Joseph Pedlosky

Geophysical Fluid Dynamics

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Preface

The content of this book is based, largely, on the core curriculum in geophysical fluid dynamics which I and my colleagues in the Department of Geophysical Sciences at The University of Chicago have taught for the past decade. Our purpose in developing a core curriculum was to provide to advanced undergraduates and entering graduate students a coherent and systematic introduction to the theory of geophysical fluid dynamics. The curriculum and the outline of this book were devised to form a sequence of courses of roughly one and a half academic years (five academic quarters) in length. The goal of the sequence is to help the student rapidly advance to the point where independent study and research are practical expectations. It quickly became apparent that several topics (e.g., some aspects of potential theory) usually thought of as forming the foundations of a fluid-dynamics curriculum were merely classical rather than essential and could be, however sadly, dispensed with for our purposes. At the same time, the diversity of interests of our students is so great that no curriculum can truly be exhaustive in such a curriculum period. It seems to me that the best that can be achieved as a compromise is a systematic introduction to some important segment of the total scope of geophysical fluid dynamics which is illustrative of its most fruitful methods. The focus of this book is thus the application of fluid mechanics to the dynamics of large-scale flows in the oceans and the atmosphere. The overall viewpoint taken is a theoretical, unified approach to the study of both the atmosphere and the oceans.

One of the key features of geophysical fluid dynamics is the need to combine approximate forms of the basic fluid-dynamical equations of motion with careful and precise analysis. The approximations are required to make any progress possible, while precision is demanded to make the progress meaningful. This combination is often the most elusive feature for the beginning student to appreciate. Therefore, much of the discussion of this book is directed towards the development of the basic notions of scaling and the subsequent derivation of systematic approximations to the equations of motion. The union of physical and intuitive reasoning with mathematical analysis forms the central theme. The ideas of geostrophic scaling, for example, are repeated several times, in various contexts, to illustrate the ideas by example.

The development of physical intuition is always a slow process for the beginner, and the book has a structure which aims to ease that important process. Chapters 1 and 2 discuss certain elementary but fundamental ideas in general terms before the complexities of scaling are required. In Chapter 3 the inviscid dynamics of a homogeneous fluid is discussed in order to expose, in the simplest context, the nature of quasigeostrophic motion. It has been my experience that the absence of the complexities necessarily associated with density stratification is a great help in penetrating quickly to rather basic concepts of potential vorticity dynamics. Rossby waves, inertial boundary currents, the β-plane, energy propagation, and wave interaction etc. are all topics whose first treatment is clearer and simpler for the fluid of constant density. Similarly, Chapter 4 describes some of the simple ideas of the influence of friction on large-scale flows in the context of a homogeneous fluid. The vexing problem of turbulence receives short shrift here. Only the simplest model of turbulent mixing is formulated. It is my view that, unsatisfactory as such a model is as a theory of turbulence, it is sufficient for the purposes to which it is generally applied in the theory of large-scale flows. Chapter 5 serves to exemplify the use of the homogeneous model in the discussion of a problem of major geophysical interest, i.e., the wind-driven ocean circulation.

Chapter 6 has two main purposes. First is the systematic development of the quasigeostrophic dynamics of a stratified fluid for flow on a sphere. Careful attention is given to the development of the β-plane model on logical and straightforward lines. I believe many of the elements of the derivation have been hitherto unfortunately obscure. The second major goal is the application of quasigeostrophic dynamics to a few problems which I feel are central to both meteorology and oceanography and whose outlines, at least, should be familiar to the serious student.

Chapter 7 is reserved for instability theory. Since the publication of the pioneering papers of Charney and Eady, instability theory has held a central position in the conceptual foundation of dynamic meteorology. Recent advances in oceanography suggest a significant role for instability theory also in oceanic dynamics. Baroclinic and barotropic instability are both discussed in Chapter 7, not exhaustively, but to the degree I feel is necessary to provide a clear picture of the basic issues. The final chapter discusses certain topics, not easily grouped into the broad categories of earlier chapters, and chosen primarily to illustrate the way in which the ideas previously developed can be extended by similar methods.

