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IMAGE FORMATION FROM COHERENCE FUNCTIONS IN ASTRONOMY

Edited by Cornelis van Schooneveld

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PROCEEDINGS OF IAU COLLOQUIUM NO. 49 ON THE FORMATION OF IMAGES FROM SPATIAL COHERENCE FUNCTIONS IN ASTRONOMY, HELD AT GRONINGEN, THE NETHERLANDS, 10–12 AUGUST 1978

Edited by

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INTRODUCTION

The IAU Colloquium No.49, on the formation of images from spatial coherence functions in astronomy, was held at Groningen, the Netherlands, during the period 10-12 August 1978. The colloquium was attended by 108 participants from 14 countries (U.S.A. 29, the Netherlands 20, U.K. 19, Germany 10, France 7, Australia 5, Canada 5, Japan 4, India 2, New Zealand 2, Sweden 2, Argentina 1, Belgium 1, Israel 1). It was sponsored by the Netherlands Foundation for Radio Astronomy, the International Astronomical Union, the Department of Education and Sciences, the Union Radio-Scientifique Internationale, the Leiden Kerkhoven-Bosscha Foundation and the State University at Groningen.

This volume contains 36 of the 37 papers presented. Nearly all papers are followed by a summary of the discussion that took place after their presentation. A few papers, published in full elsewhere, are given only as abstracts.

The majority of the papers are related to aperture synthesis in radio astronomy; a small number deal with optical astronomy and with applications in acoustics and medicine. The presentations are divided in 7 groups: aperture synthesis and its deficiencies, the problem of limited or missing phase information, techniques for processing and data display, optical interferometric methods, maximum entropy image reconstruction, other image improvement methods, and a survey of image formation from projections. Each group contains one or two invited lectures (see Table of Contents), intended as surveys of particular areas; on the average they occupy twice as many pages as the other papers.

Looking at the contents as a whole, one is struck by the fact that relatively little is said about the classic method of image formation through straightforward fourier inversion of the visibility data. On the other hand, many proposals are made for non-fourier techniques. They are prompted by the wish to cope with the phase problem or by the desire to achieve improved resolution in the image. The key to many of these methods is a simple one: the exploitation of previously neglected a priori knowledge of the brightness distribution and of the measurement errors. In spite of this simplicity we are confronted by a profusion of non-fourier image formation schemes. Some 20 different methods are listed in the Subject Index under the heading "Image reconstruction". A comparison of the relative performance of these methods - and of their combinations - is at present hampered by the fact that practically none of them has a closed form solution. More work is needed in the future to clarify the situation. Progress in this area is not only expected through astronomically oriented investigations but also through contributions from geophysics, acoustics, statistical spectrum analysis and picture processing, where similar problems exist.

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C. van SCHOONEVELD

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PART I

APERTURE SYNTHESIS METHODS

FUNDAMENTALS AND DEFICIENCIES OF APERTURE SYNTHESIS

(Invited paper)

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1. INTRODUCTION

Aperture synthesis is the method used by astronomers to determine the accurate brightness distribution of the radio sky with a resolution much better than that possible with a single large antenna. The technique, now over a decade old, utilizes a large number of connected radio antennas, some of them physically moveable, to follow a region of sky for many hours or days in order to sample the spatial coherence function of the radiation field over a sufficiently large area and with a reasonable filling factor. Landmark references for aperture synthesis are McCready et al. (1947), Stanier (1950), Christiansen and Warburton (1955), Lequeux et al. (1962), Read (1961) and Ryle and Hewish (1960).

In Section 2 the relationship between the spatial coherence function and the brightness distribution, essentially the van Cittert-Zernike Theorem, together with necessary assumptions about the radiation field, will be discussed. Aperture synthesis is a straightforward application of this theorem. The deficiencies in aperture synthesis are many and these are summarized in Section 3. A major part of this colloquium will deal with methods to correct the adverse effects of these deficiencies. Some are fundamental to the technique--e.g., the spotty sampling of the spatial coherence function. Some are related to practical considerations in the design of the array and the speed of map making at the expense of some accuracy--e.g., the use of the Fast-Fourier transform algorithm. Some are related to non-stationary effects during the course of the observations--e.g., tropospheric phase fluctuations. In Section 4 a brief description of problems in aperture synthesis that strike the author as important or annoying as well as a *potpourri* of other ideas will be mentioned.

