THE DYNAMICS OF THE UPPER OCEAN

O. M. PHILLIPS, F.R.S.

SECOND EDITION

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PREFACE

The ten years since the publication of the first edition of this book have seen many important advances in our understanding of the dynamical processes occurring in the upper ocean. For example, the clarification of the action conservation principle for waves in moving media provides a powerful new technique, widely applicable to oceanographic situations. Resonant wave interactions are now seen to be involved in energy exchanges not only among surface waves (the context in which they were first studied) but in many other situations as well. Beautifully conceived and skilfully executed measurements, particularly on oceanic internal waves, abound where previously there were almost none – what was tentative ten years ago seems now either naive or erroneous. New phenomena have been encountered and more profound questions asked.

The revisions demanded by these advances have been substantial, but the general scope of the book remains the same. The inclusion or exclusion of a particular topic is not only a matter of taste but of the breadth of the ramifications involved and of the alternative sources available. Excellent recent monographs by Turner and Kitaigorodskii describe respectively convective motions and observational aspects of air–sea interactions. The range of phenomena influenced by bottom topography is extensive and its literature is expanding so rapidly that to give it even scant justice would require more space than the present book allows.

Once again, to acknowledge all those who, directly or indirectly, have contributed to this book would be to acknowledge all my colleagues working in this area. Nevertheless, I am particularly indebted to Professor M. S. Longuet-Higgins and to Professor F. P. Bretherton for the many stimulating discussions we have enjoyed, to Mr D. Irvine for his critical reading of the manuscript of this edition and to Ms S. Burke and Ms J. Koutz for their skilful work in its preparation. Finally, it is a pleasure to acknowledge the U.S. Office of Naval Research for its patient support.

Baltimore
March 1976

O.M.P.

FROM THE PREFACE TO THE FIRST EDITION

A proper understanding of the dynamical processes that occur in the upper ocean is crucial to much of physical oceanography. During the last ten years, close contact between theory on the one hand and observation and experiment on the other has resulted in many important advances in this area. Some of these are described in papers scattered among the various journals and their connexions are not immediately apparent; others have not hitherto been published. The time seems particularly opportune, then, to attempt to provide a coherent account of the recent developments in this subject in the hope of making them accessible to a greater number of oceanographers and, at the same time, of stimulating the interest of other workers in this rewarding and fertile field of geophysical fluid mechanics. In some aspects, our understanding is already fairly detailed, but in others we are handicapped by the lack of either exploratory or definitive experiments, so that theoretical studies are necessarily rather tentative and subject to revision. Where this is so, I have tried not to conceal the fact, in the hope of stimulating others to remedy the deficit. In any event, the success or failure of this monograph will, I suppose, be measured by the degree to which it provides the basis for future developments or, possibly, dissent.

An earlier draft was awarded the Adams Prize for the years 1963–64 by the University of Cambridge.

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CHAPTER 1

INTRODUCTION

1.1. The ocean environment

From the beginnings of history, the sea has excited our fascination and respect. Whether Phoenician or Polynesian, Arab, Viking or Medieval European, the lore of the sailor and his achievements have contributed to the streams of our culture; they have provided recurrent themes in painting, in literature and in music. Today, as in time past, the sea continues to attract or repel those who know her; few are neutral.

Many generations of navigators had wondered about the ocean depths beneath them, and their ignorance bore apprehension. By the middle of the nineteenth century, the exploration of the ocean surface was virtually complete, but the depths remained unknown. One of the earliest of the great oceanographic expeditions was the circumnavigation of the earth by the *Challenger* in the eighteen-seventies. For the first time there was a systematic collection of data and specimens from not only the surface but the ocean depths as well. In later expeditions, ships with evocative names such as *Discovery*, *Meteor* and *Atlantis* made enduring contributions to the description of the world's oceans. These explorations continue – it is, for example, barely twenty years since Cromwell, Montgomery and Stroup established the existence of the Pacific equatorial undercurrent, by any account one of the largest oceanic current systems (Montgomery and Stroup, 1962). It is barely ten since systematic deep ocean bathymetry revealed the structure of rifts, fractures and trenches associated with sea floor spreading and led to the revolution in our conception of tectonic processes that is still only beginning. As a result of many such painstaking studies, our present knowledge of the oceans and of the basins wherein they are contained has accumulated.

