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网络科学与工程丛书

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# Propagation Dynamics on Complex Networks

Models, Methods  
and Stability Analysis

英文版

复杂网络传播动力学  
——模型、方法与稳定性分析

傅新楚 Michael Small 陈关荣 著

高等教育出版社  
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**傅新楚**，2001年获英国Exeter大学应用数学博士学位。1997年至2002年在英国剑桥大学、Warwick大学作高级访问学者，随后在英国Surrey大学、Exeter大学任Research Fellow，由英国国家基金EPSRC资助研究一类不连续系统的动力学问题。2002年5月回国，在上海大学数学系工作，任教授、博士生导师。先后主持国家自然科学基金项目5项，曾参加国家“攀登计划”重大项目。



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**陈关荣**，1981年获中山大学计算数学硕士学位，1987年获美国德克萨斯A&M大学应用数学博士学位。于休斯顿大学任教至2000年，现任香港城市大学电子工程系讲座教授。1996年当选为IEEE Fellow。获2012年及2008年国家自然科学二等奖、2010年何梁何利奖、2011年俄罗斯欧拉奖并获俄罗斯圣彼得堡国立大学荣誉博士学位，获5项IEEE等最佳学术杂志论文奖，是国内外30多所大学的荣誉或客座教授，现任International Journal of Bifurcation and Chaos主编。SCI他引两万多次，h指数78，被ISI评定为工程学高引用率研究人员。



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# 序

随着以互联网为代表的网络信息技术的迅速发展，人类社会已经迈入了复杂网络时代。人类的生活与生产活动越来越多地依赖于各种复杂网络系统安全可靠和有效的运行。作为一个跨学科的新兴领域，“网络科学与工程”已经逐步形成并获得了迅猛发展。现在，许多发达国家的科学界和工程界都将这个新兴领域提上了国家科技发展规划的议事日程。在中国，复杂系统包括复杂网络作为基础研究也已列入《国家中长期科学和技术发展规划纲要(2006—2020年)》。

网络科学与工程重点研究自然科学技术和社会政治经济中各种复杂系统微观性态与宏观现象之间的密切联系，特别是其网络结构的形成机理与演化方式、结构模式与动态行为、运动规律与调控策略，以及多关联复杂系统在不同尺度下行为之间的相关性等。网络科学与工程融合了数学、统计物理、计算机科学及各类工程技术科学，探索采用复杂系统自组织演化发展的思想去建立全新的理论和方法，其中的网络拓扑学拓展了人们对复杂系统的认识，而网络动力学则更深入地刻画了复杂系统的本质。网络科学既是数学中经典图论和随机图论的自然延伸，也是系统科学和复杂性科学的创新发展。

为了适应这一高速发展的跨学科领域的迫切需求，中国工业与应用数学学会复杂系统与复杂网络专业委员会偕同高等教育出版社出版了这套“网络科学与工程丛书”。这套丛书将为中国广大的科研教学人员提供一个交流最新

研究成果、介绍重要学科进展和指导年轻学者的平台，以共同推动国内网络科学与工程研究的进一步发展。丛书在内容上将涵盖网络科学的各个方面，特别是网络数学与图论的基础理论，网络拓扑与建模，网络信息检索、搜索算法与数据挖掘，网络动力学（如人类行为、网络传播、同步、控制与博弈），实际网络应用（如社会网络、生物网络、战争与高科技网络、无线传感器网络、通信网络与互联网），以及时间序列网络分析（如脑科学、心电图、音乐和语言）等。

“网络科学与工程丛书”旨在出版一系列高水准的研究专著和教材，使其成为引领复杂网络基础与应用研究的信息和学术资源。我们殷切希望通过这套丛书的出版，进一步活跃网络科学与工程的研究气氛，推动该学科领域知识的普及，并为其深入发展做出贡献。

