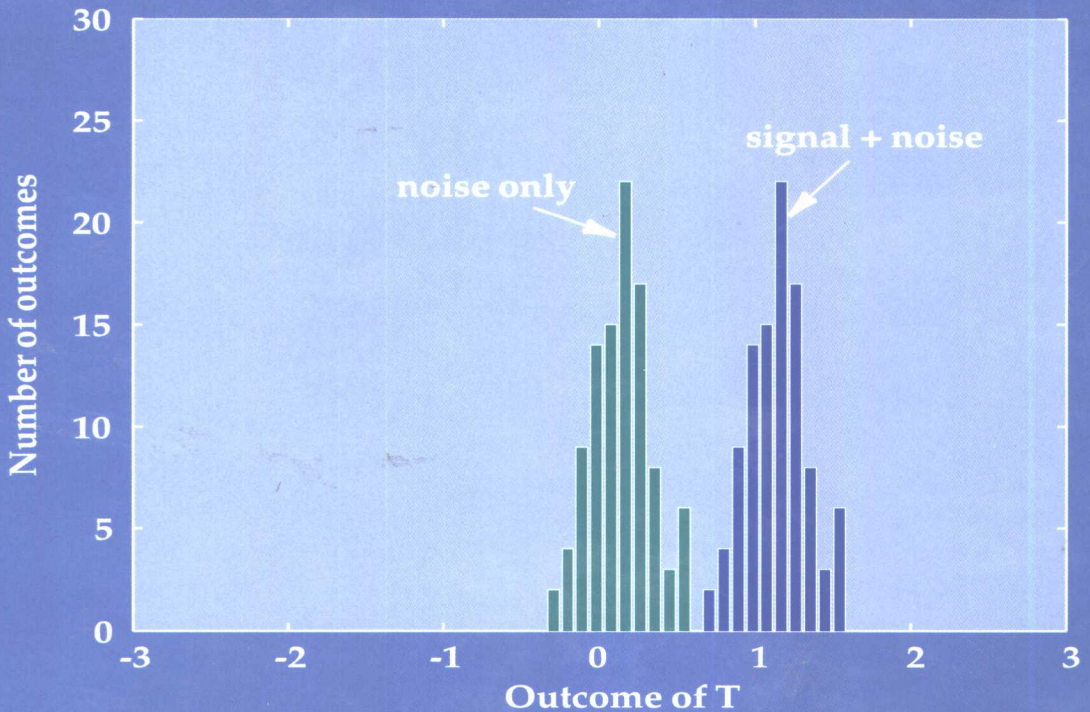


VOLUME II

FUNDAMENTALS OF STATISTICAL SIGNAL PROCESSING

DETECTION THEORY



STEVEN M. KAY

PRENTICE HALL SIGNAL PROCESSING SERIES
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**Fundamentals of
Statistical Signal Processing
Volume II
Detection Theory**

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**Fundamentals of
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Volume II
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Prentice Hall PTR
Upper Saddle River, New Jersey 07458
<http://www.phptr.com>

Library of Congress Cataloging-in-Publication Data

Kay, Steven M.
Fundamentals of statistical signal processing : detection theory
/ Steven M. Kay

p. cm. — (PH signal processing series)

Includes bibliographical references and index.

ISBN 0-13-504135-X

1. Signal processing—Statistical methods 2. Detection theory.

I. Title. II. Series: Prentice-Hall signal processing series.

TK5102.5.K379 1993

621.382'2—dc20

92-29495
CIP

Editorial/Production Supervision: *Craig Little*

Acquisitions Editor: *Bernard Goodwin*

Cover Design Director: *Jerry Votta*

Cover Design: *Wanda Lubelska*

Manufacturing Buyer: *Julia Meehan*

Marketing Manager: *Miles Williams*

Composition: *Steven M. Kay, PreTEX, Inc.*



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Upper Saddle River, New Jersey 07458

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Printed in the United States of America

10 9 8 7 6 5 4 3 2

ISBN 0-13-504135-X

Prentice-Hall International (UK) Limited, *London*

Prentice-Hall of Australia Pty. Limited, *Sydney*

Prentice-Hall Canada Inc., *Toronto*

Prentice-Hall Hispanoamericana, S.A., *Mexico*

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*To my parents
Phyllis and Jack.*