In geographical terms, the sea bed can be divided into two or three quite distinct regions. Surrounding the continental shorelines, for a distance of the order of 100 or 200 km, the water is relatively shallow, averaging some 200 m in depth. The sea floor of this continental shelf usually undulates gently, rising occasionally to the surface in off-shore banks or islands. This is the region of most concern to the returning navigator and is the most accessible with his instruments; though the least homogeneous, it is the best known. The water above the continental shelf is moved by the tides and by estuarine currents; it can be stirred throughout its depth by the wind stress at the surface. Beyond this region, the sea floor falls away to the ocean depths beyond. The transition is remarkably abrupt, the overall slope being generally of the order 0.1, but frequently corrugated by submarine canyons where locally the slope is much larger. At the foot of the continental slope lies the deep ocean bed whose depth is of the order 4000 m and which lies beneath the great majority of the ocean surface area. This again is relatively flat, the abyssal plain being broken by an occasional sea-mount or trough or by a long mid-ocean ridge with its complex of fracture zones dividing offset segments of the ridge axis. The study of the motions at these great depths involves costly instrumentation and measurement over substantial intervals of space and time, supported by national or international endeavours such as the Mid-Ocean Dynamics Experiment (MODE) of 1973 and the more recent POLYMODE.

In the open ocean above the deep sea bed, the vertical structure of the water mass is characterized by the presence of one of more thermoclines, regions of large temperature gradient within the upper few hundred metres. The thermocline is strongest in equatorial waters, where the contrast between the warmed surface layer and the cold deeper water is greatest; at increasing latitudes it weakens, even vanishing in certain polar waters. The maintenance and structure of the thermocline are central questions in an understanding of the general oceanic circulation. In the upper ocean, from the thermocline region to the surface, are found the variety of inter-related motions, internal waves, turbulence and surface waves with whose dynamics we shall be concerned.

The motions in the upper ocean provide the means for the exchange of matter, momentum and energy between the atmosphere and the underlying deep ocean. On a global scale, these exchanges produce in the oceans the general circulation pattern and at the same time constitute one of the most important factors in the world-wide distribution of climate. A proper understanding of the behaviour of the upper ocean is at the heart of these larger questions, and to this ultimate end are our efforts directed.

1.2. The development of the subject

The threads to be woven into the fabric of this subject lead far back in time. They are of three kinds. There is first the observation of the phenomena of the upper ocean, the identification, description and measurement of the various modes of motion and dynamical processes to be found under natural, uncontrolled (but hopefully, measured) conditions. Secondly, there is the experimentation in the laboratory, where it may be possible to isolate one or two of these phenomena, and to study their properties and mutual interactions. Finally, there is the analysis of these motions and their interrelations, the development of a coherent theory. It is only by drawing these threads together in a consistent way that the pattern of the subject can be discerned and its texture be felt.

In each of the three major topics involved in the dynamics of the upper ocean this process can be followed. Casual observations on surface waves and their relation to the wind have been made for time immemorial, so that it was not unnatural that the pioneers of theoretical fluid mechanics, Lagrange, Airy, Stokes and Rayleigh, sought to account for the elementary properties of surface waves in terms of perfect fluid theory. Even simple experiments allowed the frequency–wavelength relations to be compared with this theory, and the results must have been encouraging. Nevertheless, surface waves in the ocean were less simple and provided a constant reproach to the elementary theory. Rayleigh wrote: 'The basic law of the seaway is the apparent lack of any law.' In the last century, the irregularity of ocean waves defied description – this had to await the developments in probability theory made during the first forty years of this century. The problem of relating the rate of wave growth to the wind was recognized by Kelvin, but no real progress was made. By 1850, Stevenson had made observations on surface waves in a number of lakes and derived an empirical relationship between the 'greatest wave height' and the fetch (distance from the windward shore). Seventy-five years later, Jeffreys attempted to model the generation of waves by wind in a laboratory experiment. But as recently as 1956, Ursell could reasonably write: 'Wind blowing over a water surface generates waves in the water by physical processes which cannot be regarded as known.' There was no even remotely adequate theory; the results of oceanic observation and laboratory experiment seemed flatly inconsistent. The threads were either missing or disjoint.

Ursell's pessimism provoked new efforts to develop a sound theory and to make well-documented experiments and observations. These attempts are described in chapter 4 herein; whatever their shortcomings it appears that theory and observation may at last be becoming mutually relevant.

