声场声信息国家重点实验室年报

ANNUAL REPORT Volum 2

STATE KEY LABORATORY OF ACOUSTICS

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1992

中国科学院声学研究所

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中国科学院声学研究所声场声信息
国家重点实验室简介

一、 概 况

声场声信息国家重点实验室(State Key Laboratory of Acoustics, SKLA)位于北京市海淀区中关村东路，中国科学院声学研究所院内，是为创造一流的声学科研基地于1987年由国家计委批准筹建，1989年底建成，1990年2月通过国家验收，1990年9月批准正式对外开放的。在实验室筹建开始，即本着"边建设，边开放，边工作"的方针，开展和支持了多项国家自然科学基金、"863"、国家攻关、院重大基金、军工等项目，取得了多项具有国际先进水平的科研成果，其中1989年"浅海声传播损失数值预报"获中科院自然科学一等奖；1990年"简正波声场的变换与过滤的研究"、"脉冲超声在固体中散射的理论与实验研究"、"反转点会聚区理论与南海声道研究"分别获国家自然科学二、三、四等奖；1991年"汉语语音特征研究"获中科院自然科学一等奖；"数字化水声信号处理机"获中科院科技进步一等奖。1992年"典型浅海海底混响模型"获中科院自然科学二等奖；"多普勒流速剖面声呐"、"温跃层深度探测"分别获中科院科技进步二、三等奖。近年来，科技人员还在国内外有影响的学术刊物、学术会议上发表论文165篇。在九一年度国家计委委托国家基金委组织的"开放实验室运行补助费"评比中，实验室被评为甲类优秀实验室。

中科院学部委员、水声物理学家张仁和研究员任实验室主任，北方交通大学信息研究所所长袁保宗教授和中科院声学研究所吴国清研究员任副主任。中国科学院秘书长侯自强研究员任学术委员会主任。学术委员会由中外声学界著名专家学者20人组成，其中中科院学部委员2人，外籍委员4人。

实验室有固定人员29人，其中研究人员22人(高级10人，中级5人，初级7人)，技术人员5人(中级1人，初级4人)，管理人员2人。

二、 实验室研究方向

研究声波与物质的相互作用，探索物质结构对声场规律的影响以及利用声波作为探测物质结构的方法，是声学研究的基本内容。

实验室主要研究方向:

(1)声传播和声场的物理与计算机模拟

研究重点: 海洋信号场、混响场与噪声场的统一理论与数值预报方法，发展二维与三维非均匀媒质中的声传播与散射的理论，建立声场数据库与计算方法库。

(2)声场反演理论和声探测

研究重点: 非均匀媒质中分层结构的反演、声学参数的提取，各种媒质中目标物的反演与声成像，声源定位与识别，生物组织特征量的识别。

(3)声场的有源控制研究

研究重点: 在声场物理研究和计算机模拟的基础上实现有源声场控制，有源噪声和振动抵消的理论与实现方法，利用计算机控制声场特性。

(4)语言信号处理

研究重点: 语言信号的处理与分析方法，语言识别、合成与文语转换，神经网络在语言识别中的应用。

(5)声场时空处理的理论和应用

研究重点: 数字信号处理在声场研究中的应用，自适应滤波与浅海声场的自适应匹配，最佳时延估计和噪声测距，可编时域多波束成形系统，奇异值分解及其应用，数字信号处理新算法和 ASIC 芯片设计方法。

根据实验室学术委员会审定，近年将大量支持水声场的统一理论、水声学逆问题研究、超声逆散射和超声成像、汉语“人一机”语音对话系统工程、被动测距与目标识别等五大课题。

三、主要科研设备和仪器

实验室由信号处理中心和水声、空气声、语声、超声等分实验室组成。信号处理中心由 VAX8350、VAX750、VAX3500、MicroVAXⅡ、SUN4、SUN3 等构成计算机网络系统、并加入中国科学院院网，与中关村地区 30 多个研究所实现了计算机软硬件资源共享。这个系统与多种记录、测量仪器、信号分析仪、谱分析仪相连，利用有限元分析、数学分析和多种图形显示软件，可以处理现场采集的多种模拟、数字信号。