*to my in-laws
Betty and Walter.*

*and to my family
Cindy, Lisa, and Ashley*

Preface

This text is the second volume of a series of books addressing statistical signal processing. The first volume, *Fundamentals of Statistical Signal Processing: Estimation Theory*, was published in 1993 by Prentice-Hall, Inc. Henceforth, it will be referred to as [Kay-I 1993]. This second volume, entitled *Fundamentals of Statistical Signal Processing: Detection Theory*, is the application of statistical hypothesis testing to the detection of signals in noise. The series has been written to provide the reader with a broad introduction to the theory and application of statistical signal processing.

Hypothesis testing is a subject that is standard fare in the many books available dealing with statistics. These books range from the highly theoretical expositions written by statisticians to the more practical treatments contributed by the many users of applied statistics. This text is an attempt to strike a balance between these two extremes. The particular audience we have in mind is the community involved in the design and implementation of signal processing algorithms. As such, the primary focus is on obtaining optimal detection algorithms that may be implemented on a digital computer. The data sets are therefore assumed to be samples of a continuous-time waveform or a sequence of data points. The choice of topics reflects what we believe to be the important approaches to obtaining an optimal detector and analyzing its performance. As a consequence, some of the deeper theoretical issues have been omitted with references given instead.

It is the author's opinion that the best way to assimilate the material on detection theory is by exposure to and working with good examples. Consequently, there are numerous examples that illustrate the theory and others that apply the theory to actual detection problems of current interest. We have made extensive use of the MATLAB[®] scientific programming language (Version 4.2b)¹ for all computer-generated results. In some cases, actual MATLAB programs have been listed where a program was deemed to be of sufficient utility to the reader. Additionally, an abundance of homework problems has been included. They range from simple applications of the theory to extensions of the basic concepts. A solutions manual is available from the author. To aid the reader, summary sections have been provided at the beginning of each chapter. Also, an overview of all the principal detection approaches and the rationale for choosing a particular method can be found in

¹MATLAB is a registered trademark of The MathWorks, Inc.

Chapter 11. Detection based on simple hypothesis testing is described in Chapters 3–5, while that based on composite hypothesis testing (to accommodate unknown parameters) is the subject of Chapters 6–9. Other chapters address detection in non-Gaussian noise (Chapter 10), detection of model changes (Chapter 12), and extensions for complex/vector data useful in array processing (Chapter 13).

This book is an outgrowth of a one-semester graduate level course on detection theory given at the University of Rhode Island. It includes somewhat more material than can actually be covered in one semester. We typically cover most of Chapters 1–10, leaving the subjects of model change detection and complex data/vector data extensions to the student. It is also possible to combine the subjects of estimation and detection into a single semester course by a judicious choice of material from Volumes I and II. The necessary background that has been assumed is an exposure to the basic theory of digital signal processing, probability and random processes, and linear and matrix algebra. This book can also be used for self-study and so should be useful to the practicing engineer as well as the student.

The author would like to acknowledge the contributions of the many people who over the years have provided stimulating discussions of research problems, opportunities to apply the results of that research, and support for conducting research. Thanks are due to my colleagues L. Jackson, R. Kumaresan, L. Pakula, and P. Swaszek of the University of Rhode Island, and L. Scharf of the University of Colorado. Exposure to practical problems, leading to new research directions, has been provided by H. Woodsum of Sonetech, Bedford, New Hampshire, and by D. Mook and S. Lang of Sanders, a Lockheed-Martin Co., Nashua, New Hampshire. The opportunity to apply detection theory to sonar and the research support of J. Kelly of the Naval Undersea Warfare Center, J. Salisbury, formerly of the Naval Undersea Warfare Center, and D. Sheldon of the Naval Undersea Warfare Center, Newport, Rhode Island are also greatly appreciated. Thanks are due to J. Sjogren of the Air Force Office of Scientific Research, whose support has allowed the author to investigate the field of statistical signal processing. A debt of gratitude is owed to all my current and former graduate students. They have contributed to the final manuscript through many hours of pedagogical and research discussions as well as by their specific comments and questions. In particular, P. Djurić of the State University of New York proofread much of the manuscript, and S. Talwalkar of Motorola, Plantation, Florida proofread parts of the manuscript and helped with the finer points of MATLAB.