Internal waves, on the other hand, are not the creatures of our common experience that surface waves are. The simplest solutions for waves on an internal interface were given by Stokes in 1847; their properties could again be illustrated by simple experiments. Ekman, writing in the record of Nansen's North Polar Expedition of 1893–6 (vol. v, 1904, 562), described some laboratory observations on the generation of internal waves by a slowly moving ship model. There had been numerous rather cursory indications of the presence of internal waves in the ocean, but even ten years ago our acquaintance with oceanic internal waves was so limited that each isolated measurement was rare, valuable and tantalizing. But imagination, indispensable though it is, offers a poor substitute for information. The development of ingenious new instruments, the careful design of experiments and more detailed data analysis have since revealed a variety of dynamical behaviour which was hardly conceived previously and which offers a continuing challenge to theory. In the laboratory the greatest contributions have undoubtedly been made by Long (1953*a*,*b*, 1954, 1955) and have been concerned with the general problem of the excitation of internal waves by the flow over irregularities in the bed. Other interesting and important experiments by Martin, Simmons and Wunsch (1972) and McEwan (1971) have demonstrated the energy transfer among modes by resonant interaction, and the degeneration of internal waves by the formation of local patches of turbulence. With some notable exceptions, early theoretical studies were confined to the finding of eigensolutions for particular density distributions, but the scintillating variety of new observations drew interest to the dynamical mechanisms underlying them. Even now, the strands have not quite drawn together as one would wish. The processes of internal-wave generation, wave–wave interaction, reflexion from a sloping bottom and wave degeneration are partially explored, but the ways in which they combine under oceanic conditions are still seen only dimly. This is a situation to be remedied by time and patient endeavour.

The application of our ideas about turbulence to specifically oceanographical contexts has taken place only fairly recently. In 1883, Osborne Reynolds published his celebrated account of laboratory observations on turbulent ('sinuous') flow, pointing out the statistical nature of the problem and the need for describing the motion in terms of its average properties. Later, under the stimulus given by the growth of aerodynamics, the mixing length concepts were developed, in which the motion of eddies was likened to that of discrete entities such as the molecules in a gas. A close collaboration between theory and experiment during this time revealed both the success (limited though it was) of these concepts, and more important, their shortcomings. Gradually, it became evident that before any real insight into the dynamics of turbulence could be gained, a more fundamental approach would be needed. This was ultimately given by G. I. Taylor in 1935 in a series of five papers to the Royal Society; from these has developed the modern theory of turbulence. The particular case of homogeneous turbulence allows some analytical simplifications, and at the same time is approximately realizable in laboratory wind tunnels. The penetration of this part of the subject by Batchelor, Kolmogorov, Kraichnan, Townsend and others has consequently been substantial, though the central problem – the statistical mechanics of the non-linear interactions – cannot yet be considered solved. None the less, a considerable insight was gained that is relevant and valuable in a consideration of other turbulent motions. At the same time, detailed experimental studies of turbulent shear flows were being made, particularly by Townsend, Laufer and Corrsin. These gave, if nothing else, a greater appreciation of the power and limitations of similarity methods in describing such flows.

As early as 1915, G. I. Taylor had made astute observations on the structure of atmospheric turbulence, but thirty years were to elapse before suitable instruments were available to allow a detailed and systematic investigation. Since the last war, this has been a fertile field for the application and extension of our understanding of turbulence, notable contributions having been made by groups in the Soviet Union and in Australia. Oceanic turbulence, on the other hand, is less accessible and (with some shining exceptions) virtually unexplored. But from a combination of our background experience with homogeneous and with atmospheric turbulence, the use of similarity reasoning together with an appreciation of the phenomena specific to the ocean (such as the presence of breaking waves) it is possible (in chapter 6) to make a number of inferences about the structure of oceanic turbulent motions. Some of these can already be compared with direct observations; others, not yet.

These three modes of motion do not, of course, occur in isolation; their mutual interactions are of interest. When these are correctly understood, there is hope that we may have some appreciation of the rôle of the upper ocean in the dynamics of the ocean as a whole. In the following pages, each of these modes is considered in turn. In chapter 3, the elementary properties of surface waves, their conservation laws and interaction characteristics are derived as simply as possible. These are basic to an understanding of the sometimes complex problems posed by a random field of ocean waves, its generation and decay. The structure of internal waves is then described, the emphasis being not with the analytical details of the mode structure under various particular conditions but rather with the kind of physical processes that may be associated with such motions. Finally, we turn to the mechanisms and phenomena involved in turbulence in the upper ocean and to the interactions between this and the other types of motion. But first, it is necessary to state briefly the relevant governing equations in the forms to be used throughout the book, and to this is the next chapter devoted.