The task of writing a text is made especially difficult by the evident impossibility of being truly comprehensive. The limitations of size make it necessary to omit topics of interest. To begin with, certain introductory aspects of fluid mechanics, such as the derivation of the Navier–Stokes equations (which is essential to a core curriculum) are deleted. Such topics may be found already in such excellent texts as Batchelor's *Fluid Dynamics* or Sommerfeld's *Mechanics of Deformable Bodies*. In other cases, when confronted by difficult choices, I have tried to include material which illustrates principles of general utility in fluid mechanics, e.g., boundary-layer concepts and the application of multiple-time-scale ideas to nonlinear problems. In this way I believe that the problems of geophysical fluid dynamics serve additionally as an excellent vehicle for the teaching of broader dynamical concepts. For example, the relationship between group velocity and phase speed in the Rossby wave is discussed at length in Chapter 3. There is, perhaps, no more dramatic example of the distinction between the two concepts in all fluid dynamics, and it can serve as a useful example of such a distinction for students of varying fluid-dynamical interests.

Naturally, in many cases I have chosen topics for discussion on the basis of my own interest and judgement. To that extent the text is a personal expression of my view of the subject.

It was my happy good fortune as a student to have had a series of marvelous teachers of fluid dynamics. Each in their own way made the subject vivid and beautiful to me. By now, no doubt, many of their ideas and attitudes are so intimately mixed into my own view that they appear implicitly here to the benefit of the text.

It is a pleasure, however, to explicitly acknowledge the singular influence of my teacher and colleague, Professor Jule Charney. His prodigious contributions to the study of the dynamics of the atmosphere and oceans as well as his example of scholarly integrity have been a continuing source of inspiration.

This book was largely written during a sabbatical year made possible by a fellowship from the John Simon Guggenheim Foundation, as well as by the continued support of the University of Chicago. The Woods Hole Oceanographic Institution kindly provided an office for me for the year and their warm hospitality considerably eased the task of writing and preparing the original manuscript. Special thanks are due to the students of the M.I.T. Woods Hole joint program in physical oceanography, who read the evolving manuscript and made numerous helpful corrections and suggestions. Doris Haight typed the manuscript with skill, patience, and good humor.

Woods Hole
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CHAPTER 1

Preliminaries

1.1 Geophysical Fluid Dynamics

The atmosphere and the ocean have so many fluid-dynamical properties in common that the study of one often enriches our understanding of the other. Experience has also shown that the recognition of the underlying dynamical concepts applicable to both the atmosphere and the oceans is an excellent starting point for the study of either. Geophysical fluid dynamics is the subject whose concerns are the fundamental dynamical concepts essential to an understanding of the atmosphere and the oceans. In principle, though, geophysical fluid dynamics deals with all naturally occurring fluid motions. Such motions are present on an enormous range of spatial and temporal scales, from the ephemeral flutter of the softest breeze to the massive and persistent oceanic and atmospheric current systems. Indeed, even the "solid" earth itself undergoes a fluidlike internal circulation on time scales of millions of years, the surface expression of which is sea-floor spreading and continental drift. All these phenomena can properly be included within the domain of geophysical fluid dynamics. Partly for historical reasons, however, the subject has tended to focus on the dynamics of large-scale phenomena in the atmosphere and the oceans. It is on large scales that the common character of atmospheric and oceanic dynamics is most evident, while at the same time the majestic nature of currents like the Gulf Stream in the ocean and the atmospheric jet stream makes such a focus of attention emotionally compelling and satisfying. This limitation will be observed in the following discussion, which consequently provides an introductory rather than exhaustive treatment of the subject. In particular the present text does not discuss the observational and descriptive features of meteorology and oceanography, although a familiarity with such evidence is a necessity for the proper formulation of new fluid-dynamical theories. Reference will be made from time to time in the text to the description of particular phenomena for the purpose of clarifying the motive for particular lines of study.

The principles to be derived are largely theoretical concepts which can be applied to an understanding of the natural phenomena. Such principles spring most naturally from the study of model problems whose goal is the development of conceptual comprehension rather than detailed simulation of the complete geophysical phenomenon. Geophysical fluid dynamics has historically progressed by the consideration of a study sequence within a hierarchy of increasingly complex models where each stage builds on the intuition developed by the precise analysis of simpler models.

1.2 The Rossby Number

The attribute "large scale" requires a more precise definition. A phenomenon whose characteristic length scale is fifty kilometers might be considered small scale in the atmosphere, while motions of just that scale in the oceans could be considered accurately as large scale. Whether a phenomenon is to be considered a large-scale one *dynamically* depends on more than its size.