实验室设有消声室、半消声室、混响室、测量水池、实验水槽、固体声场模拟实验装置以及相应的高精度座标装置与声学测量系统，可以进行从次声到超声频率范围内的固体、空气和水中声场的实验研究与计算机模拟。

四、交流合作与人才培养

国家实验室为了贯彻“开放、流动、联合”的方针，广泛地开展了国内外的学术交流与合作。实验室与美、加、法、俄等国的研究院所签订了一系列的合作协议。在“大洋远程声传播、海洋声学层析、海底声学识别、海洋声学仪器”等方面进行合作研究，并支持了南京大学、同济大学、西北工业大学、北京理工大学、北方交通大学、哈尔滨船舶工程学院、中船总715所、中科院武汉物理所等单位的一些研究课题。尤其是利用实验室自身优势，每年组织一、二次大型的浅海或深海海上实验。

在人才培养方面，实验室已培养了43名硕士生，9名博士生。目前在读的研究生12名，50余名客座人员来实验室从事研究工作。

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SPATIAL COHERENCE IN MULTIPATH UNDERWATER ACOUSTIC CHANNELS

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I. INTRODUCTION

Whether the sound wave propagates in the deep sea or in shallow water, there are always many paths from the source to receiver because of the refraction caused by the inhomogeneity of sea–water and the reflection from boundaries. Therefore, the multipath transmission is an important characteristic of underwater acoustic channels[1,2].

The multipath transmission causes the waveform distortion and time spread of signals, and then results in coherence loss of acoustic field[3,4]. For a moving source or recriver, since the signals through different paths have different Doppler frequency–shifts, the multipath propagation makes the time and frequency coherence decrease[5,6]. As the phase and group velocities for different paths are different, the multipath propagation may cause severe bearing error for a time–delay compensated receiver array[4,7].

In general, the horizontal transverse correlation of acoustic field is mainly determined by the random fluctuation of medium, the vertical correlation is chiefly determined by the multipath transmission, and the horizontal longitudinal correlation is dependent on both the causes mentioned–above. This paper mainly discusses the vertical and longitudinal correlations due to the multipath transmission.

II. SPATIAL COHERENCE THEORY

Assumming that the ocean channel is a horizontally stratified medium, the acoustic field of a hormonic point source may be approximately expressed as

$$P(z_1,z_2,r)=\sqrt{\frac{8\pi}{r}}\sum_l \Psi_l(z_1)\Psi_l(z_2)\sqrt{\mu_l}\exp(i\mu_l r-\beta_l r+i\pi/4), \tag{1}$$

where z_1 and z_2 are the source and receiver depths, μ_l and β_l are the real and imaginary parts of a mode eigenvalue respectively, and $\Psi_l(z)$ is the eigenfunction. In this paper, the generalized phase–integral (WKBZ) approximation[8] is used to calculate the eigenfunction, i.e.,

$$\Psi_l(z)=\sqrt{\frac{2}{S_l}}\begin{cases}\dfrac{\exp(-\int_z^{\eta_l}\sqrt{\mu_l^2-k^2(y)}\ dy)}{\sqrt{2}[Eb^{2/3}(z)+4\mu_l^2-4k^2(z)]^{1/4}}, & z<\eta_l\\[2ex] \dfrac{\sin(\int_{\eta_l}^{z}\sqrt{k^2(y)-\mu_l^2}\ dy+\pi/4)}{[Eb^{2/3}(z)+k^2(z)-\mu_l^2]^{1/4}}, & \eta_l<z<\zeta_l\\[2ex] \dfrac{(-1)^l\exp(-\int_{\zeta_l}^{z}\sqrt{\mu_l^2-k^2(y)}\ dy)}{\sqrt{2}[Eb^{2/3}(z)+4\mu_l^2-4k^2(z)]^{1/4}}, & \zeta_l<z\ ,\end{cases} \tag{2}$$

where $k(z)=\omega/c(z), E=0.875$, $b(z)=|dk^2(z)/dz|$, S_l is the cycle distance of an eigenray, and η_l and ζ_l are the upper and lower turning depths of an eigenray[8].

