

# ELECTRICAL MEASUREMENTS

AND THE CALCULATION  
OF THE ERRORS INVOLVED

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## *Part II*

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BY

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## AUTHOR'S PREFACE

As all research and development work is based on accurate measurement and a knowledge of the errors involved, the author of this volume, as well as in Part I, has set himself a threefold task :

Firstly to cover the syllabus of the University of London Examination B.Sc.Eng. in Electrical Measurements and Measuring Instruments ; secondly to show how errors can be calculated, what methods to use for a particular measurement and the conditions for which a method will give the best results ; thirdly to provide a Laboratory Manual giving proved methods of measurement.

The author hopes he has had some success in this effort and will be grateful for further suggestions.

The chapter on a.c. bridges gives all the bridges that are really necessary, sufficient for any work for which an a.c. bridge is required. The author has judged it unnecessary to explain bridges which, although useful, can be done without, considering that the measurement can be done as well, if not better, on simpler bridges. The subject of Electrical Measurements is extensive enough as it is.

The book has no bibliography because part of it is the author's original work and the remainder, which is classical, is based on the lectures given by the author to B.Sc.Eng. students at the Northampton Polytechnic, London, E.C.1.

The original work of the author in this book is as follows : The calculation of the error made when assuming an impedance to be a four- or two-terminal network ; division of laboratory equipment ; division of frequency ; some tests on the C.R.O. ; some calculations relating to Lissajous figures ; calculations relating to the limit of frequency of potentials applied to the deflecting plates in a C.R.O. ; calculation of residuals and subsequent errors in bridges ; calculation of bridge sensitivity ; calculation of determination and general errors in bridges ; calculation of errors in Tuttle's circuits ; and finally, calculation of errors in a.c. potentiometers.



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direction of travel, or traverse around the loop. If when going around the loop in the direction of the traverse a generator is entered through its negative terminal, then the p.d. across the generator terminals or its e.m.f. is entered with a plus sign in equation (2); if the generator is entered through its positive terminal, then its p.d. or e.m.f. is entered in (2) with a minus sign. The sign of the p.d.s across an inductance, capacity or resistance will be as already explained.

The e.m.f. induced in the loop or part of it by a mutual inductance is entered in (2) with a plus sign if this e.m.f. produces or tends to produce a current in the direction of the traverse; otherwise it is entered with a minus sign.

If, after having calculated the currents (by means of the two laws) in the branches of the loop, some or all of the currents are found to appear with a minus sign, it simply means that at the instant considered their direction is in opposition to the arbitrary arrows drawn when the circuit was traced out.

*Example 1.* Consider fig. 1. Let the arbitrarily chosen directions of the currents be as shown by the arrows and let the p.d. across the source terminals be  $v = f(t)$  (that is a function of time). The polarity of the source is as shown at the instant considered.

There are four loops in the circuit (shown by closed arrows) ·

1. through  $baAFDb$
2. through  $ABFA$
3. through  $BDFB$
4. through  $baABDb$

Let the directions of the chosen traverses around the loops be as shown by the closed arrows. We have then for the first loop (entering the source through its negative terminal)

$$v - L \frac{di_5}{dt} - \int \frac{i_4 dt}{C} = 0; \quad v = L \frac{di_5}{dt} + \int \frac{i_4 dt}{C} \text{ (assuming } L \text{ not to have any resistance).}$$

For the second loop

$$-R_2 i_2 - R_6 i_6 + L \frac{di_5}{dt} = 0; \quad L \frac{di_5}{dt} = R_2 i_2 + R_6 i_6$$

For the third loop

$$-R_3 i_3 + \int \frac{i_4 dt}{C} + R_6 i_6 = 0; \quad \int \frac{i_4 dt}{C} = R_3 i_3 - R_6 i_6$$

For the fourth loop

$$v - R_2 i_2 - R_3 i_3 = 0; \quad v = R_2 i_2 + R_3 i_3$$

(c) KIRCHHOFF'S LAWS APPLIED TO VECTORS. Kirchhoff's laws can be applied to vectors only if the voltages and currents follow sinusoidal