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Bottom Turbulence

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BOTTOM TURBULENCE

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FOREWORD

While the atmospheric boundary layer has been extensively investigated, the marine boundary layer above the sea floor - although very similar in character - was, until recently, much less well known ; the difficulty of making measurements in the sea, near the bottom, and the cost in equipment and human effort of any single experiment, reflecting on the calibration and the quality of the models.

Bottom turbulence is however a determinant factor in such important problems as bottom friction and energy dissipation in marine circulation, sedimentation, bottom erosion, recycling of nutrients, trapping and release of pollutants, etc.. Understanding bottom turbulence is prerequisite for the development of accurate forecasting models of the marine systems which, nowadays, the extensive exploitation of the sea requires.

In the recent years, the perfection of advanced techniques and the extension of the research effort have brought new interesting results and a more comprehensive insight into the characteristics of marine turbulence in the bottom boundary layer.

Furthermore, the detection, in the bottom layer of the sea, of semi-coherent structures and the simultaneous study of the effects of the suspended sediments load have contributed, beyond the simple investigation of marine turbulence, to a better understanding of the general features of turbulence and such phenomena - still much debated - as drag reduction by additives.

The International Liège Colloquia on Ocean Hydrodynamics are organized annually. Their topics differ from one year to another and try to address, as much as possible, recent problems and incentive new subjects in physical oceanography.

Assembling a group of active and eminent scientists from different countries and often different disciplines, they provide a forum for discussion and foster a mutually beneficial exchange of information opening on to a survey of major recent discoveries, essential mechanisms, impelling question-marks and valuable suggestions for future research.

The Scientific Organizing Committee of the Eighth Colloquium saw the desirability of bringing together, on the important topic of bottom turbulence, specialists from different fields, experimentalists and modellers, hydrodynamicists and sedimentologists.

The present book which may be regarded as the outcome of the colloquium comprises the proceedings of the meeting and specially commissioned contributions on observations, parameterization and modelling of turbulence in the bottom boundary layer of the sea.

Jacques C.J. NIHOUL

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CONTENTS

THE NUMERICAL SOLUTION OF THE THREE-DIMENSIONAL HYDRODYNAMIC EQUATIONS, USING A B-SPLINE REPRESENTATION OF THE VERTICAL CURRENT PROFILE

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ABSTRACT

A numerical model is described in which vertical current structure may be determined using a new method involving expansion through the depth in terms of B-splines. By way of a first test, wind induced motion in a simple rectangular basin is computed, yielding surface elevations and vertical current profiles in good agreement with those obtained by Heaps (1971) using an integral transform method. The effect of varying eddy viscosity is investigated, considering the changes thereby produced in the wind induced vertical and horizontal circulations and in the surface and bottom currents.

INTRODUCTION

Two-dimensional finite difference models, based on the vertically-integrated equations of continuity and motion, have been used extensively in recent years to calculate tides and storm surges. This approach is satisfactory for problems where the primary aim is to calculate changes in sea surface elevation, but for problems involving water circulation, and particularly in engineering the calculation of the forces exerted by the sea on off-shore structures, a knowledge of vertical current profile is required. The use of a Laplace transform method to recover the vertical current structure from a two-dimensional vertically integrated model has been proposed by Jelesnianski (1970) and applied by Forristall (1974) to the calculation of current profiles generated by a hurricane in the Gulf of Mexico. This method is particularly suitable for determining the depth distribution of currents at a specific position for a given moment in time, but for circulation studies the size of the computations would make it less convenient.

Finite difference models with grid boxes in both the horizontal and the vertical have been used recently in circulation studies (e.g. Leendertse 1973). This model involves vertical integration over each layer, and the use of a coefficient of interfacial friction. The bottom stress, however, is expressed in terms of the current in the bottom layer, a physically more realistic assumption than that employed in many two-dimensional models where the bottom stress is related to the depth mean current. However, solutions in the vertical are only available at discrete points, and the determination of a continuous velocity profile is not possible.