The spatial correlation $\Gamma(z_1, z_2, r; d, l)$ and normalized correlation coefficient $\gamma(z_1, z_2, r; d, l)$ of acoustic field respectively are defined as

$$\Gamma(z_1,z_2,r;d,l)=\left|\overline{P(z_1,z_2,r)P^*(z_1,z_2+d,r+l)}\right| \tag{3}$$

$$\gamma(z_1,z_2,r;d,l)=\frac{\left|\overline{P(z_1,z_2,r)P^*(z_1,z_2+d,r+l)}\right|}{\left[\overline{|P(z_1,z_2,r)|^2}\,\overline{|P(z_1,z_2+d,r+l)|^2}\right]^{1/2}} \tag{4}$$

where d and l are the vertical and longitudinal seperations between two receivers respectively, and the overbar '———' denotes the range and depth–average[8].

Substituting Eqs.(1) and (2) into Eqs.(3) and (4), one get the intergral expressions of spatial correlation and correlation coefficient as follws:

$$\Gamma(z_1,z_2,r;d,l)=\left|\int_{\alpha_{oc}}^{\pi/2}G(\alpha_0)\cos(k_2 d\sin\alpha_2)e^{-ik_2 l\cos\alpha_2}\ d\alpha_0\right|, \tag{5}$$

$$\gamma(z_1,z_2,r;d,l)=\Gamma(z_1,z_2,r;d,l)/\int_{\alpha_{oc}}^{\pi/2}G(\alpha_0)d\alpha_0, \tag{6}$$

where $\alpha_{oc}=\cos^{-1}[\max(k_1,k_2)/k_0]$, α_0 is the grazing angle at the depth with the minimum velocity, α_1 and α_2 are the grazing angles at the source and receiver depths, respectively. The function $G(\alpha_0)$ is given by

$$G(\alpha_0)=\frac{2k_0^2}{rk_1k_2}\frac{\sin(2\alpha_0)\exp[-2\beta(\alpha_0)r]}{S(\alpha_0)[D(z_1)+\sin^2\alpha_1]^{1/2}[D(z_2)+\sin^2\alpha_2]^{1/2}}, \tag{7}$$

where $D(z) = 0.875\left|\frac{1}{\pi f}\frac{dc(z)}{dz}\right|^{2/3}$

The expression (5) of spatial correlation has clear physical meaning: the function $G(\alpha_0)$ denotes the angular energy density, and then the spatial correlation is the weighted integral of spatial correlations of local plane-waves. The formula shows that the spatial correlation is relative to the propagation condition, range, source and receiver depths, frequency and seperation between receivers. It is not difficult to see from Eqs.(5) and (6) that the longitudinal correlation has reciprocity, while the vertical one has not.

III. NUMERICAL SIMULATIONS AND EXPERIMENTAL RESULTS

We have used Eq.(6) to make a great number of numerical simulations, Fig.1 is an example. In Fig.1 are shown the vertical and longitudinal correlation coefficients versus the receiver depth in a shallow water with thermocline, where $f = 1$kHz, $z_1 = 10$m, $d = 1.5$ m, $l = 50$m, and $r = 5$, 10, 20, 30, 40Km. It can be seen from Fig.1 that (1) the correlations have obvious depth and range dependence, (2) the correlations received above the thermocline are obviously greater than those below the thermocline when the source is above the thermocline, (3) the more distant the range, the stronger the correlation, and (4) the longitudinal correlation is much stronger than the vertical one.

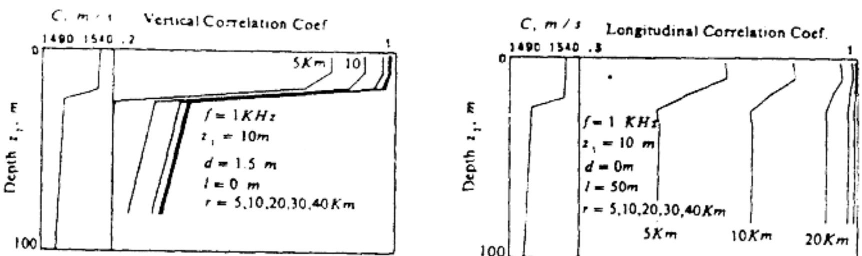

Fig 1(a)   Fig.1(b)