金芳蓉 (Fan Chung) 院士  
美国加州大学圣地亚哥分校  
二〇一一年元月

# Preface

Throughout history, epidemic diseases have been a serious threat to human health and life. In the past few years, many infectious diseases such as dengue, malaria, HIV, and SARS have captured global attention. Many of these, and others, remain a great threat, with potential for new outbreaks – particularly, for example, with a human-transmissible version of the H5N1 avian influenza. Moreover, with the development of globalized transportation, the potential for epidemic transmission has become much greater. Once a disease emerges, it will very likely diffuse globally very rapidly: 2009 H1N1 spread to some 30 countries worldwide in a relatively short period of time leaving more than 800 dead. The continual computer virus attacks on the Internet also illustrate the urgent need for knowledge about modeling, analysis, and control of epidemic dynamics on complex networks. Concerning the advance of techniques, it has become clear that more fundamental knowledge is needed within the context of mathematical and numerical studies on how epidemic dynamical networks can be modeled, analyzed and controlled. The main objective of this book is to present the state-of-the-art and recent progress in the investigation of these important topics and some related issues arising from various epidemic and information systems.

This book covers most emerging topics of epidemic dynamics on complex networks, including models, theories, methods, and global stability analysis. We also extend our discussions to include information propagation dynamics, and address topics such as how information, opinions, and rumors spread in the Internet or social networks. This work has developed from a series of research papers resulting from an on-going collaboration among the three authors and their research groups since 2006.

This is mainly a research monograph and also a textbook that can be used as either a research reference book or for a one-semester introductory course on propagation dynamics and epidemic control on complex networks for upper-division undergraduates and first-year graduates in applied mathematics, engineering, computer science, information science, communication systems, biological and life sciences, applied physics, as well as biomedical and social sciences. It covers most basic topics in the field, and therefore can serve well for self-study of these topics by graduate students and researchers interested in network science and engineering.

Throughout the text we often keep the adjective complex to reflect the historical perspective and to emphasize the nature of the subject, which is in line with



the common phrases of complex systems and complex dynamics alike, therefore it should not be seen as redundant.

We would like to take this opportunity to express our gratitude to the editor Ms Ying Liu at China Higher Education Press for her invaluable help and support throughout the writing of this book and the subsequent publication processes. We would also like to thank the editors at Wiley for their timely responses to our book proposal and for all their helpful comments aimed at improving the final product.

We would also like to acknowledge and thank Luonan Chen, Zhen Jin, Xiang Li, Zengrong Liu, Zonghua Liu, Jun-an Lu, Robert MacKay, Chi K. Tse, Binghong Wang, Xiaofan Wang, to mention just a few, and also our research group members, for their kind help and support.

Finally, we would like to thank our postgraduate students for their contributions, helpful discussions and useful suggestions during the writing of this book. Their contributions are too many to be listed individually.

The research was supported jointly by the University Grants Council of Hong Kong (HK UGC GRF PolyU5300/09E and CityU1109/12E), the Australian Research Council Future Fellowship scheme (grant number FT110100896), City University of Hong Kong, the NSFC grant 11072136, the Shanghai University Leading Academic Discipline Project “Complex Systems: Theory, Methods and Technology” (2012–2014) (Project No. A.13-0101-12-004), and a grant of “The First-class Discipline of Universities in Shanghai”. The publication of this book was supported by the China National Publishing Fund for Academic Books in Science and Technology.

# Summary

This book evolved from a series of research papers by the three authors and their students published since 2006. It covers the emerging topics of propagation dynamics on complex networks, including models, methods, and stability analysis. Throughout history, epidemic diseases have always been a serious threat to mankind's health and life, and ongoing serious virus attacks on the Internet also illustrate the emergent need for knowledge about modeling, analysis, and control in epidemic dynamics on complex networks. For advance of techniques, it has become clear that more fundamental knowledge will be needed in mathematical and numerical context about how epidemic dynamical networks can be modeled, analyzed, and controlled. The aim of this book is to report the progress made in these topics and some related issues of various epidemic systems. The book will first present a brief history of mathematical epidemiology, and epidemic modeling on complex networks. Then different epidemic models on complex networks, such as staged progression models, models with population mobility, or effective contacts, models on weighted networks, or directed networks, discrete epidemic models, stochastic SIRS epidemic models, and so on, will be discussed. Some threshold analyses by the direct method and by using spectral properties are given. Networked models for SARS and H1N1 are established by setting up plausible models for propagation of the SARS virus and avian influenza outbreaks, which provides a reality-check for the otherwise abstract mathematical models of this text, and it is shown that such models do match well the reality of current emerging diseases. Furthermore, various infectivity functions, including constant, piecewise-linear, saturated, and nonlinear cases, are considered. This book also concentrates on the cases for SIS models with an infective medium, the roles of human awareness in epidemic control, adaptive mechanism between dynamics and epidemics. Methods for epidemic control and different immunization strategies are summarized. Global stability analysis for several networked epidemic models is demonstrated. Finally, information transmission on complex networks and some differences between information and epidemic spreading are investigated.