C. van Schooneveld (ed.), Image Formation from Coherence Functions in Astronomy, 3-18.

2. FUNDAMENTALS OF APERTURE SYNTHESIS

2.1. Coherence Properties of the Radiation Field

Let $E(\bar{u},t)$ represent the radiation field at any point in space, $\bar{u}$, at any time, t. Although electromagnetic radiation is described by a vector field which satisfies Maxwell's Equations, for most astronomical applications an approximate description of the field by a scalar wave function (for example, the transverse component of the electric field) is adequate. The field fluctuates rapidly and behaves ergodically; that is, the time-average properties are well-defined and any measurement of an average property is typical of all such similar measurements.

While it is now possible at radio frequencies to follow the detailed fluctuations of the field, there is little additional information in them and most characteristics of the radiation are embodied in various average functions of the field. The intensity of the field can be easily measured using a suitable probe which responds with a signal $S(\bar{u},t)$ proportional to $E(\bar{u},t)$. The signal is first "detected" by measuring the power and then averaged over some time interval T which is long compared with the time-scale of the fluctuations. The intensity $I(\bar{u})$ is thus

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} S^*(\bar{u},t)\, S(\bar{u},t)\, dt \tag{2.1}$$

and is usually written in short-hand notation as

$$I(\bar{u}) = \langle S^*(\bar{u},t)\, S(\bar{u},t)\rangle \ . \tag{2.2}$$

As is customary when dealing with signals associated with the wave equation, a complex number, called the analytic signal, is used in place of the real signal (e.g., Steel 1967, p.22).

When two or more sources of radiation are superimposed the intensity in the region of superposition can vary. This phenomenon is called interference. In general the radiations from different sources are incoherent; i.e. their fluctuations are completely uncorrelated and, thus, they do not interfere over typical averaging time scales $t > 1/\Delta\nu$ where $\Delta\nu$ is the bandwidth of the radiation. But if the radiation from *one* source is separated into two beams, by sampling two points of the wavefront or by splitting the radiation at some point, the fluctuations in the two beams are generally correlated and they will interfere when combined.

The measure of the correlation in a radiation field between two points $\bar{u}_1$ and $\bar{u}_2$ at a time difference τ is given by

$$\Gamma(\bar{u}_1,\bar{u}_2,\tau) = \langle S^*(\bar{u}_1,t) \quad S(\bar{u}_2, t+\tau)\rangle \tag{2.3}$$

and is denoted as the *mutual coherence function* of the field. For example if we combine the signals sampled at two points $\bar{u}_1$ and $\bar{u}_2$, separated by time τ, the time-average intensity of the signal $S_s = S(\bar{u}_1,t) + S(\bar{u}_2,t)$ would be

$$\langle S_s^* \; S_s \rangle = I(\bar{u}_1) + I(\bar{u}_2) + 2 \text{ Real part } \Gamma(\bar{u}_1,\bar{u}_2,\tau). \tag{2.4}$$

The measured intensity could have any value between 0 and 4I depending on the phase and amplitude of the mutual coherence function. The last term in equation (2.4) can be isolated by multiplying, rather than adding, the two signals; the imaginary part can be obtained by introducing a 90-deg phase shift in one of the signals. Thus the mutual coherence function of a radiation field can be easily measured.

2.2. The Van Cittert-Zernike Theorem

The arrays used in radio astronomy sample many parts of the wavefront of the radiation from a source. Thus, the sum (or product) of signals from any pair of antennas is related to $\Gamma(\bar{u}_1,\bar{u}_2,\tau)$ where $\bar{u}_1$ is the position of one antenna, $\bar{u}_2$ is the position of the other antenna and τ is the time difference between the samples measured with respect to a wavefront from some arbitrary direction. If the two points coincide, $\Gamma(\bar{u},\bar{u},\tau)$ is called the *autocorrelation function* of the field at $\bar{u}$. By the Wiener-Khinchin Theorem the autocorrelation function is the Fourier transform of the power spectrum of the field (e.g., Steel 1967, p. 44). On the other hand for the special case when τ=0, $\Gamma(\bar{u}_1,\bar{u}_2,0)$ is called the *spatial coherence function* of the field.