Fig.1 Vertical and longitudinal correlation coefficients versus receiver depth

In Fig.2 are shown the measured results of longitudinal correlations in a shallow water with thermocline, where the source depth is 7 m, the receiver depths respectively are 7 and 25 m, the range is 7.6 km, the longitudinal seperation is 50 m, the signal is the narrow-band one with the carrier frequency of 630 Hz and the band of 1 / 3 Oct. The mearsured results show that the correlation coefficient for the source and receiver depths of 7 m is 0.86, while that for the source depth of 7m and the receiver depth of 25m is 0.75, the former is obviously greater than the later.

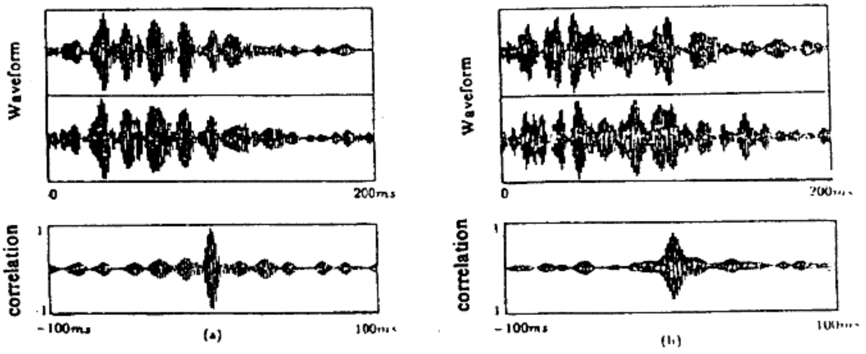

Fig.2 Longitudinal correlations at 7.6 km; f=630Hz, z_1 = 7m, d=0, l=50m, (a)z_2 = 7m, (b)z_2 = 35m.

IV. CONCLUSIONS

(1) The longitudinal correlation in multipath channels has reciprocity, while the vertical one has not.

(2) The longitudinal correlation is much stronger than the vertical one.

(3) In shallow water with thermocline the spatial correlations for both the source and receiver above the thermocline are greater than those for the source above and the receiver below the thermocline.

(4) The longitudinal and vertical correlations increase with increase of range.

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A RAY–MODE THEORY OF SURFACE–GENERATED AMBIENT NOISE IN THE SEA

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The sea ambient noise is a kind of noise backgrounds in underwater acoustical signal detection, it can reduce the performance of information receiving in the sea. Otherwise, the principal oceanological、 biological and meteorological informations are contained within ambient noise. Therefore, the environmental parameters of the sea may be abstracted by using inverse methods for research the ambient noise field[1,2].

Interaction of wind and sea–surface is one of the principal causes in generating of the sea ambient noise. If we are not consider the mechanism of generation of the ambient noise, the investigation on the ambient noise may be considered as a problem of the sound transmission in the sea caused by surface distributed noise source. In this paper the following assumptions are made:

1. The statistically independent directional sound sources are uniformly distributed on the surface[3].

2. The sea is assumed to be a stratified medium.

I. INTENSITY, SPATIAL CORRELATION AND DIRECTIONAL DENSITY FUNCTION OF AMBIENT NOISE

According to the ray–mode theory of ocean acoustics, by using the WKBZ method to solve one–dimensional wave equation, taking the smooth–average over rang and depth and by ensemble–averaging, the correlation function can be obtained as follows:

$$\langle P(A)P^*(B)\rangle = 2\pi\sigma \int^{\pi/2} \frac{\cos\alpha\sin\alpha\gamma^2(\alpha_s)J_0(k\rho\cos\alpha)\cos(kd\sin\alpha - i\ln V_b)d\alpha}{V_s(\beta+a)S(\alpha)[D(0)+\sin^2\alpha_s]^{1/2}[D(z)+\sin^2\alpha]^{1/2}}, \tag{1}$$

where $k = k(z)$, $\alpha_s = \arccos(k\cos\alpha / k(0))$, $\alpha_b = \arccos(k\cos\alpha / k(h))$,

$$D(z) = 0.875\left|\frac{2}{\omega}\frac{dc}{dz}\right|^{2/3}$$

The characteristics of ambient noise may be discussed from Eq.(1) as following.