Heaps (1971, 1976) has overcome this latter problem for both the linear and non-linear hydrodynamic equations by expanding the two components of horizontal current in terms of depth-dependent eigenfunctions with time-dependent, horizontally-dependent coefficients. Both surface and bottom boundary conditions are satisfied in the limit as the number of terms in the expansion tend to infinity. In practice, Heaps shows that the expansion converges very rapidly, yielding a technique which is particularly economic in computer time.

In this paper a method is proposed in which the two components of horizontal current are expanded in terms of the product of depth dependent functions (B-splines), and coefficients which vary with time and horizontal position. The determination of the coefficients is accomplished by substituting these expansions into the two equations of motion and minimizing the resulting residual with respect to each coefficient in a least squares sense. The surface and bottom boundary conditions are satisfied exactly by using linear combinations of B-splines.

The application of the present method to the solution of the linear three-dimensional hydrodynamic equations, assuming a rectangular basin of constant depth with a constant eddy viscosity and a constant bottom friction coefficient, yield nearly identical solutions for wind induced motion to those obtained by Heaps (1971), providing an initial confirmation of the accuracy and stability of the method. The time variation of both horizontal and vertical circulation induced by the wind is calculated for a number of cases having different eddy viscosity, and the influence of eddy viscosity upon surface and bottom currents together with the induced circulation is examined.

SOLUTION OF THE BASIC EQUATIONS USING AN EXPANSION OF B-SPLINES

For a homogeneous fluid, neglecting shear stress in the horizontal, the advective terms, and the equilibrium tide, the equations of continuity and motion may be written

$$\frac{\partial}{\partial x}\int_0^h u\,dz + \frac{\partial}{\partial y}\int_0^h v\,dz + \frac{\partial \xi}{\partial t} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} - \gamma v = -g\frac{\partial \xi}{\partial x} - \frac{1}{\rho}\frac{\partial F}{\partial z} \tag{2}$$

$$\frac{\partial v}{\partial t} + \gamma u = -g\frac{\partial \xi}{\partial y} - \frac{1}{\rho}\frac{\partial G}{\partial z} \tag{3}$$

where

- t denotes time,
- x,y,z Cartesian co-ordinates, forming a left handed set, with x and y in the horizontal plane of the undisturbed sea surface, and z measuring depth below that surface,
- h undisturbed depth of water,
- ξ elevation of the sea surface above the undisturbed level,
- u,v components of the current at depth z, in the directions of increasing x,y respectively,
- ρ the density of the water,
- γ the geostrophic coefficient, uniform and constant,
- g the acceleration due to gravity.

Also, F,G denote internal shear stresses at depth z, in the x,y directions respectively, given by

$$F = -\rho \, N \frac{\partial u}{\partial z} \ , \quad G = -\rho \, N \frac{\partial v}{\partial z} \tag{4}$$

where N is a coefficient of eddy viscosity, in general varying with x,y and z , but taken as a constant in the following analysis. Substituting (4) into (2) and (3) gives

$$\frac{\partial u}{\partial t} - \gamma v = - g \frac{\partial \xi}{\partial x} + \frac{\partial}{\partial z} \left(N \frac{\partial u}{\partial z}\right) \tag{5}$$

$$\frac{\partial v}{\partial t} + \gamma u = - g \frac{\partial \xi}{\partial y} + \frac{\partial}{\partial z} \left(N \frac{\partial v}{\partial z}\right) \tag{6}$$

To solve these equations it is necessary to specify both surface and bottom boundary conditions. At the surface,

$$- \rho \left(N \frac{\partial u}{\partial z}\right)_o = F_s \quad , \quad - \rho \left(N \frac{\partial v}{\partial z}\right)_o = G_s \tag{7}$$

where F_s, G_s denote the components of wind stress over the water surface in the x and y directions, suffix o denoting evaluation at z = 0.

Similarly at the sea bed, z = h ,

$$- \rho \left(N \frac{\partial u}{\partial z}\right)_h = F_B \quad , \quad - \rho \left(N \frac{\partial v}{\partial z}\right)_h = G_B \tag{8}$$

where G_B, F_B denote the components of bottom friction in the x and y directions.