CHAPTER 2

THE EQUATIONS OF MOTION

2.1. Specification of the motion

A fluid motion is usually specified in one of two ways. The descriptions are of course equivalent, the choice being a matter not of principle but of convenience.

In an Eulerian description of the motion, physical quantities such as the velocity $\boldsymbol{u}$, pressure p and density ρ are regarded as functions of position $\boldsymbol{x}$ and time t. Thus $\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x}, t)$ and $\rho = \rho(\boldsymbol{x}, t)$ represent the velocity and density of the fluid at prescribed points in space–time. The partial derivative with respect to $\boldsymbol{x}$ or t represents the gradient in the field at a given instant or the rate of change at a given point.

Alternatively, the fluid elements can be identified by their position $\boldsymbol{a}$ at some initial instant t_0, and the motion specified by the subsequent position and velocity of these fluid elements. This is usually known as a Lagrangian specification of the motion, the independent variables being the initial co-ordinates $\boldsymbol{a}$ and the elapsed time $t - t_0$. Thus

$$\boldsymbol{x} = \boldsymbol{x}(\boldsymbol{a}, t - t_0); \quad \boldsymbol{x}(\boldsymbol{a}, 0) = \boldsymbol{a}. \tag{2.1.1}$$

The velocity of a fluid element is the time derivative of its position:

$$\boldsymbol{u}(\boldsymbol{a}, t - t_0) = \frac{\partial}{\partial t}\boldsymbol{x}(\boldsymbol{a}, t - t_0), \tag{2.1.2}$$

so that

$$\boldsymbol{x} - \boldsymbol{a} = \int_{t_0}^{t} \boldsymbol{u}(\boldsymbol{a}, t - t_0)\, \mathrm{d}t. \tag{2.1.3}$$

The fluid acceleration is the time derivative $\mathrm{d}\boldsymbol{u}/\mathrm{d}t$ of the velocity of a fluid element as it moves in space. The total time derivative, or the derivative 'following the motion', can be expressed in Eulerian terms as

$$\frac{\mathrm{d}}{\mathrm{d}t}=\frac{\partial}{\partial t}+(\boldsymbol{u}\cdot\nabla), \tag{2.1.4}$$

the sum of the time rate of change at a fixed point and a convective rate of change.

Most of the dynamical problems discussed in this book are posed and solved more simply with an Eulerian specification. Many instruments, measuring fluid properties at a fixed point, provide Eulerian information directly. On the other hand, in questions of diffusion or mass transport, the motion of the fluid elements is of interest and a Lagrangian specification of the problem may be more natural. In observation, the marking of fluid elements by dye or other tracer gives Lagrangian information. It is sometimes difficult to relate the two specifications, particularly when the flow is unsteady; the general problem of transformation from one to the other in an arbitrary motion has, in fact, not been solved. However, in some particular motions, such as the surface waves described in chapter 3, the transformation is fairly simple and the Lagrangian characteristics can be inferred readily from the Eulerian solution.

In the upper ocean, many of the dynamical phenomena have length scales much smaller than the radius of the earth. For these, it is sufficient to use a system of orthogonal Cartesian co-ordinates (x, y, z) fixed with respect to the earth; the z-axis will be taken vertically upwards. When the notation of Cartesian tensors is used, the co-ordinates $x_i=(x_1, x_2, x_3)$ will be used interchangeably, with x_3 corresponding to z. The velocity components are accordingly (u, v, w) or (u_1, u_2, u_3). Sometimes, it is convenient to consider the horizontal velocity components separately from the vertical one; the notation $\boldsymbol{u}=(q_\alpha, w)$ being then used, with the Greek suffix α taking values 1, 2 so that $\boldsymbol{q}=(u, v)=(u_1, u_2)$.