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Contents

1	Introduction	1
1.1	Detection Theory in Signal Processing	1
1.2	The Detection Problem	7
1.3	The Mathematical Detection Problem	8
1.4	Hierarchy of Detection Problems	13
1.5	Role of Asymptotics	14
1.6	Some Notes to the Reader	15
2	Summary of Important PDFs	20
2.1	Introduction	20
2.2	Fundamental Probability Density Functions and Properties	20
2.2.1	Gaussian (Normal)	20
2.2.2	Chi-Squared (Central)	24
2.2.3	Chi-Squared (Noncentral)	26
2.2.4	F (Central)	28
2.2.5	F (Noncentral)	29
2.2.6	Rayleigh	30
2.2.7	Rician	31
2.3	Quadratic Forms of Gaussian Random Variables	32
2.4	Asymptotic Gaussian PDF	33
2.5	Monte Carlo Performance Evaluation	36
2A	Number of Required Monte Carlo Trials	45
2B	Normal Probability Paper	47
2C	MATLAB Program to Compute Gaussian Right-Tail Probability and its Inverse	50
2D	MATLAB Program to Compute Central and Noncentral χ^2 Right- Tail Probability	52
2E	MATLAB Program for Monte Carlo Computer Simulation	58

3	Statistical Decision Theory I	60
3.1	Introduction	60
3.2	Summary	60
3.3	Neyman-Pearson Theorem	61
3.4	Receiver Operating Characteristics	74
3.5	Irrelevant Data	75
3.6	Minimum Probability of Error	77
3.7	Bayes Risk	80
3.8	Multiple Hypothesis Testing	81
3A	Neyman-Pearson Theorem	89
3B	Minimum Bayes Risk Detector - Binary Hypothesis	90
3C	Minimum Bayes Risk Detector - Multiple Hypotheses	92
4	Deterministic Signals	94
4.1	Introduction	94
4.2	Summary	94
4.3	Matched Filters	95
4.3.1	Development of Detector	95
4.3.2	Performance of Matched Filter	101
4.4	Generalized Matched Filters	105
4.4.1	Performance of Generalized Matched Filter	108
4.5	Multiple Signals	112
4.5.1	Binary Case	112
4.5.2	Performance for Binary Case	114
4.5.3	M-ary Case	119
4.6	Linear Model	122
4.7	Signal Processing Examples	125
4A	Reduced Form of the Linear Model	139
5	Random Signals	141
5.1	Introduction	141
5.2	Summary	141
5.3	Estimator-Correlator	142
5.4	Linear Model	154
5.5	Estimator-Correlator for Large Data Records	165
5.6	General Gaussian Detection	167
5.7	Signal Processing Example	169
5.7.1	Tapped Delay Line Channel Model	169
5A	Detection Performance of the Estimator-Correlator	183

6	Statistical Decision Theory II	186
6.1	Introduction	186
6.2	Summary	186
6.2.1	Summary of Composite Hypothesis Testing	187
6.3	Composite Hypothesis Testing	191
6.4	Composite Hypothesis Testing Approaches	197
6.4.1	Bayesian Approach	198
6.4.2	Generalized Likelihood Ratio Test	200
6.5	Performance of GLRT for Large Data Records	205
6.6	Equivalent Large Data Records Tests	208
6.7	Locally Most Powerful Detectors	217
6.8	Multiple Hypothesis Testing	221
6A	Asymptotically Equivalent Tests - No Nuisance Parameters	232
6B	Asymptotically Equivalent Tests - Nuisance Parameters	235
6C	Asymptotic PDF of GLRT	239
6D	Asymptotic Detection Performance of LMP Test	241
6E	Alternate Derivation of Locally Most Powerful Test	243
6F	Derivation of Generalized ML Rule	245
7	Deterministic Signals with Unknown Parameters	248
7.1	Introduction	248
7.2	Summary	248
7.3	Signal Modeling and Detection Performance	249
7.4	Unknown Amplitude	253
7.4.1	GLRT	254
7.4.2	Bayesian Approach	257
7.5	Unknown Arrival Time	258
7.6	Sinusoidal Detection	261
7.6.1	Amplitude Unknown	261
7.6.2	Amplitude and Phase Unknown	262
7.6.3	Amplitude, Phase, and Frequency Unknown	268
7.6.4	Amplitude, Phase, Frequency, and Arrival Time Unknown	269
7.7	Classical Linear Model	272
7.8	Signal Processing Examples	279
7A	Asymptotic Performance of the Energy Detector	297
7B	Derivation of GLRT for Classical Linear Model	299