This book covers most basic topics in the field, and therefore can serve well for self-study of the subjects by graduate students and researchers interested in network science and dynamical systems, and related interdisciplinary fields.

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Motivation and background	1
1.2	A brief history of mathematical epidemiology	2
1.2.1	Compartmental modeling	3
1.2.2	Epidemic modeling on complex networks	4
1.3	Organization of the book	5
	References	6
<b>2</b>	<b>Various epidemic models on complex networks</b>	<b>10</b>
2.1	Multiple stage models	10
2.1.1	Multiple susceptible individuals	11
2.1.2	Multiple infected individuals	12
2.1.3	Multiple-staged infected individuals	13
2.2	Staged progression models	13
2.2.1	Simple-staged progression model	14
2.2.2	Staged progression model on homogenous networks	14
2.2.3	Staged progression model on heterogenous networks	15
2.2.4	Staged progression model with birth and death	16
2.2.5	Staged progression model with birth and death on homogenous networks	16
2.2.6	Staged progression model with birth and death on heterogenous networks	16
2.3	Stochastic SIS model	17
2.3.1	A general concept: Epidemic spreading efficiency	18
2.4	Models with population mobility	19
2.4.1	Epidemic spreading without mobility of individuals	20
2.4.2	Spreading of epidemic diseases among different cities	20
2.4.3	Epidemic spreading within and between cities	21
2.5	Models in meta-populations	22

## ii CONTENTS

2.5.1	Model formulation	22
2.6	Models with effective contacts	24
2.6.1	Epidemics with effectively uniform contact	25
2.6.2	Epidemics with effective contact in homogenous and heterogenous networks	26
2.7	Models with two distinct routes	26
2.8	Models with competing strains	28
2.8.1	SIS model with competing strains	28
2.8.2	Remarks and discussions	30
2.9	Models with competing strains and saturated infectivity	31
2.9.1	SIS model with mutation mechanism	31
2.9.2	SIS model with super-infection mechanism	33
2.10	Models with birth and death of nodes and links	33
2.11	Models on weighted networks	34
2.11.1	Model with birth and death and adaptive weights	36
2.12	Models on directed networks	38
2.13	Models on colored networks	40
2.13.1	SIS epidemic models on colored networks	41
2.13.2	Microscopic Markov-chain analysis	42
2.14	Discrete epidemic models	44
2.14.1	Discrete SIS model with nonlinear contagion scheme	44
2.14.2	Discrete-time epidemic model in heterogenous networks	45
2.14.3	A generalized model	46
	References	47
<b>3</b>	<b>Epidemic threshold analysis</b>	<b>53</b>
3.1	Threshold analysis by the direct method	53
3.1.1	The epidemic rate is $\beta/n_i$ inside the same cities	63
3.1.2	Epidemics on homogenous networks	65
3.1.3	Epidemics on heterogenous networks	66
3.2	Epidemic spreading efficiency threshold and epidemic threshold	69
3.2.1	The case of $\lambda_1 \neq \lambda_2$	71
3.2.2	The case of $\lambda_1 = \lambda_2$	74
3.2.3	Epidemic threshold in finite populations	75
3.2.4	Epidemic threshold in infinite populations	75
3.3	Epidemic thresholds and basic reproduction numbers	76
3.3.1	Threshold from a self-consistency equation	77
3.3.2	Threshold unobtainable from a self-consistency equation	78
3.3.3	Threshold analysis for SIS model with mutation	80
3.3.4	Threshold analysis for SIS model with super-infection	83
3.3.5	Epidemic thresholds for models on directed networks	86
3.3.6	Epidemic thresholds on technological and social networks	87
3.3.7	Epidemic thresholds on directed networks with immunization	89
3.3.8	Comparisons of epidemic thresholds for directed networks with immunization	90

