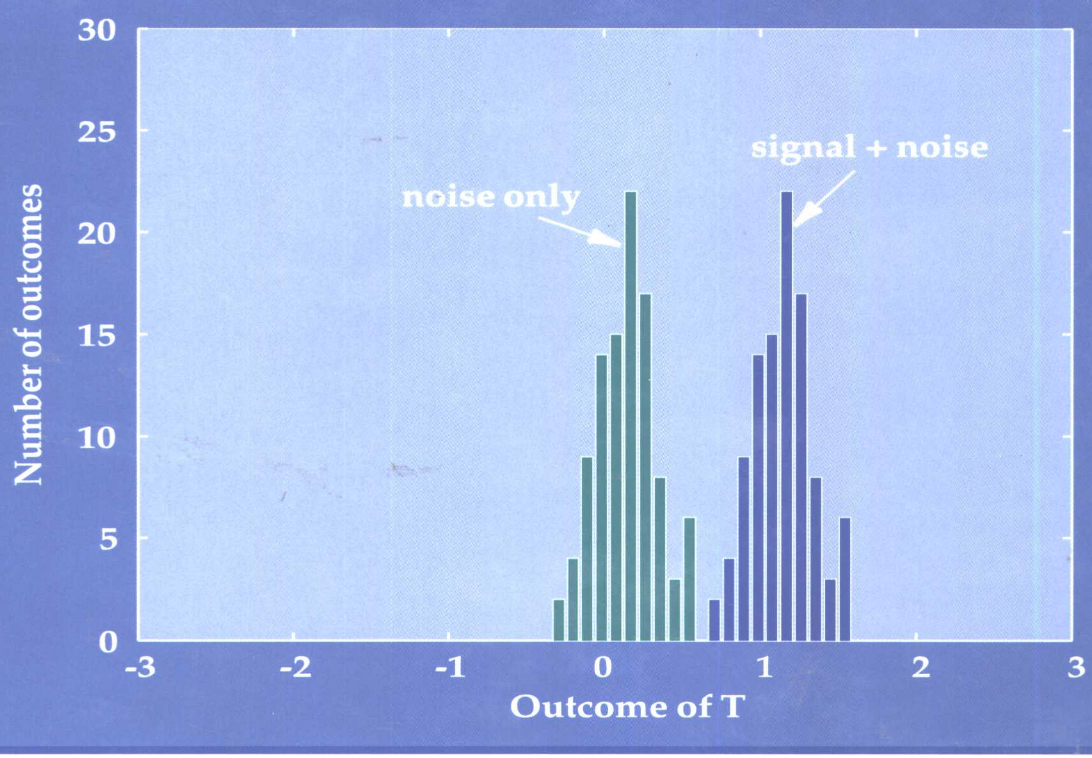
VOLUME II

FUNDAMENTALS OF STATISTICAL SIGNAL PROCESSING

DETECTION THEORY



STEVEN M. KAY

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Fundamentals of Statistical Signal Processing

Volume II

Detection Theory

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Fundamentals of Statistical Signal Processing

Volume II

Detection Theory

Steven M. Kay
University of Rhode Island

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To my parents
Phyllis and Jack,

to my in-laws
Betty and Walter,

and to my family
Cindy, Lisa, and Ashley

Preface

This text is the second volume of a series of books addressing statistical signal processing. The first volume, *Fundamentals of Statistical Signal Processing: Estimation Theory*, was published in 1993 by Prentice-Hall, Inc. Henceforth, it will be referred to as [Kay-I 1993]. This second volume, entitled *Fundamentals of Statistical Signal Processing: Detection Theory*, is the application of statistical hypothesis testing to the detection of signals in noise. The series has been written to provide the reader with a broad introduction to the theory and application of statistical signal processing.

Hypothesis testing is a subject that is standard fare in the many books available dealing with statistics. These books range from the highly theoretical expositions written by statisticians to the more practical treatments contributed by the many users of applied statistics. This text is an attempt to strike a balance between these two extremes. The particular audience we have in mind is the community involved in the design and implementation of signal processing algorithms. As such, the primary focus is on obtaining optimal detection algorithms that may be implemented on a digital computer. The data sets are therefore assumed to be samples of a continuous-time waveform or a sequence of data points. The choice of topics reflects what we believe to be the important approaches to obtaining an optimal detector and analyzing its performance. As a consequence, some of the deeper theoretical issues have been omitted with references given instead.

It is the author's opinion that the best way to assimilate the material on detection theory is by exposure to and working with good examples. Consequently, there are numerous examples that illustrate the theory and others that apply the theory to actual detection problems of current interest. We have made extensive use of the MATLAB® scientific programming language (Version 4.2b)[1] for all computer-generated results. In some cases, actual MATLAB programs have been listed where a program was deemed to be of sufficient utility to the reader. Additionally, an abundance of homework problems has been included. They range from simple applications of the theory to extensions of the basic concepts. A solutions manual is available from the author. To aid the reader, summary sections have been provided at the beginning of each chapter. Also, an overview of all the principal detection approaches and the rationale for choosing a particular method can be found in

[1]MATLAB is a registered trademark of The MathWorks, Inc.

