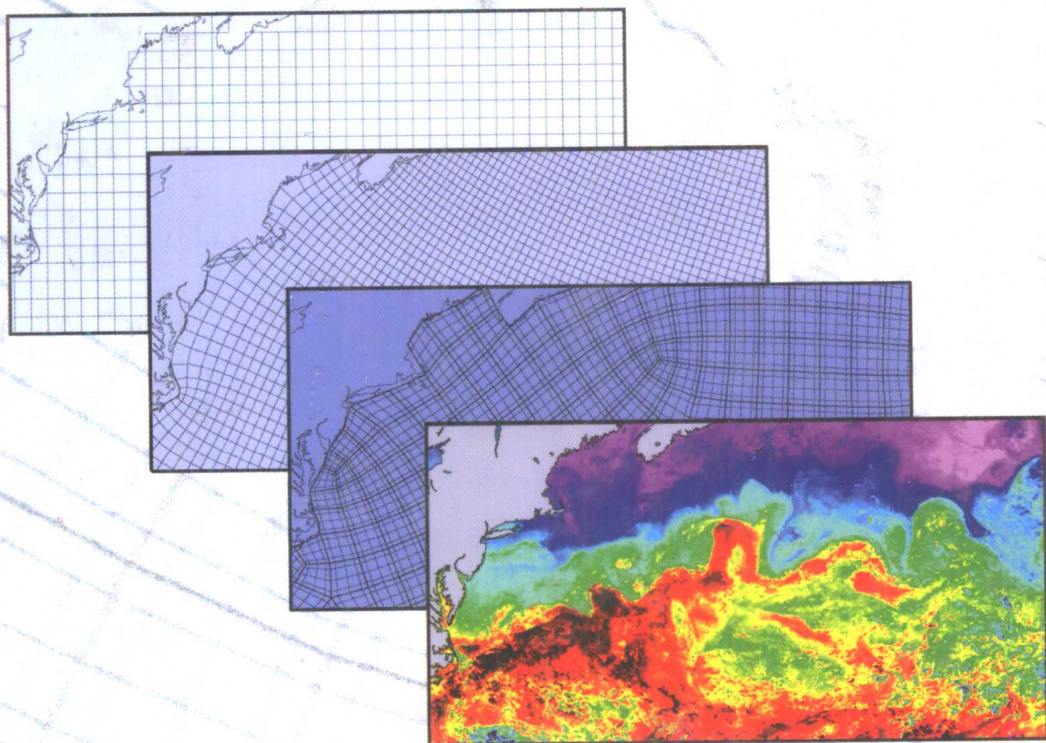




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NUMERICAL OCEAN CIRCULATION MODELING



Dale B. Haidvogel
Aike Beckmann

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Dale B. Haidvogel has been a leader in the development and application of alternative numerical ocean circulation models for nearly two decades. Since receiving his PhD in Physical Oceanography from the Massachusetts Institute of Technology and the Woods Hole Oceanographic Institution in 1976, his research activities have spanned the range from idealized studies of fundamental oceanic processes to the realistic modeling of coastal and marine environments. He currently holds the position of Professor II in the Institute of Marine and Coastal Sciences at Rutgers, the State University of New Jersey.

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To our daughters, Ilona and Annika.

Preface

Until recently, algorithmic sophistication in and diversity among regional and basin-scale ocean circulation models were largely non-existent. Despite significant strides being made in computational fluid dynamics in other fields, including the closely related field of numerical weather prediction, ocean circulation modeling, by and large, relied on a single class of models which originated in the late 1960's. Over the past decade, the situation has changed dramatically. First, systematic development efforts have greatly increased the number of available models. Secondly, enhanced interest in ocean dynamics and prediction on all scales, together with more ready access to high-end workstations and supercomputers, has guaranteed a rapidly growing international community of users. As a result, the algorithmic richness of existing models, and the sophistication with which they have been applied, has increased significantly.

In such a rapidly evolving field, it would be foolhardy to attempt a definitive review of all models and their areas of application. Our interest in composing this volume is more modest yet, we feel, more important. In particular, we seek to review the fundamentals upon which the practice of ocean circulation modeling is based, to discuss and to contrast the implementation and design of four models which span the range of current algorithms, and finally to explore and compare the limitations of each model class with reference to both realistic modeling of basin-scale oceanic circulation and simple two-dimensional idealized test problems.

The latter are particularly timely. With the expanded variety and accessibility of today's ocean models, it is now natural to ask which model might be best for a given application. Unfortunately, no systematic com-

parison among available large-scale ocean circulation models has ever been conducted. Replicated simulations in realistic basin-scale settings are one means of providing comparative information. Nonetheless, they are expensive and difficult to control and to quantify. The alternative — the development of a set of relatively inexpensive, process-oriented test problems on which model behavior can be assessed relative to known and quantifiable standards of merit — represents an important and complementary way of gaining experience on model performance and behavior.