For the purpose of this text large-scale motions are those which are significantly influenced by the earth's rotation. An important measure of the significance of rotation for a particular phenomenon is the Rossby number, which we define as follows. Let L be a characteristic length scale of the *motion*. Figure 1.2.1, for example, shows a typical wave pattern observed in the pressure field of the troposphere. A typical and appropriate length scale of the motion, i.e., one that characterizes the horizontal spatial variations of the dynamical fields, could be the distance between a pressure peak and a succeeding trough. Similarly let U be a horizontal velocity scale characteristic of the motion. In Figure 1.2.1 L would be O(1,000 km), while U would be O(20 m s^{-1}).*

The time it takes a fluid element moving with speed U to traverse the distance L is L/U. If that period of time is much less than the period of rotation of the earth, the fluid can scarcely sense the earth's rotation over the time scale of the motion. For rotation to be important, then, we anticipate

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* The symbol O( ) is used in two quite separate ways in this text. The statement that the functions $f(x)$ and $g(x)$ are in the relation $f(x) = O(g(x))$ (in some limit) implies that $f(x)/g(x) \rightarrow$ constant in that limit in a formal asymptotic sense. The symbol will also be used to mean that a variable quantity, in this case U, has a size exemplified by the value following the ordering symbol. No limit or approximation criterion is implied in the latter case. The two usages are distinct and the particular context will show clearly which is meant.

that

$$\frac{L}{U} \geq \Omega^{-1}, \tag{1.2.1}$$

or, equivalently,

$$\varepsilon = \frac{U}{2\Omega L} \leq 1. \tag{1.2.2}$$

The nondimensional parameter ε is the Rossby number. Large-scale flows are defined as those with sufficiently large L for ε to be order one or less. For the earth $\Omega = 7.3 \times 10^{-5}\ \mathrm{s}^{-1}$. For the L and U given above, $\varepsilon = 0.137$ and we can expect the earth's rotation to be important.

Such estimates must often be more refined. For planetary motions we shall see that it is really only the component of the planetary rotation perpendicular to the earth's surface which naturally enters the estimate of ε. Hence (1.2.2) could seriously underestimate the Rossby number for phenomena in low latitudes. Such elaborations and qualifications will be taken up later.

Note that the smaller the characteristic velocity is, the smaller L can be and yet still qualify for a large-scale flow. The Gulf Stream has velocities of order 100 $\mathrm{cm\ s}^{-1}$. Although its characteristic horizontal scale as shown in Figure 1.2.2 is only O(100 km), the associated Rossby number is 0.07. Although the use of the local normal component of the earth's rotation would double this value at a latitude of 30°, it is still clear that such currents meet the criterion of large-scale motion.

Now these considerations have been essentially kinematic. However, the important dynamical consequence of even a moderately small Rossby number follows from the fact that small ε implies that large-scale motions are slow compared to the velocity imposed by the solid-body rotation of the earth. To a first approximation—i.e., to $O(\varepsilon)$—the atmosphere and oceans rotate with the planet with small but significant deviations which we, also rotating with the earth, identify as winds and currents. It is useful to recognize explicitly that the interesting motions are small departures from solid-body rotation by describing the motions in a rotating coordinate frame which kinematically eliminates the rigid rotation. In a frame rotating at a rate Ω only the deviations from solid-body rotation will be seen. Since such a rotating frame is an accelerating rather than an inertial frame, certain well-known "inertial forces" will be sensed, i.e., the centrifugal force and the subtle and important Coriolis force. We shall see that whenever the Rossby number is small, the Coriolis force is a dominant participant in the balance of forces. The study of the dynamics of large scale oceanic or atmospheric motions must include the Coriolis force to be geophysically relevant, and once the Coriolis force is included a host of subtle and fascinating dynamical phenomena are possible.

(a)