Chapter 11. Detection based on simple hypothesis testing is described in Chapters 3–5, while that based on composite hypothesis testing (to accommodate unknown parameters) is the subject of Chapters 6–9. Other chapters address detection in nonGaussian noise (Chapter 10), detection of model changes (Chapter 12), and extensions for complex/vector data useful in array processing (Chapter 13).

This book is an outgrowth of a one-semester graduate level course on detection theory given at the University of Rhode Island. It includes somewhat more material than can actually be covered in one semester. We typically cover most of Chapters 1–10, leaving the subjects of model change detection and complex data/vector data extensions to the student. It is also possible to combine the subjects of estimation and detection into a single semester course by a judicious choice of material from Volumes I and II. The necessary background that has been assumed is an exposure to the basic theory of digital signal processing, probability and random processes, and linear and matrix algebra. This book can also be used for self-study and so should be useful to the practicing engineer as well as the student.

The author would like to acknowledge the contributions of the many people who over the years have provided stimulating discussions of research problems, opportunities to apply the results of that research, and support for conducting research. Thanks are due to my colleagues L. Jackson, R. Kumaresan, L. Pakula, and P. Swaszek of the University of Rhode Island, and L. Scharf of the University of Colorado. Exposure to practical problems, leading to new research directions, has been provided by H. Woodsum of Sonetech, Bedford, New Hampshire, and by D. Mook and S. Lang of Sanders, a Lockheed-Martin Co., Nashua, New Hampshire. The opportunity to apply detection theory to sonar and the research support of J. Kelly of the Naval Undersea Warfare Center, J. Salisbury, formerly of the Naval Undersea Warfare Center, and D. Sheldon of the Naval Undersea Warfare Center, Newport, Rhode Island are also greatly appreciated. Thanks are due to J. Sjogren of the Air Force Office of Scientific Research, whose support has allowed the author to investigate the field of statistical signal processing. A debt of gratitude is owed to all my current and former graduate students. They have contributed to the final manuscript through many hours of pedagogical and research discussions as well as by their specific comments and questions. In particular, P. Djurić of the State University of New York proofread much of the manuscript, and S. Talwalkar of Motorola, Plantation, Florida proofread parts of the manuscript and helped with the finer points of MATLAB.

Steven M. Kay
University of Rhode Island
Kingston, RI 02881
Email: kay@ele.uri.edu

Contents

Chapter 1

Introduction

1.1 Detection Theory in Signal Processing

Modern detection theory is fundamental to the design of electronic signal processing systems for decision making and information extraction. These systems include

1. Radar

2. Communications

3. Speech

4. Sonar

5. Image processing

6. Biomedicine

7. Control

8. Seismology,

and all share the common goal of being able to decide when an event of interest occurs and then to determine more information about that event. The latter task, information extraction, is the subject of the first volume [Kay 1993]. The former problem, that of decision making, is the subject of this book and is broadly termed *detection theory*. Other names associated with it are *hypothesis testing* and *decision theory*. To illustrate the problem of detection as applied to signal processing, we briefly describe the first three of these systems.

In radar we are interested in determining the presence or absence of an approaching aircraft [Skolnik 1980]. To accomplish this task we transmit an electromagnetic pulse, which if reflected by a large moving object, will indicate the presence of an aircraft. If an aircraft is present, the received waveform will consist of the reflected pulse (at some time later) and noise due to ambient radiation and the receiver electronics. If an aircraft is not present, then only noise will be present. It is the function of the signal processor to decide whether the received waveform consists of noise only (no aircraft) or an echo in noise (aircraft present). As an example, in Figure 1.1a we have depicted a radar and in Figure 1.1b a typical received waveform for the two possible scenarios. When an echo is present, we see that the character of the received waveform is somewhat different, although possibly not by much. This is because the received echo is attenuated due to propagation loss and possibly distorted due to the interaction of multiple reflections. Of course, if the aircraft is detected, then it is of interest to determine its bearing, range, speed, etc. Hence, detection is the first task of the signal processing system while the second task is information extraction. Estimation theory provides the foundation for the second task and has already been described in Volume I [Kay-I 1993]. The optimal detector for the radar problem is the Neyman-Pearson detector, which is described in Chapter 4. A more practical detector which accommodates signal uncertainties, however, is discussed in Chapter 7.

