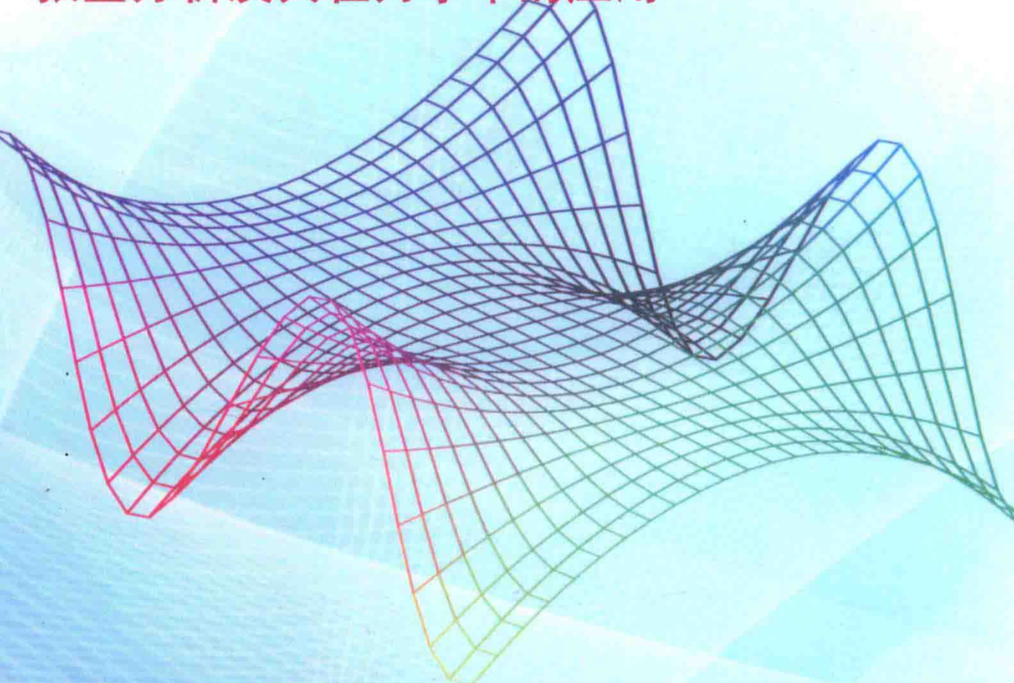
Tensor Analysis with Applications in Mechanics

张量分析及其在力学中的应用



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Foreword

Every science elaborates tools for the description of its objects of study. In classical mechanics we make extensive use of vectorial quantities: forces, moments, positions, velocities, momenta. Confining ourselves to a single coordinate frame, we can regard a vector as a fixed column matrix. The definitive trait of a vector quantity, however, is its objectivity; a vector does not depend on our choice of coordinate frame. This means that as soon as the components of a force are specified in one frame, the components of that force relative to any other frame can be found through the use of appropriate transformation rules.

But vector quantities alone do not suffice for the description of continuum media. The stress and strain at a point inside a body are also objective quantities; however, the specification of each of these relative to a given frame requires a square matrix of elements. Under changes of frame, these elements transform according to rules different from the transformation rules for vectors. Stress and strain tensors are examples of tensors of the second order. We could go on to cite other objective quantities that occur in the mechanics of continua. The set of elastic moduli associated with Hooke's law comprise a tensor of the fourth order; as such, these moduli obey yet another set of transformation rules. Despite the differences that exist between the transformation laws for the various types of objective quantities, they all fit into a unified scheme: the theory of tensors.