8	Random Signals with Unknown Parameters	302
8.1	Introduction	302
8.2	Summary	302
8.3	Incompletely Known Signal Covariance	303
8.4	Large Data Record Approximations	311
8.5	Weak Signal Detection	314
8.6	Signal Processing Example	315
8A	Derivation of PDF for Periodic Gaussian Random Process	332
9	Unknown Noise Parameters	336
9.1	Introduction	336
9.2	Summary	336
9.3	General Considerations	337
9.4	White Gaussian Noise	341
9.4.1	Known Deterministic Signal	341
9.4.2	Random Signal with Known PDF	343
9.4.3	Deterministic Signal with Unknown Parameters	345
9.4.4	Random Signal with Unknown PDF Parameters	349
9.5	Colored WSS Gaussian Noise	350
9.5.1	Known Deterministic Signals	350
9.5.2	Deterministic Signals with Unknown Parameters	353
9.6	Signal Processing Example	358
9A	Derivation of GLRT for Classical Linear Model for σ^2 Unknown . . .	371
9B	Rao Test for General Linear Model with Unknown Noise Parameters	375
9C	Asymptotically Equivalent Rao Test for Signal Processing Example .	377
10	NonGaussian Noise	381
10.1	Introduction	381
10.2	Summary	381
10.3	NonGaussian Noise Characteristics	382
10.4	Known Deterministic Signals	385
10.5	Deterministic Signals with Unknown Parameters	392
10.6	Signal Processing Example	400
10A	Asymptotic Performance of NP Detector for Weak Signals	410
10B	Rao Test for Linear Model Signal with IID NonGaussian Noise . . .	413

11 Summary of Detectors	416
11.1 Introduction	416
11.2 Detection Approaches	416
11.3 Linear Model	427
11.4 Choosing a Detector	433
11.5 Other Approaches and Other Texts	437
12 Model Change Detection	439
12.1 Introduction	439
12.2 Summary	439
12.3 Description of Problem	440
12.4 Extensions to the Basic Problem	445
12.5 Multiple Change Times	449
12.6 Signal Processing Examples	455
12.6.1 Maneuver Detection	455
12.6.2 Time Varying PSD Detection	460
12A General Dynamic Programming Approach to Segmentation	469
12B MATLAB Program for Dynamic Programming	471
13 Complex/Vector Extensions, and Array Processing	473
13.1 Introduction	473
13.2 Summary	473
13.3 Known PDFs	474
13.3.1 Matched Filter	474
13.3.2 Generalized Matched Filter	478
13.3.3 Estimator-Correlator	479
13.4 PDFs with Unknown Parameters	484
13.4.1 Deterministic Signal	484
13.4.2 Random Signal	486
13.5 Vector Observations and PDFs	486
13.5.1 General Covariance Matrix	490
13.5.2 Scaled Identity Matrix	491
13.5.3 Uncorrelated from Temporal Sample to Sample	491
13.5.4 Uncorrelated from Spatial Sample to Sample	492
13.6 Detectors for Vector Observations	492
13.6.1 Known Deterministic Signal in CWGN	492
13.6.2 Known Deterministic Signal and General Noise Covariance	495

