. $\int \sqrt{e^t}\,dt$

〔解〕 設 $v=\frac{1}{2}t$；$dv=\frac{1}{2}dt$

$\int \sqrt{e^t}dt = \int e^{\frac{1}{2}t}dt = 2\int e^{\frac{1}{2}t}(\frac{1}{2}dt) = 2\sqrt{e^t}+C$

8. $\int \frac{4dt}{\sqrt{e^t}}$

〔解〕 設 $v=-\frac{1}{2}t$；$dv=-\frac{1}{2}dt$

$$\int \frac{4dt}{\sqrt{e^t}}=4\int e^{-\frac{1}{2}t}dt=-8\int e^{-\frac{1}{2}t}\left(-\frac{1}{2}dt\right)$$

$$=-8e^{-\frac{1}{2}t}+C=-\frac{8}{\sqrt{e^t}}+C$$

9. $\int e^{\sin\theta}\cos\theta d\theta$.

〔解〕 設 $v=\sin\theta$；$dv=\cos\theta d\theta$

$$\int e^{\sin\theta}\cos\theta d\theta=e^{\sin\theta}+C.$$

10. $\int e^{\tan\theta}\sec^2\theta d\theta$

〔解〕 設 $v=\tan\theta$；$dv=\sec^2\theta d\theta$

$$\int e^{\tan\theta}\sec^2\theta d\theta=e^{\tan\theta}+C$$

11. $\int a^x e^x dx$.

〔解〕 $\int a^x e^x dx=\int (ae)^x dx=\frac{(ae)^x}{\ln ae}+C=\frac{a^x e^x}{\ln a+1}+C.$

12. $\int (e^{5x}+a^{5x})dx$.

〔解〕 $\int (e^{5x}+a^{5x})dx=\int e^{5x}dx+\int a^{5x}dx=\frac{1}{5}e^{5x}+\frac{1}{5}\frac{a^{5x}}{\ln a}+C$

13. $\int_0^2 e^{\frac{x}{2}}dx$

〔解〕 設 $v=\frac{x}{2}$；$dv=\frac{1}{2}dx$

$$\int_0^2 e^{\frac{x}{2}}dx=2\int_0^2 e^{\frac{x}{2}}\cdot\frac{1}{2}dx=2\left(e^{\frac{x}{2}}\right]_0^2=2(e-1)$$

14. $\int_0^{\frac{1}{2}} \frac{dx}{e^{2x}}$

〔解〕設 $v=-2x$；$dv=-2\,dx$

$$\int_0^{\frac{1}{2}} \frac{dx}{e^{2x}} = \int_0^{\frac{1}{2}} e^{-2x}\,dx = -\frac{1}{2}\int_0^{\frac{1}{2}} e^{-2x}(-2\,dx)$$

$$= -\frac{1}{2}\left[e^{2x}\right]_0^{\frac{1}{2}} = \frac{1}{2}(1-e)$$

15. $\int_0^1 4^x dx$

〔解〕$\int_0^1 4^x\,dx = \left[\frac{4^x}{\ln 4}\right]_0^1 = \frac{3}{\ln 4}$.

16. $\int_1^2 \frac{dx}{3^x}$

〔解〕設 $v=-x$；$dv=-dx$

$$\int_1^2 \frac{dx}{3^x} = \int_1^2 3^{-x}\,dx = \int_1^2 3^{-x}(-dx) = -\left[\frac{3^{-x}}{\ln 3}\right]_1^2$$

$$= -\left[\frac{1}{\ln 3\cdot 3^x}\right]_1^2 = \frac{1}{3\ln 3} - \frac{1}{9\ln 3} = \frac{2}{9\ln 3}$$

17. $\int (e^{\frac{x}{a}} + e^{-\frac{x}{a}})\,dx$.

〔解〕$\int (e^{\frac{x}{a}} + e^{-\frac{x}{a}})\,dx = \int e^{\frac{x}{a}}\,dx + \int e^{-\frac{x}{a}}\,dx$

$$= a e^{\frac{x}{a}} - a e^{-\frac{x}{a}} + C = a(e^{\frac{x}{a}} - e^{-\frac{x}{a}}) + C.$$

18. $\int (e^{2x} - e^{-2x})\,dx$

〔解〕$\int (e^{2x} - e^{-2x})\,dx = \int e^{2x}\,dx - \int e^{-2x}\,dx$

$$= \frac{1}{2}e^{2x} + \frac{1}{2}e^{-2x} + C.$$

19. $\int (e^{\frac{x}{a}} - e^{-\frac{x}{a}})^2 dx$

〔解〕 $\int (e^{\frac{x}{a}} - e^{-\frac{x}{a}})^2 dx = \int (e^{\frac{2x}{a}} - 2 + e^{-\frac{2x}{a}}) dx$

$= \frac{a}{2} e^{\frac{2x}{a}} - 2x - \frac{a}{2} e^{-\frac{2x}{a}} + C = \frac{a}{2}(e^{\frac{2x}{a}} - e^{-\frac{2x}{a}}) - 2x + C$

20. $\int \frac{(e^x + 4) dx}{e^x}$

〔解〕 $\int \frac{(e^x + 4) dx}{e^x} = \int dx + \int \frac{4}{e^x} dx = x - 4e^{-x} + C$