1. DIRECTIONAL DENSITY FUNCTION $N(\alpha,Z)$

By using Euler's formula, equation (1) can be written as

$$\langle P(A)P^*(B)\rangle = \int_{-\pi/2}^{\pi/2} 2\pi\cos\alpha N(\alpha,z)J_0(k\rho\cos\alpha)e^{ikd\sin\alpha}\,d\alpha, \tag{2}$$

in which $N(\alpha,z)$is the dirtectional density function obtained by ray–mode theory, it is equal to

$$N(\alpha,z) = \frac{\sigma\sin|\alpha|\gamma^2(\alpha_s)}{2V(\beta+a)S(\alpha)[D(0)+\sin^2\alpha_s]^{1/2}[D(z)+\sin^2\alpha]^{1/2}}\begin{cases}1 & \alpha<0\\ V_b^2 & \alpha>0\end{cases} \tag{3}$$

when the frequency is low and the grazing angle α is great enough, Eq.(3) can be simplified as

$$N(\alpha,z) = \frac{1-V^2}{2V|\ln V|}N_r(\alpha,z), \tag{4}$$

where the function $N_r(\alpha,\ z)$ is the directional density function deduced by the ray theory, it may be shown to be[2]

$$N_r(\alpha,z) = \frac{\sigma\gamma^2(\alpha_s)}{(1-V^2)\sin\alpha_s}\begin{cases}1 & \alpha<0\\ V_b & \alpha>0\end{cases} \tag{5}$$

The factor $(1-V^2)/(2V|\ln V|)$ in Eq.(4) is not great than 1 dB. It will be shown that the function $N(\alpha,z)$ obtained by ray–mode theory can be applied to large grazing angles.

2. SPATIAL CORRELATION COEFFICIENT $\Gamma(d,\rho;z)$

According to the definition of spatial correlation coefficient

$$\Gamma(d,\rho;z) = \frac{\langle P(A)P^*(B)\rangle}{[\langle P(A)P^*(A)\rangle\langle P(B)^*(B)\rangle]^{1/2}} \tag{6}$$

As a result, we get from Eq.(1)

$$\Gamma(d,\rho;z) = \frac{\int_{-\pi/2}^{\pi/2}\cos\alpha N(\alpha,z)J_0(k\rho\cos\alpha)e^{ikd\sin\alpha}\,d\alpha}{\int_{-\pi/2}^{\pi/2}\cos\alpha N(\alpha,z)d\alpha} \tag{7}$$

3. INTENSITY OF THE AMBIENT NOISE

Let $d=\rho=0$, we get the intensity from Eq.(1)

$$I(z) = \pi\sigma\int^{\pi/2}\frac{(1+V_b^2)\cos\alpha\sin\alpha\gamma^2(\alpha_s)}{V(\beta+a)S(\alpha)[D(0)+\sin^2\alpha_s]^{1/2}[D(z)+\sin^2\alpha]^{1/2}}\,d\alpha \tag{8}$$

II. NUMERICAL RESULTS

1. SHALLOW WATER WITH THERMOCLINE

The sound speed above and below the thermocline are taken as 1530 m / s and 1500 m / s, respectively. The thickness of the thermocline layer is 5 m. $V_s = 1$, $V_b = 0.4$, and $a = 0$. The noise sources distributed on the sea surface are assumed to be dipole, i. e., $\gamma(\alpha_s) = \sin\alpha_s$. We calculated the directional density function $N(\alpha,z)$ from Eq.(3). Figure 1 shows the dependence of the function $N(\alpha,z)$ on the grazing angle α. It is seen from Fig.1 that the function $N(\alpha,z)$ above and below the thermocline differ from each other, the value of function $N(\alpha,z)$ below the thermocline will equal to zero when $|\alpha| < \alpha_c$.