3.3.9	Thresholds for colored network models	93
3.3.10	Thresholds for discrete epidemic models	96
3.3.11	Basic reproduction number and existence of a positive equilibrium	97
	References	98
<b>4</b>	<b>Networked models for SARS and avian influenza</b>	<b>101</b>
4.1	Network models of real diseases	101
4.2	Plausible models for propagation of the SARS virus	102
4.3	Clustering model for SARS transmission: Application to epidemic control and risk assessment	108
4.4	Small-world and scale-free models for SARS transmission	114
4.5	Super-spreaders and the rate of transmission	118
4.6	Scale-free distribution of avian influenza outbreaks	124
4.7	Stratified model of ordinary influenza	130
	References	136
<b>5</b>	<b>Infectivity functions</b>	<b>139</b>
5.1	A model with nontrivial infectivity function	140
5.1.1	Epidemic threshold for SIS model with piecewise-linear infectivity	141
5.1.2	Piecewise smooth and nonlinear infectivity	142
5.2	Saturated infectivity	143
5.3	Nonlinear infectivity for SIS model on scale-free networks	143
5.3.1	The epidemic threshold for SIS model on scale-free networks with nonlinear infectivity	144
5.3.2	Discussions and remarks	148
	References	148
<b>6</b>	<b>SIS models with an infective medium</b>	<b>150</b>
6.1	SIS model with an infective medium	150
6.1.1	Homogenous complex networks	151
6.1.2	Scale-free networks: The Barabási–Albert model	152
6.1.3	Uniform immunization strategy	156
6.1.4	Optimized immunization strategies	157
6.2	A modified SIS model with an infective medium	159
6.2.1	The modified model	159
6.2.2	Epidemic threshold for the modified model with an infective medium	160
6.3	Epidemic models with vectors between two separated networks	162
6.3.1	Model formulation	162
6.3.2	Basic reproduction number	164
6.3.3	Sensitivity analysis	166
6.4	Epidemic transmission on interdependent networks	167
6.4.1	Theoretical modeling	168

6.4.2	Mathematical analysis of epidemic dynamics	172
6.4.3	Numerical analysis: Effect of model parameters on the basic reproduction number	174
6.4.4	Numerical analysis: Effect of model parameters on infected node densities	177
6.5	Discussions and remarks	179
	References	181
<b>7</b>	<b>Epidemic control and awareness</b>	<b>184</b>
7.1	SIS model with awareness	184
7.1.1	Background	185
7.1.2	The model	186
7.1.3	Epidemic threshold	190
7.1.4	Conclusions and discussions	191
7.2	Discrete-time SIS model with awareness	192
7.2.1	SIS model with awareness interactions	193
7.2.2	Theoretical analysis: Basic reproduction number	195
7.2.3	Remarks and discussions	197
7.3	Spreading dynamics of a disease-awareness SIS model on complex networks	198
7.3.1	Model formulation	198
7.3.2	Derivation of limiting systems	200
7.3.3	Basic reproduction number and local stability	201
7.4	Remarks and discussions	201
	References	203
<b>8</b>	<b>Adaptive mechanism between dynamics and epidemics</b>	<b>207</b>
8.1	Adaptive mechanism between dynamical synchronization and epidemic behavior on complex networks	207
8.1.1	Models of complex dynamical network and epidemic network	209
8.1.2	Models of epidemic synchronization and its analysis	210
8.1.3	Local stability of epidemic synchronization	212
8.1.4	Global stability of epidemic synchronization	214
8.2	Interplay between collective behavior and spreading dynamics	216
8.2.1	A general bidirectional model	217
8.2.2	Global synchronization and spreading dynamics	218
8.2.3	Stability of global synchronization and spreading dynamics	220
8.2.4	Phase synchronization and spreading dynamics	226
8.2.5	Control of spreading networks	227
8.2.6	Discussions and remarks	227
	References	228
<b>9</b>	<b>Epidemic control and immunization</b>	<b>231</b>
9.1	SIS model with immunization	231