Although we direct this book primarily towards students of the marine sciences and others who wish to get started in numerical ocean circulation modeling, the central themes (derivation of the equations of motion, parameterization of subgridscale processes, approximate solution procedures, and quantitative model evaluation) are common to other disciplines such as meteorology and computational fluid dynamics. The level of presentation has been chosen to be accessible to any reader with a graduate-level appreciation of applied mathematics and the physical sciences.

Ocean Models Today

There are, at present, within the field of ocean general circulation modeling four classes of numerical models which have achieved a significant level of community management and involvement, including shared community development, regular user interaction, and ready availability of software and documentation via the World Wide Web. These four classes are loosely characterized by their respective approaches to spatial discretization and vertical coordinate treatment.

The development of the first oceanic general circulation model (OGCM) is typically credited to Kirk Bryan at the Geophysical Fluid Dynamics Laboratory (GFDL) in the late 1960's. Following then-common practices, the GFDL model was originally designed to utilize a geopotential (z-based) vertical coordinate, and to discretize the resulting equations of motion using low-order finite differences. Beginning in the mid-1970's, significant evolution in this model class began to occur based on the efforts of Mike Cox (GFDL) and Bert Semtner (now at the Naval Postgraduate School). At present, variations on this first OGCM are in place at Harvard University (the Harvard Ocean Prediction System, HOPS), GFDL (the Modular Ocean Model, MOM), the Los Alamos National Laboratory (the Parallel Ocean Program, POP), the National Center for Atmospheric Research (the

NCAR Community Ocean Model, NCOM), and other institutions.

During the 1970's, two competing approaches to vertical discretization and coordinate treatment made their way into ocean modeling. These alternatives were based respectively on vertical discretization in immiscible layers ("layered" models) and on terrain-following vertical coordinates ("sigma" coordinate models). The former envisions the ocean as being made up of a set of non-mixing layers whose interface locations adjust in time as part of the dynamics; the latter assumes coordinate surfaces which are fixed in time, but follow the underlying topography (and are therefore not geopotential surfaces for non-flat bathymetry). In keeping with 1970's-style thinking on algorithms, both these model classes used (and continue to use) low-order finite difference schemes similar to those employed in the GFDL-based codes.

Today, several examples of layered and sigma-coordinate models exist. The former category includes models designed and built at the Naval Research Lab (the Navy Layered Ocean Model, NLOM), the University of Miami (the Miami Isopycnic Coordinate Ocean Model, MICOM), GFDL (the Hallberg Isopycnic Model, HIM), the Max Planck Institute in Hamburg, FRG (the OPYC model), and others. In the latter class are POM (the Princeton Ocean Model), SCRUM (the S-Coordinate Rutgers University Model), and GHERM (the GeoHydrodynamics and Environmental Research Model), to name the most widely used in this class.

More recently, OGCM's have been constructed which make use of more advanced, and less traditional, algorithmic approaches. Most importantly, models have been developed based upon Galerkin finite element schemes – *e.g.*, the triangular finite element code QUODDY (Dartmouth University) and the spectral finite element code SEOM (Rutgers). These differ most fundamentally in the numerical algorithms used to solve the equations of motion, and their use of unstructured (as opposed to structured) horizontal grids.

General Description of Contents

The goals of this volume are, first, to present a concise review of the fundamentals upon which numerical ocean circulation modeling is based; second, to give extended descriptions of the range of ocean circulation models currently in use; third, to explore comparative model behavior with reference to a set of quantifiable and inexpensive test problems; and lastly, to

demonstrate how these principles and issues arise in a particular basin-scale application.

Our focus is the modeling of the basin-scale to global ocean circulation, including wind-driven and thermohaline phenomena, on spatial scales of the Rossby deformation radius and greater. Smaller-scale processes (mesoscale eddies and rings, sub-mesoscale vortices, convective mixing, and turbulence; coastal, surface and bottom boundary layers) are not explicitly reviewed. It is assumed from the outset that such small-scale processes must be parameterized for inclusion of their effects on the larger-scale motions.

The related concepts of *approximation* and *parameterization* are central themes throughout our exposition. As we emphasize, the equations of motion conventionally applied to “solve for” the behavior of the ocean have been obtained via a complex (though systematic) series of dynamical approximations, physical parameterizations, and numerical assumptions. Any or all of these approximations and parameterizations may be consequential to the quality of the resulting oceanic simulation. It is therefore important for new practitioners of oceanic general circulation modeling to be aware of sources of solution sensitivity and potential trouble. We provide many examples of each.

