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# Atmospheric Dynamics

John Green

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## Atmospheric Dynamics

In his book, John Green presents a unique personal insight into the fundamentals of fluid mechanics and atmospheric dynamics. Generations of students have benefited from his lectures, and this book, many years in the making, is the result of his wide teaching and research experience.

The theory of fluid flow has developed to such an extent that very complex mathematics and models are currently used to describe it, but many of the fundamental results follow from relatively simple considerations: these classic principles are derived here in a novel, distinctive, and at times even idiosyncratic, way. In order to resist blindly following the ever more complex computer simulations of atmospheric dynamics, the author advocates that students need to keep in touch with these fundamental derivations and thought processes. These self-same lower level derivations can then often be useful in developing strategies for creation of more complex systems.

John Green's book is an introduction to fluid mechanics in the atmosphere for students and researchers that are already familiar with the subject, but who wish to extend their knowledge and philosophy beyond the currently popular development of conventional undergraduate instruction.

JOHN GREEN graduated in mathematics from Imperial College, London, where he also received a PhD from the Meteorology Department, and went on to become Reader in Meteorology. He is now a lecturer in fluid dynamics and mathematical modelling of natural processes at the School of Environmental Sciences, University of East Anglia. He has published extensively, mainly in the *Royal Meteorological Society Quarterly Journal* and the popular journal *Weather*, and has been awarded the Buchan prize of the Royal Meteorological Society for work on baroclinic instability and thunderstorms.

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## Introduction

This volume is the condensation of lectures given to students at Imperial College Department of Meteorology, while it existed, and to students of Environmental Science at the University of East Anglia when the department at Imperial College was closed down. Any good sense it might contain is almost certainly attributable to colleagues. Eric Eady opened my eyes to many fascinating phenomena, though when I taught students I began to realise that such communication is perhaps more of a two-way process than I imagined at the time. It is one thing to speculate on an interpretation, but another when this stimulates a response. Recalling a technical difficulty to a non-technical partner is perhaps a simple illustration of the suggested interaction. There is also a lot of Frank Ludlam here. He asked for explanations of mathematical theories that he could understand without having to go through the detailed mathematics. I remember an early encounter when he described his method of evaluating integrals. Simple, he said, 'you just move the variables, one by one, through the integral sign 'till you are left with an integral that you *can* do'. Aspects of this collaboration can be seen in his book on *Clouds and Storms*, which has some of me in it, but which for political reasons was forgone: Frank died before his wonderful book was published. The philosophy here is almost all that of Professor Peter Sheppard, who tormented, teased, threatened, sometimes disillusioned, many generations of students, for his criticisms were accurate and acid. If your theory could stand up to him, it would stand up anywhere. 'Where's your nose Green' I still hear him say, for he knew when innovation was essential too. Finally there were many students, and occasionally I recollect that it was a student who made some nice observation, not me. I distinctly remember one, and I remember who, when asked what we knew about ozone

said 'ignites with a glowing taper'. A beautiful comment on the inadequacy of our textbooks to prepare us for the environment.

Collaboration with diverse colleagues and students encourages a degree of interpretation that is contrary to much mathematics. However, when you give up proof, in favour of suggestion, it allows you to see what you have gained in the proof. It is common experience that as a proof becomes more rigorous, the domain to which it can be applied becomes more restricted. I think there is a role for more general, less specific analysis. This reduces me to being dissatisfied with most of the things I have published, except where I have been able to say that they have led to some sort of tangible enlightenment. Thus you will find even quite trivial manipulations leading to speculation as to another interpretation.

There are essentially no references here. Whenever I tried to read the literature I would quickly go off on an interesting tangent, and never get to the end of the article to be able to incorporate it in my vocabulary. That is a bad thing for a student to do, but it might be more forgivable than to read up a subject before studying it. One is readily convinced by the word of the current authority, and it always seemed to me to be much more fun to go off on some untrodden path than to compete with current researchers exploiting the latest theory trying to get the next result first. It is also important to have prior feeling about the topic you are reading about to compare with what the author is telling you. They may be telling you something you already know in incomprehensible language. An unbiassed opinion is almost as irrelevant as random motion.

There is not much in the way of illustrative material either. It is so easy to say 'here is an example of the phenomena being described' upon which the attentive student goes away with this thumbprint. What really happens is that there never is such a good example, all the real ones go off imperfectly, and there is no typical example. Thus I would rather the student had the notion that there were some stylised things that real phenomena might remind him of. As an example of deception, there are two speculations about wave propagation in the atmosphere. One is that influences are propagated downstream, the other that they are propagated along great circles. If we draw a polar projection, we see propagation along great circles, and in an equatorial projection, latitudinal trains of waves.

Meteorology has reached a complex position. We have access to a great bank of data and the computing power to analyse it. I came into meteorology through 'there is a wind on the heath brother, all good things', and am inspired by that feeling, and it seems to me that we are in danger of being unable to follow through from that feeling to modern data and analysis. I hope some people will be able to savour, more acutely, that connection.