A second application is in the design of a digital communication system. An example is the binary phase shift keyed (BPSK) system as shown in Figure 1.2a used to communicate the output of a digital data source that emits a "0" or "1" [Proakis 1989]. The data bit is first modulated, then transmitted, and at the receiver, demodulated and then detected. The modulator converts a 0 to the waveform $s_0(t) = \cos 2\pi F_0 t$ and a 1 to $s_1(t) = \cos(2\pi F_0 t + \pi) = -\cos 2\pi F_0 t$ to allow transmission through a bandpass channel whose center frequency is F_0 Hz (such as a microwave link). The phase of the sinusoid indicates whether a 0 or 1 has been sent. In this problem, the function of the detector is to decide between the two possibilities, as in the radar problem, although now, we always have a signal present – the question is *which* signal. Typical received waveforms are shown in Figure 1.2b. Since the sinusoidal carrier has been extracted by the demodulator, all that remains at the detector input is the baseband signal, either a positive or negative pulse. This signal is usually distorted due to limited channel bandwidth and is also corrupted by additive channel noise. The solution to this problem is given in Chapter 4.

Another application is in speech recognition where we wish to determine which word was spoken from among a group of possible words [Rabiner and Juang 1993]. A simple example is to discern among the digits "0", "1", ..., "9". To recognize a spoken digit using a digital computer we would need to *match* the spoken digit with some stored digit. For example, the waveforms for the spoken digits 0 and 1 are shown in Figure 1.3. They have been repeated three times by the same speaker. Note that the waveform changes slightly for each utterance of the same word. We may think of this change as "noise," although it is actually the natural variability of speech. Given an utterance, we wish to decide if it is a 0 or 1. More generally, we would need to decide among the ten possible digits. Such a problem is a generalization of that for radar and for digital communications in which only one of two possible choices need be made. The solution to this problem is discussed in Chapter 4.

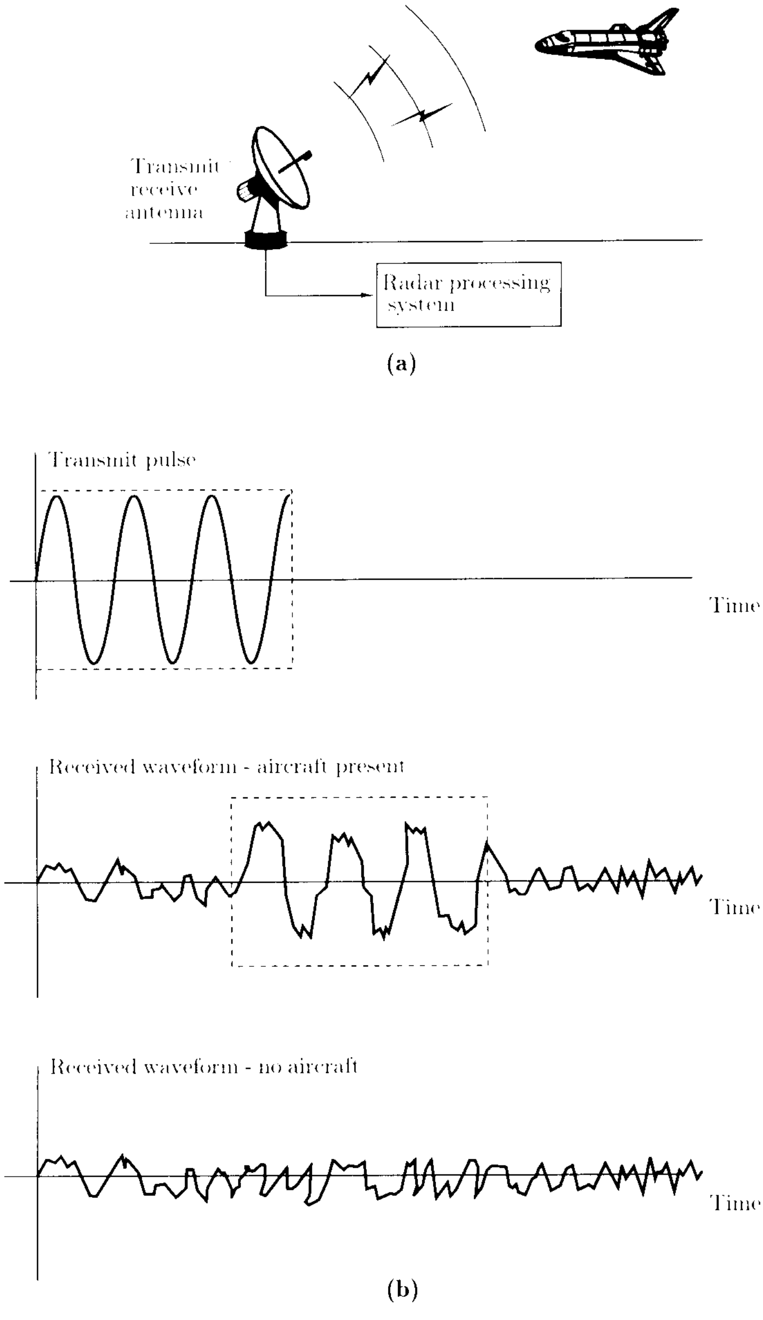

Figure 1.1. Radar system (a) Radar (b) Radar waveforms.