9.1.1	Proportional immunization	231
9.1.2	Targeted immunization	232
9.1.3	Acquaintance immunization	233
9.1.4	Active immunization	234
9.2	Edge targeted strategy for controlling epidemic spreading on scale-free networks	235
9.3	Remarks and discussions	237
	References	239
<b>10</b>	<b>Global stability analysis</b>	<b>240</b>
10.1	Global stability analysis of the modified model with an infective medium	240
10.2	Global dynamics of the model with vectors between two separated networks	241
10.2.1	Global stability of the disease-free equilibrium and existence of the endemic equilibrium	243
10.2.2	Uniqueness and global attractivity of the endemic equilibrium	245
10.3	Global behavior of disease transmission on interdependent networks	247
10.3.1	Existence and global stability of the endemic equilibrium for a disease-awareness SIS model	248
10.4	Global behavior of epidemic transmissions	250
10.4.1	Stability of the model equilibria	250
10.4.2	Stability analysis for discrete epidemic models	252
10.4.3	Global stability of the disease-free equilibrium	256
10.4.4	Global attractiveness of epidemic disease	257
10.5	Global attractivity of a network-based epidemic SIS model	260
10.5.1	Positiveness, boundedness and equilibria	260
10.5.2	Global attractivity of the model	262
10.5.3	Remarks and discussions	263
10.6	Global stability of an epidemic model with birth and death and adaptive weights	264
10.6.1	Global dynamics of the model	264
10.6.2	Discussions and remarks	266
10.7	Global dynamics of a generalized epidemic model	268
10.7.1	Model formulation	268
10.7.2	Global dynamics of the model	270
10.7.3	Discussions and remarks	273
	References	274
<b>11</b>	<b>Information diffusion and pathogen propagation</b>	<b>277</b>
11.1	Information diffusion and propagation on complex networks	277
11.1.1	Information diffusion on complex networks	278

11.1.2	Some essential differences between information propagation and epidemic spreading	280
11.2	Interplay between information of disease spreading and epidemic dynamics	281
11.2.1	Preliminaries	281
11.2.2	Theoretical analysis of the model	282
11.3	Discussions and remarks	284
	References	286
<b>Appendix A Proofs of theorems</b>		<b>289</b>
A.1	Transition from discrete-time linear system to continuous-time linear system	289
A.2	Proof of Lemma 6.1	291
A.3	Proof of Theorem 10.4	291
A.4	Proof of Theorem 10.3	292
A.5	Proof of Theorem 10.42	296
<b>Appendix B Further proofs of results</b>		<b>302</b>
B.1	Eigenvalues of the matrix $\tilde{F}$ in (6.27)	302
B.2	The matrix $\Gamma$ in (6.32)	304
B.3	Proof of (7.6) in Chapter 7	305
B.4	The positiveness of $\sigma'$ : proof of $\sigma' > 0$ in Section 9.1.2	306
B.5	The relation between $\Lambda$ and $\kappa$ in Section 9.1.3	308
<b>Index</b>		<b>311</b>



# 1

## Introduction

In this chapter we provide a brief introduction to the remainder of the book.

The uninitiated may require a broader background to the topic of complex networks. Rather than overburden our current presentation, we refer interested readers to some good introductory books and papers [1–14] for more background information on complex networks and network science.

### 1.1 Motivation and background

Throughout history, infectious diseases have always been a serious threat to human health and life. It is therefore of great practical significance to study epidemic transmission and then to take effective measures to prevent and control them. Toward this end, much research has fallen within the field of epidemiology, which uses mathematical modeling as an analytical approach. Traditionally, epidemic models were based on uniformly mixing populations, which are unable to characterize epidemic propagation in large-scale social contact networks with disparate heterogeneity. However, the fact that most population-based epidemics spread through physical interactions raises contact networks as a basic tool for mathematical description of contagion dynamics. In the last decade, spurred by the availability of real data and the maturation of network theory, there has been a burst of research on network-based epidemic transmission [15–26].

Beyond ordinary infection diseases, recurring computer virus attacks (as well as computer worms and other malware vectors) on the Internet also illustrate the urgent need for knowledge about modeling, analysis and control of epidemic dynamics on complex networks.

The World Health Organization (WHO) announced in 2012 [27] that some time in the next couple of years Guinea worm will become only the second known