Tensor theory not only relieves our memory from a huge burden, but enables us to carry out differential operations with ease. This is the case even in curvilinear coordinate systems. Through the unmatched simplicity and brevity it affords, tensor analysis has attained the status of a general language that can be spoken across the various areas of continuum physics. A full comprehension of this language has become necessary for those working in electromagnetism, the theory of relativity, or virtually any other field-theoretic discipline. More modern books on physical subjects invariably contain appendices in which various vector and tensor identities are listed. These may suffice when one wishes to verify the steps in a development, but can leave one in doubt as to how the facts were established or, *a fortiori*, how they could be adapted to other circumstances. On the other hand, a comprehensive treatment of tensors (e.g., involving a full excursion into multilinear algebra) is necessarily so large as to be flatly inappropriate for the student or practicing engineer.

Hence the need for a treatment of tensor theory that does justice to the subject and is friendly to the practitioner. The authors of the present book have met these objectives with a presentation that is simple, clear, and sufficiently detailed. The concise text explains practically all those formulas needed to describe objects in three-dimensional space. Occurrences in physics are mentioned when helpful, but the discussion is kept largely independent of application area in order to appeal to the widest possible audience. A chapter on the properties of curves and surfaces has been included; a brief introduction to the study of these properties can be considered as an informative and natural extension of tensor theory.

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(1920–2001)

Preface

The first edition of this book was written for students, engineers, and physicists who must employ tensor techniques. We did not present the material in complete generality for the case of n-dimensional space, but rather presented a three-dimensional version (which is easy to extend to n dimensions); hence we could assume a background consisting only of standard calculus and linear algebra.

We have decided to extend the book in a natural direction, adding two chapters on applications for which tensor analysis is the principal tool. One chapter is on linear elasticity and the other is on the theory of shells and plates. We present complete derivations of the equations in these theories, formulate boundary value problems, and discuss the problem of uniqueness of solutions, Lagrange's variational principle, and some problems on vibration. Space restrictions prohibited us from presenting an entire course on mechanics; we had to select those questions in elasticity where the role of tensor analysis is most crucial.

We should mention the essential nature of tensors in elasticity and shell theory. Of course, to solve a certain engineering problem, one should write things out in component form; sometimes this takes a few pages. The corresponding formulas in tensor notation are quite simple, allowing us to grasp the underlying ideas and perform manipulations with relative ease. Because tensor representation leads quickly and painlessly to component-wise representation, this technique is ideal for presenting continuum theories to students.

The first five chapters are largely unmodified, aside from some new problem sets and material on tensorial functions needed for the chapters on elasticity. The end-of-chapter problems are supplementary, whereas the integrated exercises are required for a proper understanding of the text.

In the first edition we used the term *rank* instead of *order*. This was common in the older literature. In the newer literature, the term "rank" is often assigned a different meaning.

Because the book is largely self-contained, we make no attempt at a comprehensive reference list. We merely list certain books that cover similar material, that extend the treatment slightly, or that may be otherwise useful to the reader.

We are deeply grateful to our World Scientific editor, Mr. Tjan Kwang Wei, for his encouragement and support.

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Preface to the First Edition

Originally a vector was regarded as an arrow of a certain length that could represent a force acting on a material point. Over a period of many years, this naive viewpoint evolved into the modern interpretation of the notion of vector and its extension to tensors. It was found that the use of vectors and tensors led to a proper description of certain properties and behaviors of real natural objects: those aspects that do not depend on the coordinate systems we introduce in space. This independence means that if we define such properties using one coordinate system, then in another system we can recalculate these characteristics using valid transformation rules. The ease with which a given problem can be solved often depends on the coordinate system employed. So in applications we must apply various coordinate systems, derive corresponding equations, and understand how to recalculate results in other systems. This book provides the tools necessary for such calculation.

Many physical laws are cumbersome when written in coordinate form but become compact and attractive looking when written in tensorial form. Such compact forms are easy to remember, and can be used to state complex physical boundary value problems. It is conceivable that soon an ability to merely formulate statements of boundary value problems will be regarded as a fundamental skill for the practitioner. Indeed, computer software is slowly advancing toward the point where the only necessary input data will be a coordinate-free statement of a boundary value problem; presumably the user will be able to initiate a solution process in a certain frame and by a certain method (analytical, numerical, or mixed), or simply ask the computer algorithm to choose the best frame and method. In this way, vectors and tensors will become important elements of the macro-language for the next generation of software in engineering and applied mathematics.