13.6.3	Known Deterministic Signal in Temporally Uncorrelated Noise	495
13.6.4	Known Deterministic Signal in Spatially Uncorrelated Noise . . .	496
13.6.5	Random Signal in CWGN	496
13.6.6	Deterministic Signal with Unknown Parameters in CWGN . . .	499
13.7	Estimator-Correlator for Large Data Records	501
13.8	Signal Processing Examples	508
13.8.1	Active Sonar/Radar	510
13.8.2	Broadband Passive Sonar	515
13A	PDF of GLRT for Complex Linear Model	526
A1	Review of Important Concepts	529
A1.1	Linear and Matrix Algebra	529
A1.1.1	Definitions	529
A1.1.2	Special Matrices	531
A1.1.3	Matrix Manipulation and Formulas	533
A1.1.4	Theorems	535
A1.1.5	Eigendecomposition of Matrices	536
A1.1.6	Inequalities	537
A1.2	Random Processes and Time Series Modeling	537
A1.2.1	Random Process Characterization	538
A1.2.2	Gaussian Random Process	540
A1.2.3	Time Series Models	541
A2	Glossary of Symbols and Abbreviations	
	(Vols. I & II)	545

Chapter 1

Introduction

1.1 Detection Theory in Signal Processing

Modern detection theory is fundamental to the design of electronic signal processing systems for decision making and information extraction. These systems include

1. Radar
2. Communications
3. Speech
4. Sonar
5. Image processing
6. Biomedicine
7. Control
8. Seismology,

and all share the common goal of being able to decide when an event of interest occurs and then to determine more information about that event. The latter task, information extraction, is the subject of the first volume [Kay 1993]. The former problem, that of decision making, is the subject of this book and is broadly termed *detection theory*. Other names associated with it are *hypothesis testing* and *decision theory*. To illustrate the problem of detection as applied to signal processing, we briefly describe the first three of these systems.

In radar we are interested in determining the presence or absence of an approaching aircraft [Skolnik 1980]. To accomplish this task we transmit an electromagnetic pulse, which if reflected by a large moving object, will indicate the presence of an aircraft. If an aircraft is present, the received waveform will consist of the reflected

pulse (at some time later) and noise due to ambient radiation and the receiver electronics. If an aircraft is not present, then only noise will be present. It is the function of the signal processor to decide whether the received waveform consists of noise only (no aircraft) or an echo in noise (aircraft present). As an example, in Figure 1.1a we have depicted a radar and in Figure 1.1b a typical received waveform for the two possible scenarios. When an echo is present, we see that the character of the received waveform is somewhat different, although possibly not by much. This is because the received echo is attenuated due to propagation loss and possibly distorted due to the interaction of multiple reflections. Of course, if the aircraft is detected, then it is of interest to determine its bearing, range, speed, etc. Hence, detection is the first task of the signal processing system while the second task is information extraction. Estimation theory provides the foundation for the second task and has already been described in Volume I [Kay-I 1993]. The optimal detector for the radar problem is the Neyman-Pearson detector, which is described in Chapter 4. A more practical detector which accommodates signal uncertainties, however, is discussed in Chapter 7.

A second application is in the design of a digital communication system. An example is the binary phase shift keyed (BPSK) system as shown in Figure 1.2a used to communicate the output of a digital data source that emits a “0” or “1” [Proakis 1989]. The data bit is first modulated, then transmitted, and at the receiver, demodulated and then detected. The modulator converts a 0 to the waveform $s_0(t) = \cos 2\pi F_0 t$ and a 1 to $s_1(t) = \cos(2\pi F_0 t + \pi) = -\cos 2\pi F_0 t$ to allow transmission through a bandpass channel whose center frequency is F_0 Hz (such as a microwave link). The phase of the sinusoid indicates whether a 0 or 1 has been sent. In this problem, the function of the detector is to decide between the two possibilities, as in the radar problem, although now, we always have a signal present – the question is *which* signal. Typical received waveforms are shown in Figure 1.2b. Since the sinusoidal carrier has been extracted by the demodulator, all that remains at the detector input is the baseband signal, either a positive or negative pulse. This signal is usually distorted due to limited channel bandwidth and is also corrupted by additive channel noise. The solution to this problem is given in Chapter 4.