## Chapter 1

### Description of atmospheric motion systems

And God created great whales, and every living thing that moveth, which the waters brought forth abundantly after their kind, and every winged fowl after his kind: and God saw that it was good.

#### 1.1 Introduction

We usually notice phenomena; events isolated in space, but most of this book will rely on a wave formalism. This chapter explores some of the relations between the two forms of description. We are going to describe a great variety of motion systems, with broad classes, but with each member of each class different. Moreover, the definition of the class depends, to some extent, on the reason we are trying to classify the phenomenon. A cumulus cloud is fairly well defined, in the sense that two independent observers will (usually) agree that a specific cloud should be put in that broad category. There will be some debate as to whether this specimen is young or old, becoming congested, and such subtleties are important as indicators of the future development of the convection. The particular one we see now is an individual, and studied for a specific purpose; it might make a gust that will disturb my boat, cover up the sun, precipitate soon, carry aphids/momentum/water vapour, into the higher levels of the troposphere; a whole host of things that will affect the way I look at it.

Usually, we are concerned with relations between things; like the organisation that causes cumulonimbus to occur in particular regions of a frontal zone; frontal zones to appear in fairly well defined regions of a weather system; weather systems appear in certain zones of the circulation of larger scale. We

become aware of the importance of the spaces between the more obvious parts of a system.

It is comforting to see individual motion-systems. A depression, with its region of closed isobars on the surface chart appears quite well defined, and we can therefore speak of its position now, and where it and its associated weather will be tomorrow. But on the 500 mb chart, which is more representative of the bulk of the atmosphere, the closed isobaric centre is usually absent, and is replaced by a trough in the streamlines which is less well defined and less spectacular. Moreover, the air currently near (including above) the surface low, is moving in response to forces that have been organised for a few days, during which time air has travelled several thousand kilometers from a wide variety of directions. In some sense, the scale of this system is, at least for some purposes, very much larger than that of its most obvious surface feature.

The beautiful crisp edge of a cumulus cloud defines the limit of penetration of potentially warm moist air from below. Clear air must be moved out of its way (by the action of the pressure field) so there must be descent of dry, negatively buoyant, air outside the cloud. This is an important aspect of the circulation, for it is this descent which leads to the general rise in temperature of the layer of air into which the cloud penetrates. Moreover it is one mechanism by which one cloud interacts with another to give organisation on the scale of the cloud field. This way we get the notion that, at least for some purposes, motion systems occupy all the available space. For them, spectral analysis, using for example sinusoidal functions (Fourier series) might be useful. These allow uniform representation all over space, uniquely separate scale from intensity, and help us to identify some relevant physical processes.

Sinusoidal functions are not so convenient for describing systems that are truly isolated in space. For example a single updraught of width  $a$  needs all wavenumbers up to several times  $a^{-1}$  together with their phase relations, for its description. Here we will be concerned mainly with the long-time evolution of the atmosphere, therefore with the interaction between smaller and larger scales of motion. Thus individual motion systems acquire a rather transient nature and we tend to regard the wavelike description as more fundamental than the phenomenological even though sometimes less fun. The contrast between the 'wave' and 'particle' description of light, where some phenomena are more easily described in terms of one idealisation rather than the other, is a useful analogy.

## 1.2 Spectrum of motion

The vast majority of the kinetic energy of the atmosphere is contained in the zonal mean of the zonal component of the flow. This represents a sort of

flywheel, in which energy is stored and may be available for other motion systems to make use of, but which itself does little. In broad terms the wind increases from being small near the ground to an average of some  $20\text{--}30\text{ms}^{-1}$  in the same sense as the rotation of the earth, near the tropopause, especially in middle latitudes. See Figure 15.1 and the discussion of Section 15.2 for further details. Superimposed on this are wavelike systems, on which we now direct our attention.

Motion of wavelengths from a few mm, severely affected by viscous forces, to the 40 000 km of the size of the earth are of interest to us. Because of this great range of scales it is convenient to use a logarithmic scale for the wavelength, and adjust the amplitude scale so that it still represents energy. The log-scale has the additional advantage that we can equally well label the axis with spatial wavenumber, which is usually a more convenient mathematical parameter than wavelength, and with some further assumption even put a rough temporal–frequency scale on as well.