We would like to thank the editorial staff at World Scientific — especially Mr. Tjan Kwang Wei and Ms. Sook-Cheng Lim — for their assistance in the production of this book. Professor Byron C. Drachman of Michigan State University commented on the manuscript in its initial stages. Lastly, Natasha Lebedeva and Beth Lannon-Cloud deserve thanks for their patience and support.

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Contents

PART 1

Tensor Analysis

Chapter 1

Preliminaries

1.1 The Vector Concept Revisited

The concept of a vector has been one of the most fruitful ideas in all of mathematics, and it is not surprising that we receive repeated exposure to the idea throughout our education. Students in elementary mathematics deal with vectors in component form — with quantities such as

$$\mathbf{x} = (2, 1, 3)$$

for example. But let us examine this situation more closely. Do the components $2, 1, 3$ determine the vector $\mathbf{x}$? They surely do if we specify the basis vectors of the coordinate frame. In elementary mathematics these are supposed to be mutually orthogonal and of unit length; even then they are not fully characterized, however, because such a frame can be rotated. In the description of many common phenomena we deal with vectorial quantities like forces that have definite directions and magnitudes. An example is the force your body exerts on a chair as you sit in front of the television set. This force does not depend on the coordinate frame employed by someone writing a textbook on vectors somewhere in Russia or China. Because the vector $\mathbf{f}$ representing a particular force is something objective, we should be able to write it in such a form that it ceases to depend on the details of the coordinate frame. The simplest way is to incorporate the frame vectors $\mathbf{e}_i$ $(i = 1, 2, 3)$ explicitly into the notation: if $\mathbf{x}$ is a vector we may write

$$\mathbf{x} = \sum_{i=1}^{3} x_i \mathbf{e}_i. \tag{1.1}$$

Then if we wish to change the frame, we should do so in such a way that $\mathbf{x}$ remains the same. This of course means that we cannot change only the frame vectors $\mathbf{e}_i$: we must change the components x_i correspondingly. So the components of a vector $\mathbf{x}$ in a new frame are not independent of those in the old frame.

1.2 A First Look at Tensors

In what follows we shall discuss how to work with vectors using different coordinate frames. Let us note that in mechanics there are objects of another nature. For example, there is a so-called tensor of inertia. This is an objective characteristic of a solid body, determining how the body rotates when torques act upon it. If the body is considered in a Cartesian frame, the tensor of inertia is described by a 3×3 matrix. If we change the frame, the matrix elements change according to certain rules. In textbooks on mechanics the reader can find lengthy discussions on how to change the matrix elements to maintain the same objective characteristic of the body when the new frame is also Cartesian. Although the tensor of inertia is objective (i.e., frame-independent), it is not a vector: it belongs to another class of mathematical objects. Many such *tensors of the second order* arise in continuum mechanics: tensors of stress, strain, etc. They characterize certain properties of a body at each point; again, their "components" should transform in such a way that the tensors themselves do not depend on the frame. The precise meaning of the term *order* will be explained later.

For both vectors and tensors we can introduce various operations. Of course, the introduction of any new operation should be done in such a way that the results agree with known special cases when such familiar cases are met. If we introduce, say, dot multiplication of a tensor by a vector, then in a Cartesian frame the operation should resemble the multiplication of a matrix by a column vector. Similarly, the multiplication of two tensors should be defined so that in a Cartesian frame the operation involves matrix multiplication. To this end we consider *dyads* of vectors. These are quantities of the form

$$\mathbf{e}_i \mathbf{e}_j .$$

A tensor may then be represented as

$$\sum_{i,j} a_{ij} \mathbf{e}_i \mathbf{e}_j$$

where the a_{ij} are the components of the tensor. We compare with equation (1.1) and notice the similarity in notation.