Another application is in speech recognition where we wish to determine which word was spoken from among a group of possible words [Rabiner and Juang 1993]. A simple example is to discern among the digits “0”, “1”, ..., “9”. To recognize a spoken digit using a digital computer we would need to *match* the spoken digit with some stored digit. For example, the waveforms for the spoken digits 0 and 1 are shown in Figure 1.3. They have been repeated three times by the same speaker. Note that the waveform changes slightly for each utterance of the same word. We may think of this change as “noise,” although it is actually the natural variability of speech. Given an utterance, we wish to decide if it is a 0 or 1. More generally, we would need to decide among the ten possible digits. Such a problem is a generalization of that for radar and for digital communications in which only one of two possible choices need be made. The solution to this problem is discussed in Chapter 4.

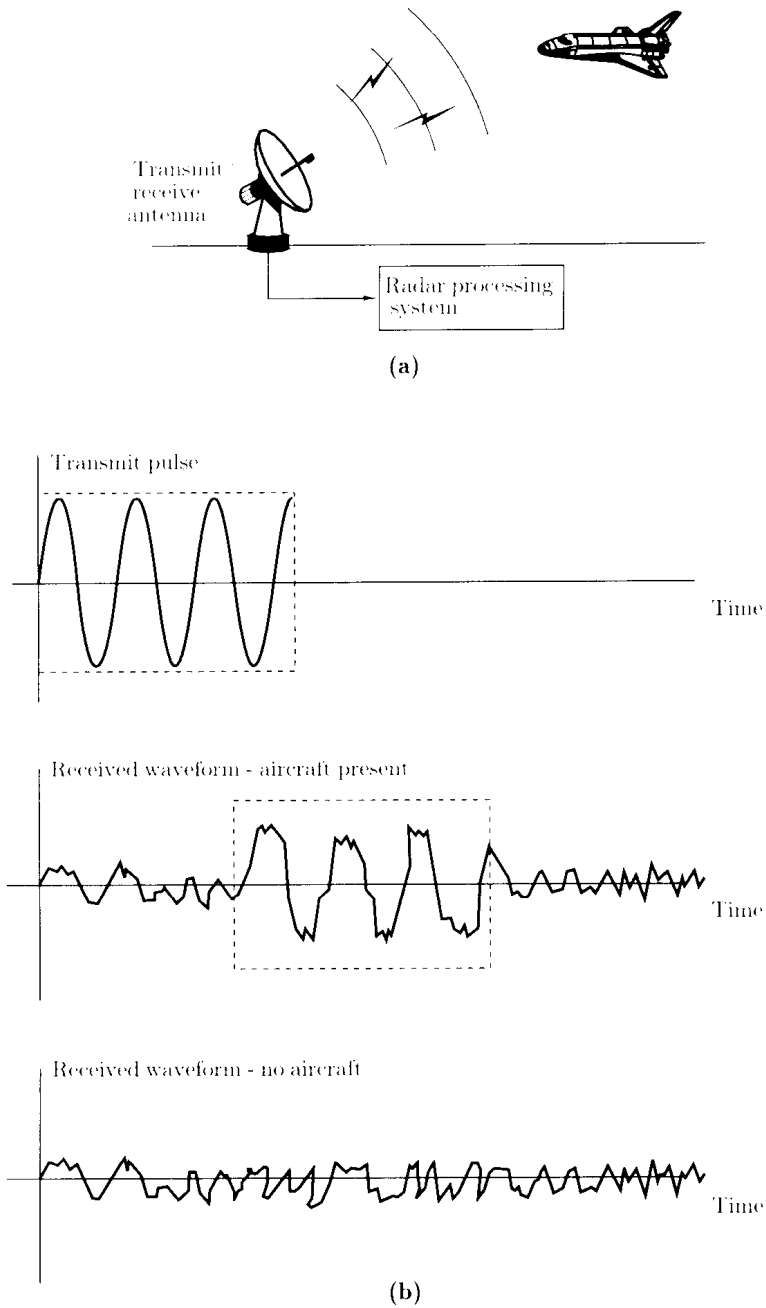


Figure 1.1. Radar system (a) Radar (b) Radar waveforms.