Chapter 1 offers a brief introduction to the derivation of the oceanic equations of motion (the hydrostatic primitive equations) and various often-used approximate systems. Beginning with the traditional equations for conservation of mass, momentum, mechanical energy and heat, we show how these equations are modified within a rotating, spherical coordinate system. These continuous equations have many conservation properties; conservation of angular momentum, vorticity, energy and enstrophy are discussed. Various approximations are necessary to arrive at the accepted equations of oceanic motion. We review the arguments for the traditional, Boussinesq, and hydrostatic approximations, and the assumption of incompressibility, and how they relate to conservation properties such as energy and angular momentum. Lastly, additional approximations yield further-simplified systems including the beta-plane, quasigeostrophic and shallow water equations.

Chapter 2 discusses why we cannot solve the oceanic equations of motion directly. Instead, we must find approximate solutions using discrete numerical solution procedures. Two levels of discretization are involved — the approximation of functions and the approximation of equations; we review a variety of approaches to each. Solutions of the discretized equations

of motion can differ, sometimes dramatically, from the solutions of the original continuous equations. Sources of approximation error, with illustrative examples drawn from the one-dimensional heat and wave equations, are given. Alternative approaches to time differencing (e.g., explicit-in-time, implicit-in-time and semi-implicit) are also reviewed.

Additional numerical considerations arise when seeking solutions in two or more spatial dimensions (Chapter 3). Among these are the occurrence of tighter time-stepping stability restrictions, the need for fast solution procedures for elliptic boundary value problems, and the possibility of horizontally staggered gridding of the dependent variables. The latter is of particular interest in that different choices for the horizontal lattice have direct effects on numerical approximation errors and discrete conservation properties. As an example of these effects, the propagation characteristics of a variety of wave phenomena (inertial-gravity, planetary waves) are examined on several traditional staggered grids, showing the types of numerical approximation errors that can occur.

Four well-studied ocean models of differing algorithmic design are described in detail in Chapter 4. Among these are examples utilizing alternate vertical coordinates (geopotential, isopycnal, and topography-following), horizontal discretizations (unstaggered, staggered grids), methods of approximation (finite difference, finite element), and approximation order (low-order, high-order). The semi-discrete equations of motion are given for each model, as well as a brief summary of model-specific design features.

Chapter 5 describes why the “complete” equations of motion derived in Chapter 1 are not really complete. Because of omitted, though potentially important, interactions between resolved and unresolved scales of motion (the “closure problem”), we must specify parameterizations for these unresolved phenomena. Processes for which alternative parameterizations have been devised include vertical mixing at the surface and bottom oceanic boundaries, lateral transport and mixing by subgridscale eddies and turbulence, convective overturning, and topographic form stress. The origin and form of these parameterizations are reviewed.

Simple two-dimensional test problems are introduced in Chapter 6 to demonstrate the range of behaviors which can be obtained with the four models of Chapter 4 even under idealized circumstances. The process-oriented problems address a range of processes relevant to the large-scale ocean circulation including wave propagation and interaction (equatorial Rossby soliton), wind forcing (western boundary currents), effects of strat-

ification (adjustment of a vertical density front), and the combined effects of steep topography and stratification (downslope flow, alongslope flow). Substantial sensitivity to several numerical issues is demonstrated, including choice of vertical coordinate, subgridscale parameterization, and spatial discretization.

Chapter 7 examines the current state of the art in non-eddy-resolving modeling of the North Atlantic Ocean. After a brief review of simulation strategies and validation measures, we describe three recent multi-institutional programs which have sought to model the North Atlantic and to understand numerical and model-related dependencies. Taken together, these programs provide further illustration of the controlling influences of the numerical approximations and physical parameterizations employed in the model formulation. Nonetheless, model validation against known observational measures shows that, with care, numerical simulation of the North Atlantic Basin can be made with a considerable degree of skill.

Finally, Chapter 8 speculates briefly on promising directions for ocean circulation modeling, in particular the prospects for novel new spatial approximation treatments.

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The early chapters in this book are an abbreviated version of lecture notes developed over the past 20 years for graduate-level courses in ocean dynamics and modeling. The first author thanks the Woods Hole Oceanographic Institution, the Naval Postgraduate School, the Johns Hopkins University and Rutgers University for their support of this instructional development. The test problems described in Chapter 6 have benefitted from the encouragement and support of Terri Paluszkiewicz and the Pacific Northwest National Laboratory. The authors also acknowledge the Institute of Marine and Coastal Sciences of Rutgers University and the Alfred-Wegener-Institute for logistical and financial support during the completion of this monograph. Discussions with, and helpful comments by, several colleagues have significantly improved this volume. We are particularly grateful for the insightful suggestions made by Claus Böning, Eric Chassignet and Joachim Dengg. Lastly, we note with thanks the many technical contributions of Kate Hedström, Hernan Arango and Mohamed Iskandarani.

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