The relationship between the spatial coherence function and the brightness distribution of an extended source is given by the van Cittert-Zernike Theorem. A derivation is given by Born and Wolf (1964, p. 510) or Steel (1967, p. 47). The theorem applies only if the radiation from the source is incoherent; that is the fluctuations of the radiation from different elements of the source are statistically independent.

For most astronomical applications, a radio source can be considered in the far field and the van Cittert-Zernike Theorem reduces to

$$\Gamma(\bar{u},\tau) = \int_{\nu=0}^{\infty} \left\{ \exp(i2\pi\nu\tau) \int_{\bar{\ell}} I(\bar{\ell},\nu)\exp(i2\pi\frac{\nu}{c}\,\bar{u}\cdot\bar{\ell})d\bar{\ell} \right\} d\nu \tag{2.5}$$

where $\bar{u}$ = the separation of the sampled field points = $\bar{u}_1 - \bar{u}_2$, independent of origin
$\bar{\ell}$ = the direction cosine to an element of the source
ν = the frequency of the radiation
c = the speed of light
$I(\bar{\ell},\nu)$ = the brightness distribution of the source
$\Gamma(\bar{u},\tau)$ = the mutual coherence function.

The actual measurement of the spatial coherence function $\Gamma(\bar{u},0)$ in typical radio astronomy applications is complicated by the following: 1) the radiation probes (antennas) sample the field over a finite area with relative sensitivity $a(\bar{u})$, usually called the grading of the antenna; 2) the bandwidth of radiation accepted is finite with a relative sensitivity given by $B(\nu)$; 3) all signals must be combined with delay τ=0 with respect to some arbitrary direction, usually denoted as the phase center or map center. With these practical considerations, the measured response of the correlation of the signals separated by $\bar{u}$ is called the *visibility function* and is given by

$$V(u,v,w) = \int_{\nu=0}^{\infty} B(\nu) \left\{ \int_{\ell,m} \frac{1}{n} A(\ell,m,\nu)\ I(\ell,m,\nu) \exp\left\{ i2\pi\frac{\nu}{c}\ [u\ell+vm+w(1-n)]\right\}\ d\ell dm \right\} d\nu \tag{2.6}$$

We have resolved $\bar{\ell}$, the direction cosine to the source with respect to the phase center and $\bar{u}$, the separation of the two samples of radiation into the usual components

$\bar{\ell} = (\ell,m,n)$ where the n-axis is parallel to the phase center direction, the ℓ- and m-axes define the plane parallel to n, ℓ directed to the east and m directed to the north. [Because the radio source is confined to the unit sphere, $n^2 = 1-\ell^2-m^2$.]

$\bar{u} = (u,v,w)$ where $u\|\ell$, $v\|m$, and $w\|n$.

The effect of the antenna grading is the multiplicative factor $A(\ell,m,\nu)$, called the antenna field pattern. If conventional radio telescopes are used as the radiation probe, A is near zero beyond a radius ℓ_{max} so that little information about $I(\ell,m,\nu)$ is measured beyond a *field of view* of radius ℓ_{max}.

2.3. Solution for the Brightness Distribution

It is clear from equation (2.6) that the visibility function measures something like a Fourier component of the brightness distribution. Aperture synthesis is, essentially, a methodical technique whereby a large number of samples in (u,v,w) are measured by 1) correlating the signals from all pairs (or at least all non-redundant pairs) of antennas in the multi-element array, 2) physically moving antennas to change the spacings between pairs of antennas, and 3) utilizing the rotation of the Earth to change the source-array aspect. With sufficient sampling, it is then possible to estimate the brightness distribution from the measured set of visibility functions.