In a conventional Fourier series the amplitude represents energy per unit wavenumber,

$$F(x) = \int_{-\infty}^{+\infty} f(k) e^{ikx} dk \quad (1.1)$$

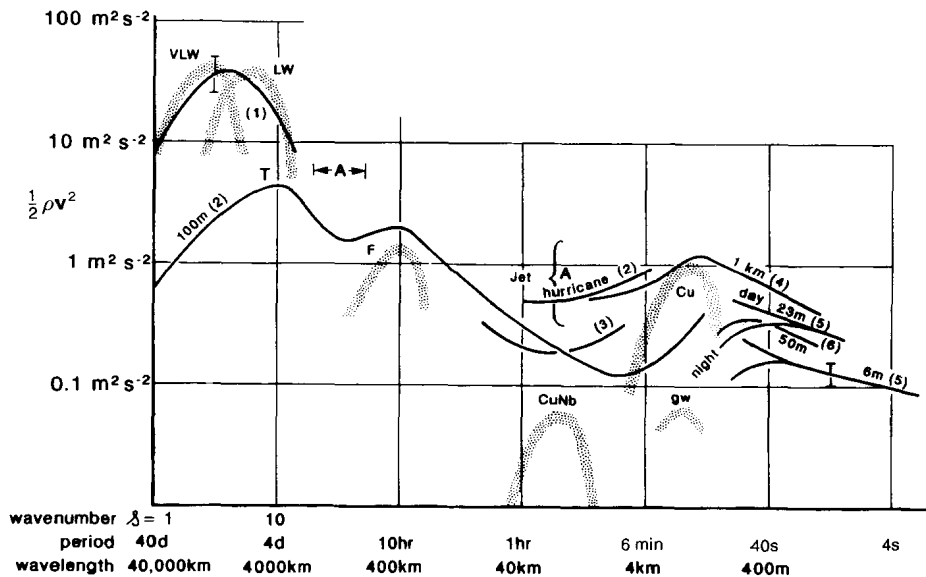
which defines the Fourier coefficient  $f$ , where,

$$\frac{1}{2} \overline{F^2(x)} = \int_{-\infty}^{+\infty} \frac{1}{2} f^2(k) dk = \int_{-\infty}^{+\infty} \frac{1}{2} k f^2(k) \frac{dk}{k} \quad (1.2)$$

defines the contribution of each wavenumber to the energy; Parseval's theorem about the multiplication of Fourier series is relevant. The first form of equation 1.2 makes the area under the curve proportional to energy if  $k$  is the ordinate, the second form if  $\log k$  is the ordinate. This amounts to the first form being the energy within unit wavenumber of the wavenumber of interest; the second the energy within wavenumber  $k$  of the wavenumber of interest.

For many scales of motion the specific kinetic energy  $\frac{1}{2} \rho \mathbf{v}^2$  is fairly independent of height so this, normalised to density at sea level, is what is plotted in Figure 1.1. We see a broad peak in kinetic energy centred on global wavenumber 6, corresponding to a wavelength of some 6000 km, or wavenumber  $10^{-6} \text{ m}^{-1}$ , or period of one week. The energy in wavenumbers 1 to 3 is mostly in slow-moving waves whose dynamics is largely governed by the variation of Coriolis parameter with latitude. These we will refer to as 'very long waves', and are idealised as Rossby waves. Energy in the range 3 to 6 to 12 is associated with travelling disturbances which we identify with the 'long waves' and 'cyclone waves' as Eady called them, or depressions or weather systems, of middle latitudes. The small peak at wavenumber 1/100 km, we associate with the transverse scale of fronts, the peak at 1/1 km with cumulus convection, and the tail to smaller scales with the transfer of energy to shorter wavelengths.





**Figure 1.1** Variation of the kinetic energy with scale for tropospheric motion. The wavelength scale is logarithmic, and is labelled with wavelength and with wavenumber, the former being more usual for smaller scales of motion, the latter for larger. A rough temporal-frequency scale is indicated using an advection speed of  $10 \text{ m s}^{-1}$ . There is one value, marked T, for the tropical experiment of Professor Sheppard, and anomalous values in curly brackets taken in a mid-latitude jet stream. The fuzzy curves represent theoretical expectations due to very long waves, long waves, fronts, cumulonimbus, cumulus, and gravity waves.

Most of the data (see references) comes from dense groups of observing stations set up in middle latitudes in the 1950s and 1960s. There are reasons for thinking that large-scale motion in the tropics is similar in character to that in middle latitudes and the values T and A may represent the long wave and frontal scale for those regions. Curve (2) is from a time-series at 100 m so the horizontal kinetic energy is, because of friction, less than that typical of the large-scale analyses, but with fronts better defined near the ground maybe this part is more reliable. Curve (3) is from a time series taken specifically to identify the meso-scale minimum. Region marked Jet are for turbulent conditions just below the jet stream, and are about the shortest scale observed for the free atmosphere. Curves (4, 5 and 6) show convergence towards a viscous dissipation scale near the ground.

The fuzzy curves show the contributions to the spectrum from 'well defined' motion systems, like very long waves, long waves, fronts, cumulo-nimbus, cumulus and gravity waves, whose appearance is supported by theoretical considerations treated later. We notice that the lines in this 'theoretical' spectrum are much better defined than in the real one. Figure 1.2 shows the corresponding spectrum for vertical motion. Notice that the units are four orders of magnitude