Assuming a slip condition at the sea bed :

$$F_B = k \rho u_h \quad , \quad G_B = k \rho v_h \tag{9}$$

where k is a constant coefficient, (8) gives ,

$$\left(N \frac{\partial u}{\partial z}\right)_h + k u_h = 0 \quad , \quad \left(N \frac{\partial v}{\partial z}\right)_h + k v_h = 0 \tag{10}$$

A no slip bottom boundary condition, namely $u_h = v_h = 0$, when employed with a coefficient of eddy viscosity which varies near the sea bed, is used in an extension of the present paper (Davies 1976a). However, for constant eddy viscosity, the relationships given by (10) are appropriate.

Expanding the two components of velocity in terms of depth dependent functions $M_r(z)$ (4'th order B-splines) gives

$$u(x,y,z,t) = \sum_{r=1}^{m} A_r(x,y,t)M_r(z) \qquad (11)$$

and

$$v(x,y,z,t) = \sum_{r=1}^{m} B_r(x,y,t)M_r(z)$$

The B-splines have a number of particularly useful features which make them a good choice as a set of basis functions. They have been used extensively for the accurate fitting of numerical data (Powell 1970), and yield very accurate solutions when used in solving linear hydrodynamic equations (Davies 1976b) and non-linear partial differential equations (Davies 1976c). The incorporation of boundary conditions is particularly easy due to the piecewise nature of the functions.

Points along the z axis, at which the B-spline changes from a zero to a non-zero function are termed knots, λ_r. A fourth order B-spline M_r being non-zero over the interval $\lambda_{r-4} \leq z \leq \lambda_r$ though at the points λ_{r-4} and λ_r , provided these knots are single, M_r and its derivatives vanish. For example, Fig. 1 shows the region $0 \leq z \leq h$ divided into ten interior knot segments, corresponding to m = 13 in equations (11) and (12), with knots at $0 = \lambda_o < \lambda_1 < \lambda_2 \ldots < \lambda_9 < \lambda_{10} = h$. In order to support the fourth order B-splines additional knots are required at $\lambda_{-3} < \lambda_{-2} < \lambda_{-1} < 0$ and $h < \lambda_{11} < \lambda_{12} < \lambda_{13}$. From this diagram it is obvious that only the first three B-splines, M_1 , M_2 and M_3 and the last three M_{11} , M_{12} , M_{13} are non-zero at the boundaries z = 0 and z = h respectively, enabling boundary conditions to be readily incorporated.

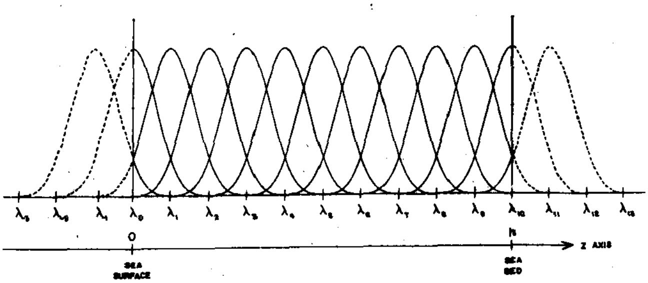

FIG. 1. : Distribution of B-splines and associated knots with depth.

For constant eddy viscosity N, the surface boundary condition for the u component of current becomes using (7) and (11),

$$\left(\frac{\partial u}{\partial z}\right)_o = T_x = \sum_{r=1}^{m} A_r(x,y,t)V_r(z) \qquad (13)$$

where

$$T_x = -\frac{F_s}{\rho N} \qquad \text{and} \qquad V_r(z) = \left.\frac{d\,M_r(z)}{dz}\right|_{z=0}$$

The positions of the knots may be chosen arbitrarily. A uniform distribution was in fact used, and in this case V_2 is zero. Thus only the derivatives of M_1 and M_3 are non-zero at the surface boundary giving,

$$T_x = A_1V_1 + A_3V_3 \qquad (14)$$

Rearranging (14) gives

$$A_1 = (T_x - A_3V_3)/V_1 \qquad (15)$$

The bottom boundary condition for the u component of current yields,

$$N\sum_{r=1}^{m} A_r(x,y,t)\left.\frac{dM_r(z)}{dz}\right|_{z=h} + k\sum_{r=1}^{m} A_r(x,y,t)M_r(z)\Big|_{z=h} = 0 \qquad (16.1)$$