However, an explicit expression for the brightness distribution can only be obtained with two further simplifications. In equation (2.6) the frequency ν and the angular coordinate $\bar{\ell}$ are coupled in the exponential term and cannot in general be separated. A quasi-monochromatic approximation applies when the change in phase of the exponential term is negligible over the radiation bandwidth $\Delta\nu$, which gives the condition

$$\frac{\Delta\nu}{c} u_{max} \ell_{max} << 1 \tag{2.7}$$

where u_{max} = maximum baseline of the array
ℓ_{max} = maximum angular extent of the emission, usually limited by the field pattern of the antenna.

We may then define an *apparent brightness distribution* $I'(\ell,m)$ given by

$$I'(\ell,m) = \int_{\nu} B(\nu) \frac{1}{n} A(\ell,m,\nu) I(\ell,m,\nu) d\nu. \tag{2.8}$$

The apparent brightness distribution is now a function of the array parameters as well as the radiation from the sky. In order to make any Fourier inversion of equation (2.6) meaningful the apparent brightness distribution must be invariant for all antenna pairs and be unchanged for the duration of the observations. This is accomplished, in part, by building an array with identical antennas and supporting electronics. Stringent mechanical and electrical tolerances are also necessary to achieve the necessary stability of I' to permit accurate aperture synthesis. Particular problems will be discussed in Section 4.

The explicit expression of the apparent brightness distribution in terms of the visibility function can now be written

$$I'(\ell,m) \delta(n-n') = \int V(u,v,w) \exp\left\{-i2\pi\frac{\nu_0}{c} (u\ell+vm+w[1-n])\right\} du dv dw \tag{2.9}$$

where $n' = (1-l^2-m^2)^{\frac{1}{2}}$ so that the emission is confined to the unit sphere, δ is the Dirac-delta function and ν_0 is the average frequency of I'.

The assumptions made in obtaining equation (2.9) are:

(1) The radiation is ergodic
(2) The radiation is incoherent across the source
(3) The radio source is in the far field
(4) The signals of each pair are correlated with $\tau=0$
(5) The quasi-monochromatic approximation applies
(6) $I'(\ell,m)$ is well-defined and stationary.

The breakdown of assumptions 1 and 2 would be surprising except for non-natural signals. The limitation imposed by the far field assumption can be lessened if we know the distance to the radio source and can correct for the effects of the spherical wavefront in the measured visibility. Many of the problems of high resolution, high frequency aperture synthesis arise from a breakdown of assumption 4 because of the variable differential delay across an array caused by the propagation of radiation in the troposphere of the Earth. If assumption 5 is not valid, then there is a loss of information about the brightness distribution at large angular separations from the phase center. The degree of validity of assumption 6 is a large measure of the ultimate accuracy in which the brightness distribution can be recovered from the visibility function.

The apparent brightness distribution and the visibility function are three-dimensional Fourier pairs; however, since I' is only defined on the unit sphere, it is often possible to reduce the dimensionality of the inversion formula to two. For many aperture synthesis applications the term $w(1-n)\ \nu_0/c << 1$ anywhere in the field of view and can be neglected. The three dimensional inversion can also be avoided if the sampled visibility function is coplanar over the entire set of observations. In this case $w=au+bv$ where a and b are arbitrary constants and a redefinition of ℓ and m gives the relation

$$I'(\ell',m') = \int V(u,v,w) \exp\left\{-i2\pi\frac{\nu_0}{c}\ (u\ell'+vm')\right\} du dv \qquad (2.10)$$

with ℓ' and m' now not quite direction cosines with respect to the phase center. Brouw (1971) was the first to show that only an east-west array (or an array on a plane perpendicular to the rotation axis of the earth) describes a coplanar aperture under earth-rotation synthesis. Other ground-based arrays may, at any instant, sample the visibility function on a plane and may use a two-dimensional transform for data taken over a short duration. However, the combination of maps over many hours of earth-rotation synthesis require a continual redefinition of ℓ and m. For ground-based arrays which are not east-west and larger than several kilometers, the curvature of the earth invalidates the coplanar property of the array.

The measurement of the polarization characteristics of the radiation can be obtained by using probes which are sensitive to the various components of the field. No additional problems are