$= x - \frac{4}{e^x} + C$

21. $\int x(e^{x^2} + 2) dx$

〔解〕 $\int x(e^{x^2} + 2) dx = \int x e^{x^2} dx + 2\int x dx$

$= \frac{1}{2} e^{x^2} + x^2 + C.$

22. $\int \frac{(e^{\sqrt{x}} - 3) dx}{\sqrt{x}}$

〔解〕 $\int \frac{(e^{\sqrt{x}} - 3) dx}{\sqrt{x}} = \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx - 3\int \frac{dx}{\sqrt{x}}$

$= 2e^{\sqrt{x}} - 6\sqrt{x} + C.$

23. $\int (e^x + 1)^2 dx$

〔解〕 $\int (e^x + 1)^2 dx = \int (e^{2x} + 2e^x + 1) dx$

$= \frac{1}{2} e^{2x} + 2e^x + x + C.$

24. $\int \frac{e^x dx}{\sqrt{e^x + 1}}$

〔解〕 設 $v = e^x + 1$, $dv = e^x dx$

$$\int \frac{e^x dx}{\sqrt{e^x+1}} = 2\sqrt{e^x+1} + C$$

25. $\int \frac{e^{2x}dx}{2e^{2x}+3}$

〔解〕 設 $v = 2^{2x}+3$； $dv = 4e^{2x}dx$.

$$\int \frac{e^{2x}dx}{2e^{2x}+3} = \frac{1}{4}\int \frac{4e^{2x}dx}{2e^{2x}+3} = \frac{1}{4}\ln(2e^{2x}+3)+C.$$

26. $\int_0^1 \frac{(2e^{2x}+3)\,dx}{e^{2x}}$

〔解〕 $\int_0^1 \frac{(2e^{2x}+3)\,dx}{e^{2x}} = \int_0^1 2dx + \int_0 \frac{3}{e^{2x}}dx$

$$= \left[2x - \frac{3}{2}\frac{1}{e^{2x}}\right]_0^1 = 2 - \frac{3}{2}\left(\frac{1}{7.4}-1\right) = 2+1.3 = 3.3$$

27. $\int_0^1 \frac{(4-e^x)\,dx}{e^x}$

〔解〕 $\int_0^1 \frac{(4-e^x)\,dx}{e^x} = \int_0^1 \frac{4}{e^x}dx - \int_0^1 dx = \left[-4e^{-x}-x\right]_0^1$

$$= \left(-\frac{4}{e}-1\right)+4 = 1.525.$$

28. $\int_0^1 \frac{e^x dx}{\sqrt{e^x+3}}$

〔解〕 設 $v = e^x+3$　　$dv = e^x dx$

$$\int_0^1 \frac{e^x dx}{\sqrt{e^x+3}} = \left[2\sqrt{e^x+3}\right]_0^1 = 2\sqrt{e+3} - 4 = 0.76.$$

29. $\int_0^1 (e^x+e^{-x})^2 dx$.

〔解〕 $\int_0^1 (e^x+e^{-x})^2 dx = \int_0^1 (e^{2x}+2+e^{-2x})\,dx$.

$$=\left[\frac{1}{2}e^{2x}+2x-\frac{1}{2}e^{-2x}\right]_0^1=\left[\frac{1}{2}(e^2-e^{-2})+2\right]$$

$$=\frac{1}{2}(7.254)+2=5.627.$$

30. $\int_0^2 (1+e^{\frac{x}{2}})^2\,dx$

〔解〕 $\int_0^2 (1+e^{\frac{x}{2}})^2\,dx=\int_0^2 (1+2e^{\frac{x}{2}}+e^x)\,dx$

$$=\left[x+4e^{\frac{x}{2}}+e^x\right]_0^2=(2+4e+e^2)-5=15.24.$$

31. $\int_1^2 (2-\frac{1}{e^x})^2\,dx$

〔解〕 $\int_1^2 (2-\frac{1}{e^x})^2\,dx=\int_1^2 (4-\frac{4}{e^x}+\frac{1}{e^{2x}})\,dx$

$$=\left[4x+\frac{4}{e^x}-\frac{1}{2}\frac{1}{e^{2x}}\right]_1^2=(4+\frac{2}{e^2}-\frac{1}{2}\frac{1}{e^4})-(\underbrace{1+\frac{2}{e}-\frac{1}{2}}_{(\frac{1}{e^2})})$$

$$=(4+\frac{1}{7.4}-\frac{1}{2}\frac{1}{55})-(1+\frac{2}{2.71}-\frac{1}{2}\cdot\frac{1}{7.4})=4.26-1.47$$

$$=2.79$$

習題解答 P－264

作出下列積分式：

1. $\int \cos mx\,dx$.

〔解〕 設 $v=mx$, $dv=m\,dx$

$$\int \cos mx\,dx=\frac{1}{m}\int \cos mx\,(m\,dx)=\frac{1}{m}\sin mx+C$$

2. $\int \sin\frac{2x}{3}\,dx$

〔解〕 設 $v=\frac{2x}{3}$ $dv=\frac{2}{3}\,dx$