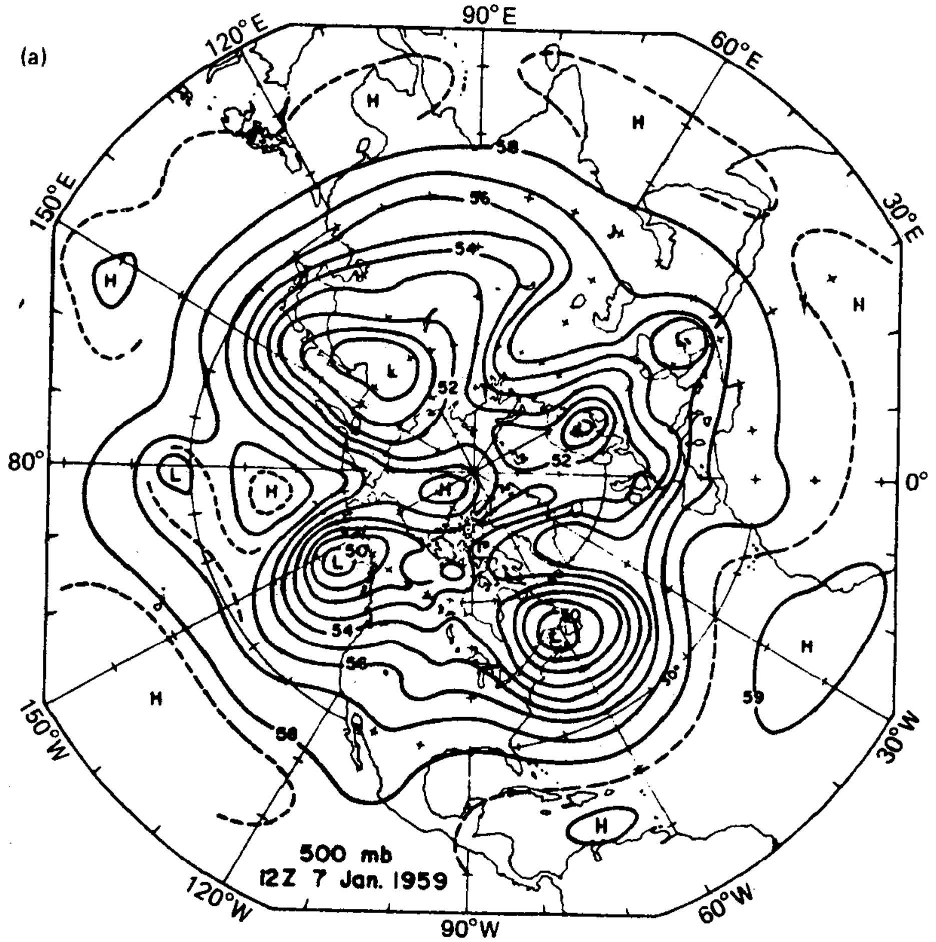


Figure 1.2.1(a) Isolines of constant pressure (isobars) at a level which is above roughly one-half the atmosphere's mass. The isobars very nearly mark the streamlines of the flow (Palmén and Newton, 1969).