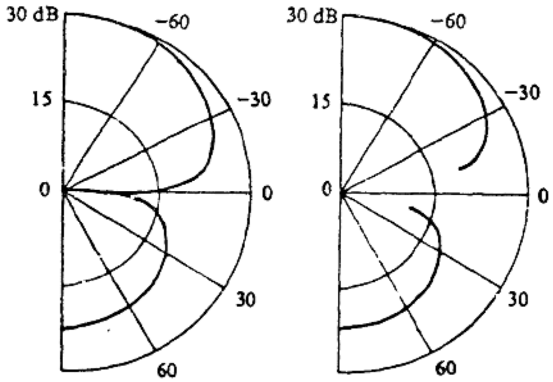

Fig.1 Directional density function $N(\alpha,z)$ of surface –generated noise in shallow water with thermocline.
a. Above thermocline b. Below thermocline

2. UNDERWATER SOUND CHANNEL WITH BILINEAR PROFILE

We assume that the axis of the sound channel is located at the depth above 1000 m, the gradients of sound speed above and below the axis are equal to $-4\times 10^{-5}\mathrm{m}^{-1}$ and $1\times 10^{-5}\mathrm{m}^{-1}$, respectively, $c(1000) = 1480$ m / s. The water depth is 6000 m. The dependence of the noise intensity I(z) on the depth z is obtained from Eq.(8), it is shown in Fig.2.

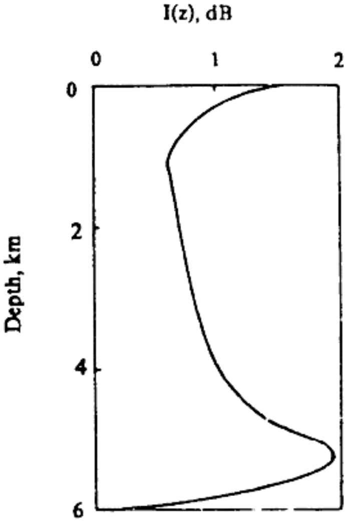

Fig.2]Noise intensity $I(z)$ versus depth z in a underwater sound channel with bilinear profile.

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THE WKBZ ADIABATIC MODE APPROACH TO SOUND PROPAGATION IN GRADUALLY RANGE–DEPENDENT CHANNELS

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I. INTRODUCTION

In most regions of the ocean the vertical gradient of sound velocity is about a thousand times of the horizontal one, so the ocean is often considered as a plane–stratified medium[1]. However, in the area of the convergence of cold and warm currents or in the regions with synoptic eddies the horizontal variation of sound velocity can not be neglected. Even in the area of the ocean with a little horizontal variation the influence of the horizontal variation on the long–distance propagation must be taken into account due to accumulative effect.

Many methods for calculating the acoustic field in the ocean with horizontal variation have been developed, such as parabolic equation method[2], harizontal ray theory[3], Gaussian beam approach[4] and so on. A WKBZ mode approach is proposed in Ref.5, and it can be used for fast and accurate calculation of the field of waveguide modes in a horizontal channel. In this paper, a WKBZ adiabatic mode approach is developed, and the approach may be used for gradually range–dependent channels and it takes the bottom interaction into account.

II. WKBZ ADIABATIC MODE THEORY

Assume that the ocean medium varies with horizontal range so gradually that the mode coupling may be neglected, resulting in the "adiabatic approximation". Under the condition of adiabatic approximation, the acoustic field of a hormonic point source may be expressed as[6]

$$P(r,z_1,z_2)=\sqrt{\frac{8\pi}{r}}\,e^{i\pi/4}\sum_l \psi_l(z_1,0)\psi_l(z_2,r)[v_l(0)/\sqrt{v_l(r)}]\times\exp[i\int_0^r v_l(x)dx] \qquad (1)$$

where $v_l(0)$ and $v_l(r)$ are the "local" eigenvalues of modes, $\psi_l(z_1,0)$ and $\psi_l(z_2,r)$ are the "local" eigenfunctions of modes, respectively at the source and receiver positions.

For definiteness, we suppose that the velocity near the sea–bottom is greater than that