2.2. The equations of motion

The motion of a fluid is governed by the conservation laws of mass and momentum, by the equation of state and the laws of thermodynamics. The first of these is the conservation of mass,

$$\frac{\mathrm{d}\rho}{\mathrm{d}t}+\rho\nabla\cdot\boldsymbol{u}=0. \tag{2.2.1}$$

In virtue of (2.1.4) this can be expressed alternatively as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0. \tag{2.2.2}$$

If the density of a fluid element does not change (though it may differ for different elements), (2.2.1) simplifies to

$$\nabla \cdot \boldsymbol{u} = 0. \tag{2.2.3}$$

The momentum equation, referred to axes at rest relative to the rotating earth, takes the form

$$\rho \frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} + \rho \boldsymbol{\Omega} \times \boldsymbol{u} + \nabla p - \rho \boldsymbol{g} = \boldsymbol{f}. \tag{2.2.4}$$

The first term represents the mass-acceleration and the second the Coriolis force, in which $\boldsymbol{\Omega}$ is the rotation vector, or twice the earth's angular velocity. Its magnitude

$$\Omega = |\boldsymbol{\Omega}| = 2\pi/12\ \mathrm{h}^{-1} = 1.46 \times 10^{-4}\ \mathrm{sec}^{-1},$$

is for the purposes of this book, constant. In the gravitational term, $\boldsymbol{g} = (0, 0, -g)$ represents the apparent gravitational acceleration, or the true (central) gravitational acceleration modified by the small ('centrifugal') contribution normal to the axis of the earth's rotation. The direction of $\boldsymbol{g}$ defines the local vertical; its magnitude varies throughout the ocean from its mean value of approximately 981 cm sec^{-2} by less than 0.3%, and for dynamical purposes it can be considered constant.

The term f on the right of equation (2.2.4) represents the resultant of all other forces acting on unit volume of the fluid. The most important of these arises from the molecular viscosity. In almost all oceanic circumstances where viscous effects are important, the water can be regarded as an isotropic, incompressible Newtonian fluid, and the stress tensor

$$p_{ij} = -p\delta_{ij} + 2\mu e_{ij}, \tag{2.2.5}$$

where δ_{ij} is the unit tensor ($\delta_{ij} = 1$ if $i = j$, and vanishes otherwise), μ is the viscosity of the fluid† and

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{2.2.6}$$

† For water at 10 °C, $\mu \sim 1.3 \times 10^{-2}$ g cm^{-1} sec^{-1}.

is the rate of strain tensor. The *frictional* force per unit volume is therefore

$$f_i = 2\mu\partial e_{ij}/\partial x_j = \mu\partial^2 u_i/\partial x_j^2, \qquad (2.2.7)$$

from the incompressibility condition (2.2.3).

If L is the differential length scale of a given motion in which the velocity varies in magnitude by U, the ratio $R = \rho UL/\mu$ (the Reynolds number) represents the relative magnitudes of the inertial and viscous terms in the momentum equation. In many oceanic motions, the Reynolds number is very large, and the viscous term is often quite negligible over most of the field of motion. Viscous forces can be important only in narrow regions of the flow, or in very small-scale motions where the velocity changes U are confined to distances of the order $\mu/\rho U$. In such regions, the local inertial and viscous forces are comparable; they are exemplified by the interfacial layer between the air and the water, by the regions of high shear associated with the smallest eddies of turbulence and by the small-scale motion of capillary waves on the ocean surface. From the analytical point of view, the occurrence of these local regions in many cases is characteristic of solutions of singular perturbation problems in which a small parameter (here R^{-1}, in dimensionless form) multiplies the term of highest order (the viscous term) in the governing equation.

Two alternative forms of the momentum equation (2.2.4) are of interest. If the continuity equation (2.2.2) is multiplied by u_i and added to (2.2.4), there results

$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_i u_j) + \varepsilon_{ijk}\rho\Omega_j u_k + \frac{\partial p}{\partial x_i} - \rho g_i = f_i, \qquad (2.2.8)$$

in the notation of Cartesian tensors. This expresses the force balance directly in terms of the rate of change of the momentum and the divergence of the momentum flux $\rho u_i u_j - p_{ij}$. Again, if $\boldsymbol{\omega} = \nabla \times \boldsymbol{u}$ is the vorticity of the fluid, the vector identity

$$\boldsymbol{\omega} \times \boldsymbol{u} = (\nabla \times \boldsymbol{u}) \times \boldsymbol{u} = \boldsymbol{u} \cdot \nabla \boldsymbol{u} - \nabla(\tfrac{1}{2}u^2)$$

enables (2.2.4) to be expressed as

$$\rho\frac{\partial \boldsymbol{u}}{\partial t} + \rho(\boldsymbol{\Omega} + \boldsymbol{\omega}) \times \boldsymbol{u} + \nabla p + \rho\nabla(\tfrac{1}{2}u^2) - \rho \boldsymbol{g} = \boldsymbol{f}. \qquad (2.2.9)$$