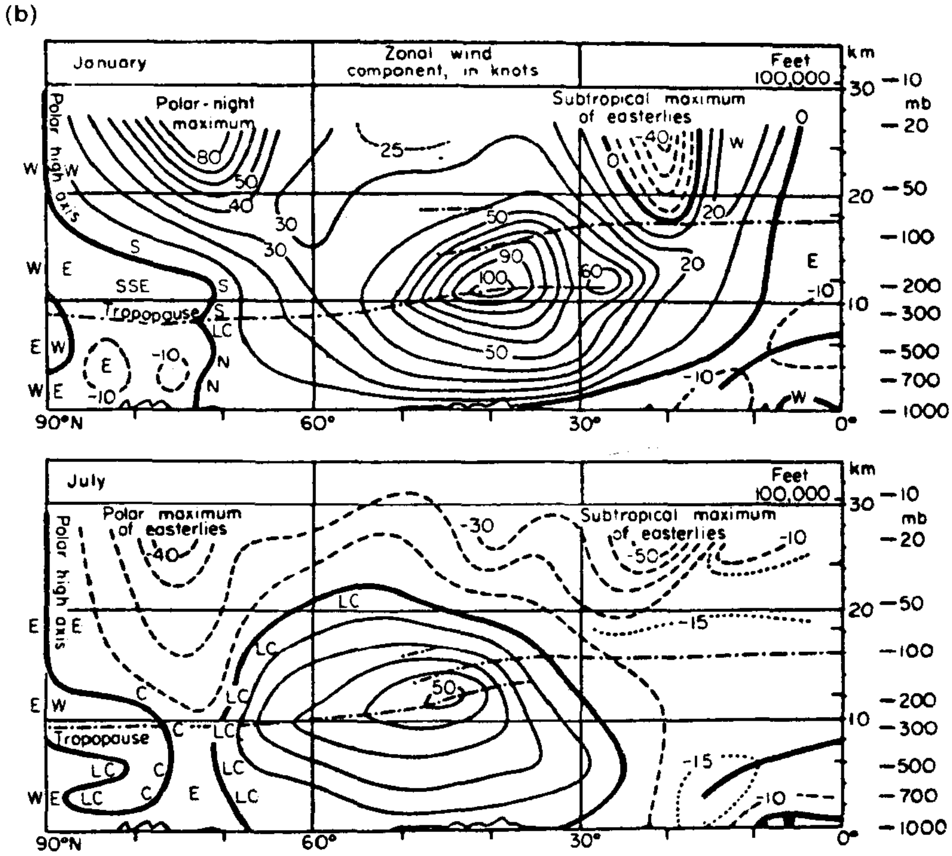
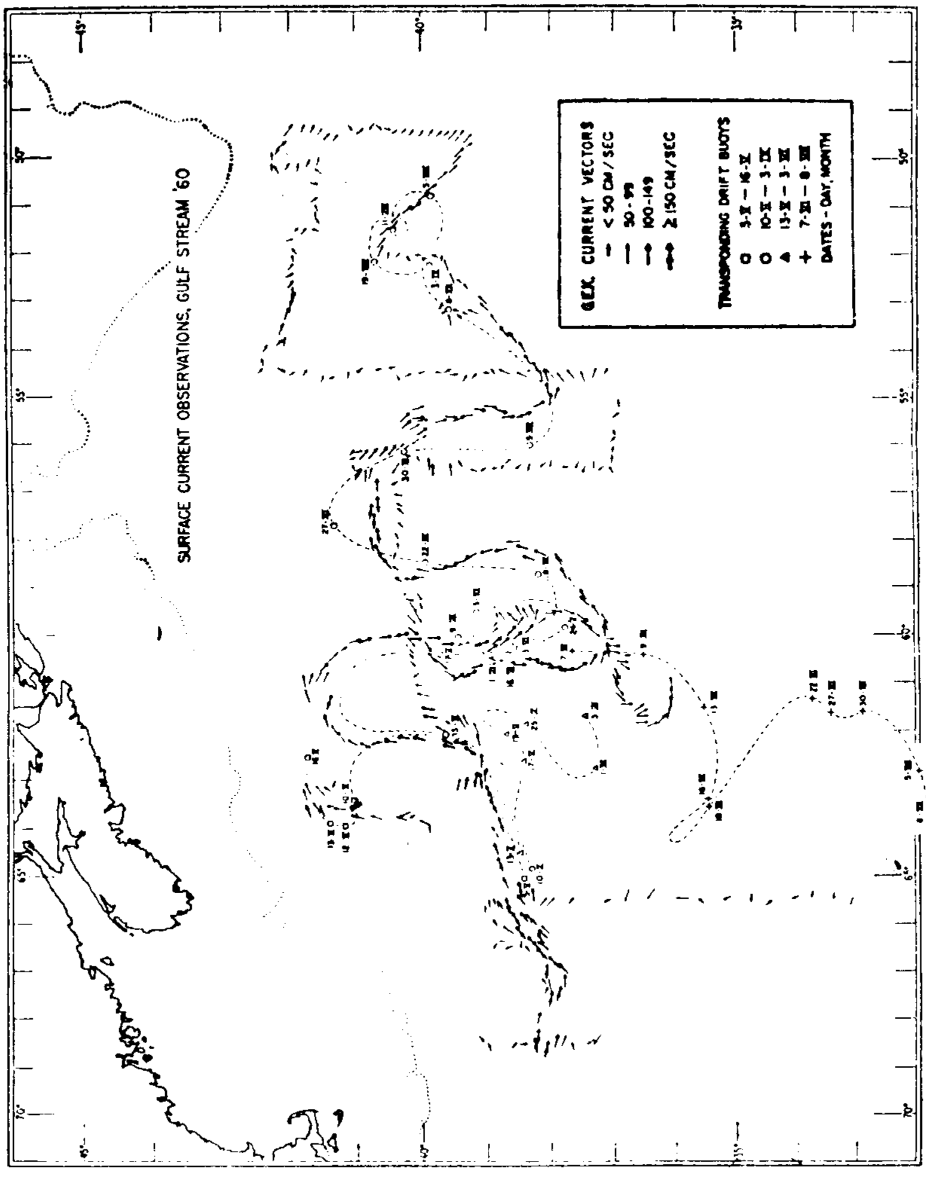
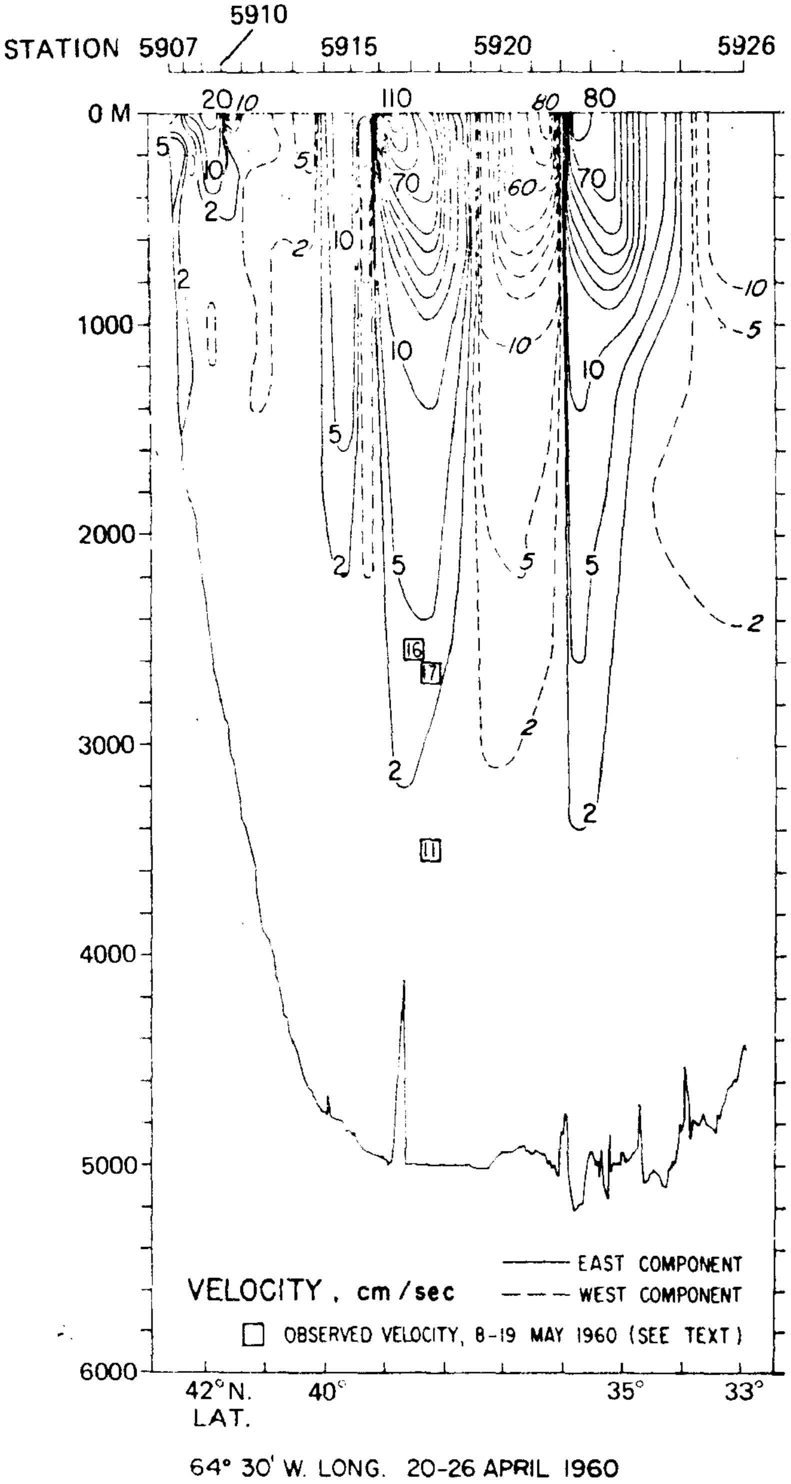


Figure 1.2.1(b) Cross section of the zonal wind (i.e., along latitude circles) showing the distribution of wind speed. (One knot ~ 50 cm s^{-1}) (Palmén and Newton, 1969, after Kochanski, 1955).


SURFACE CURRENT OBSERVATIONS, GULF STREAM '60
G.E.K. CURRENT VECTORS
< 50 CM/SEC
50-99
100-149
≥ 150 CM/SEC
TRANSPONDING DRIFT BUOYS
DATES - DAY, MONTH
45°
40°
35°
50°
55°
60°
65°
70°

(a)



(b).

Figure 1.2.2 **(a)** (*Facing page*) The path of the Gulf Stream as revealed by surface observations, and **(b)** a cross section through the Stream which displays the structure of the current velocity (Fuglister 1963).