In this case M_{m-2}, M_{m-1}, M_m, $\frac{dM_{m-2}}{dz}$ and $\frac{dM_m}{dz}$ are non-zero at the boundary $z = h$ giving

$$N(A_{m-2}V_{m-2}+A_mV_m) + k(A_{m-2}W_{m-2}+A_{m-1}W_{m-1}+A_mW_m) = 0 \qquad (16.2)$$

where

$$V_r = \left.\frac{dM_r(z)}{dz}\right|_{z=h} \qquad \text{and} \qquad W_r = M_r(z)\Big|_{z=h}$$

Rearranging (16.2) gives

$$A_m = \frac{A_{m-2}(-NV_{m-2} - kW_{m-2}) - A_{m-1}k\,W_{m-1}}{NV_m + k\,W_m} \qquad (17)$$

Substituting (15) and (17) into (11) and rearranging gives

$$u(x,y,z,t) = \frac{T_x}{V_1}M_1(z) + A_2M_2(z) + A_3\big(M_3(z) - C_1M_1(z)\big)$$

$$+ \sum_{r=4}^{m-3} A_rM_r(z) + A_{m-2}\big(M_{m-2}(z) - C_2M_m(z)\big) + A_{m-1}\big(M_{m-1}(z) - C_3M_m(z)\big) \qquad (18)$$

where

$$C_1 = V_3/V_1 \ , \ C_2 = (NV_{m-2} + k\, W_{m-2})/(NV_m + k\, W_m)$$

and

$$C_3 = k\, W_{m-1}/(NV_m + k\, W_m)$$

This can be written as

$$u(x,y,z,t) = \frac{T_x}{V_1} M_1(z) + A_2 M_2(z) + A_3 \overline{M}_3(z) + \sum_{r=4}^{m-3} A_r M_r(z) + A_{m-2}\overline{M}_{m-2}(z) + A_{m-1}\overline{M}_{m-1}(z) \tag{19}$$

where

$$\overline{M}_3(z) = M_3(z) - C_1 M_1(z) \ , \ \overline{M}_{m-2}(z) = M_{m-2}(z) - C_2 M_m(z)$$

and

$$\overline{M}_{m-1}(z) = M_{m-1}(z) - C_3 M_m(z)$$

A similar expression to (19) can be derived for the v component of current namely,

$$v(x,y,z,t) = \frac{T_y}{V_1} M_1(z) + B_2 M_2(z) + B_3 \overline{M}_3(z) + \sum_{r=4}^{m-3} B_r M_r(z) + B_{m-2}\overline{M}_{m-2}(z) + B_{m-1}\overline{M}_{m-1}(z) \tag{20}$$

where

$$T_y = -\, G_s/\rho N$$

Substituting (19) and (20) into equations (5) and (6), dropping the bar on the 3, m-2 and m-1 terms gives, for eddy viscosity N independent of z ,

$$\frac{dT_x}{dt}\frac{M_1}{V_1}(z) + \sum_{r=2}^{m-1}\frac{dA_r}{dt} M_r(z) - \gamma\left(\frac{T_y}{V_1} M_1(z) + \sum_{r=2}^{m-1} B_r M_r(z)\right) + g\frac{\partial\xi}{\partial x} - N\left(\frac{T_x}{V_1}\frac{d^2M_1(z)}{dz^2} + \sum_{r=2}^{m-1} A_r \frac{d^2M_r(z)}{dz^2}\right) = R_1 \tag{21}$$

and

$$\frac{dT_y}{dt}\frac{M_1(z)}{V_1} + \sum_{r=2}^{m-1}\frac{dB_r}{dt} M_r(z) + \gamma\left(\frac{T_x}{V_1} M_1(z) + \sum_{r=2}^{m-1} A_r M_r(z)\right) + g\frac{\partial\xi}{\partial y} - N\left(\frac{T_y}{V_1}\frac{d^2M_1(z)}{dz^2} + \sum_{r=2}^{m-1} B_r \frac{d^2M_r(z)}{dz^2}\right) = R_2 \tag{22}$$