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热烈庆祝

北京航空航天大學

建校四十周年

（1952——1992）

祝贺

北航校庆40周年

为了发展经济，消费不能超前。但，
为了发展经济，教育必须超前。
教育与科研紧密相联，为了社会发展，
在思想上必须时时改革创新；
在行动上必须时时力求创新。
只有创新才能发展，
只有创新才能前进，
只有创新，社会主义的中国才能
全面跃居世界先进之林，成为
引导世界劳动人民航向
共产主义理想目标的灯塔。

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发扬传统
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Ionospheric Total Electron Content Derived from Global Positioning System (GPS) Observations

Tan Zhanzhong

(Department of Electronic Engineering
Beijing University of Aeronautics and Astronautics)

J.Johansson and B.O.Rönnäng

(Dept. of Radio and Space Science
Onsala Space Observatory
Chalmers University of Technology)

Abstract This report summarizes the fundamental principles of estimating the ionospheric Total Electron Contents (TEC) from dual frequency Global Positioning System (GPS) data. Software to aquire the information from the GPS data files and to calculate the TEC along the satellite to receiver path and in the zenith direction are described. Examples of estimated diurnal TEC variations are given and compared with the actual modelled values transmitted by the GPS satellites. An error analysis shows that the mean TEC−values obtained from P−code data differ from the modelled values by as much as 2×10^{17} el / m^2. The conclusion is that high quality and well calibrated GPS receivers are needed and that phase information should be included in order to improve our understanding of the ionospheric degradation of precise satellite positioning.

1 Introduction

Radio positioning and navigation systems based on space techniques, such as the global satellite positioning systems GPS and GLONASS and the Very Long Baseline Interferometry (VLBI) technique are affected by the ionosphere. The ionosphere can cause considerable errors in the estimated positions due to its dispersive nature and variability [1,2] . These errors could be reduced by applying an ionospheric model or even better by simultaneous measurements of the electron content of the ionospthere. In the later case it is obvious that the satellite navigation technique also provides an efficient tool for ionospheric studies.

In this report we will introduce the fundamental principles and methods of using GPS receivers to study the ionosphere, present software to estimate the total electron content (TEC) of the ionosphere from GPS data, discuss the accuracy of the ionospheric delay estimate. This paper presents a short introduction to ionosspheric radio wave propagation as well as the appropriate equations needed to extract TEC. It also includes the software to

(1) acquire the useful data from the files produced by the GPS receiver;

(2) estimate the total electron content in the direction of the GPS satellite denoted TEC, the total electron content in the zenith direction denoted TEC_z and its geographic coordinates φ_i, λ_i;

(3) calculate TEC_{zm}, i.e. the total electron content in the zenith direction obtained from the ionospheric model used by the GPS receivers:

(4) compare TEC_{zm} with the measured TEC_z

There are several advantages of using GPS receivers to study the ionosphere:

(1) The measured *TEC* is the contribution from all ionospheric layers, *D, E, F1, F2*, topside *F*, and the plasmasphere, since the height of GPS satellites is about 20000 km above the earth's surface.

(2) We are able to monitor and measure the ionospheric *TEC* in real time.

(3) The observations can be done any time and anywhere on the earth, including high latitude places and the equatorial region, as there are always enough satellites seen by the GPS receivers.

This research may help us to understand the ionospheric behaviour in a certain area. It also helps us to estimate the ionospheric error of a single frequency receiver in which the ionospheric influence usually is corrected for by means of a model. The impact of ignoring higher–order terms in the ionospheric delay compensation could also be studied — an important topic in the case of the low elevation observations that are needed to improve the vertical coordinate estimation.

2 Ionospheric radio wave propagation

the velocity of radio waves depends on the propagation medium. The phase velocity is given by

$$v_p = \frac{c}{n_p} \tag{1}$$

where c is the speed of light in vacuum, and n_p is the phase refraction index of the medium.

In the case of the ionosphere the phase refraction index n_p depends on many factors including the frequency of the radio wave, the charged particle density (electrons and ions), particle collisions, the earth's magnetic field. If we neglect the interaction between the radio wave and the heavy ions, n_p becomes [3]

$$n_p^2 = 1 - \frac{X}{1 - iZ - \frac{(Y\sin\theta)^2}{2(1-X-iZ)} \pm \left(\frac{(Y\sin\theta)^4}{4(1-X-iZ)^2} + (Y\cos\theta)^2 \right)^{0.5}} \tag{2}$$

$$X = \frac{f_p^2}{f_0^2}; \qquad f_p = \frac{N_e^2}{(4\pi^2 \epsilon_0 m)^{0.5}} \tag{3}$$

$$Y = \frac{f_h}{f_0} \qquad f_h = \frac{eB}{2\pi m} \tag{4}$$

$$Z = \frac{f_c}{2\pi f_0} \tag{5}$$

where f_p is the plasma frequency of the ionosphere; f_h is the gyro frequency of the electrons in the earth's magnetic field; f_c is the collision frequency; f_0 is the carrier frequency of the radio wave; N_e is the electron density of the ionosphere; e is the charge of the electron; m is the mass of the electron; ϵ_0 is the dielectric constant of free space; B is the magnetic induction of the earth's magnetic field; and θ is the angle between the Earth's magnetic field and the propagation direction of the radio wave. The plus and minus signs in (2) correspond to the refractive indices of the two circular polarizations.

In the case of GPS f_0 is either 1575.42 MHz (f_1) or 1227.60 MHz (f_2). Maximum f_p–,f_h–varues of the ionosphere are of the order of 10 MHz, anc 1.4 MHz, respectively. f_c is even smaller. Thus $f_0>>f_p$, $f_0>>f_h$, and $f_0>>f_c$, and equation (2) can be simplified to

$$n_p = 1 - \frac{f_p^2}{2f_0^2} \pm \frac{f_p^2 f_h \cos\theta}{2_0^3} \tag{6}$$

If we totally neglect the effect of the earth's magnetic field, a reasonable assumption in our case as the second term is approximately 10^4times larger than the third term, (6) can be further simplified to

$$n_p = 1 - \frac{f_p^2}{2f_0^2} \tag{7}$$

The phase velocity can now be written as

$$v_p = C\left(1 + \frac{f_p^2}{2f_0^2}\right) \tag{8}$$

Experssion (8) shows that when a radio wave travels in the ionosphere,its phase velocity is larger than the speed of light in vacuum c.

3 The group velocity and the group delay

Let us assume a wave packet at point S expressed as

$$u_s(0,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} A(k) e^{-jwt} dk \tag{9}$$

where $k = w / v_p = 2\pi / \lambda$ is the wave number.

We also assume that this wave packet travels a distance L in the ionosphere from S to R and that the electron density along the path is constant. The signal at point R can be writted as

$$u_R(L,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} A(k) e^{-j(wt-kL)} dk \tag{10}$$

where k is a function of ω as the ionosphere is dispersive.

If $A(k)$ is sharply peaked around $k = k_0 = w_0 / v_p$, we can solve eq. (10) through a linear approximation of $w(k)$ around k_0. With

$$w = w_0 + (k - k_0) \left.\frac{dw}{dk}\right|_{w_0} \tag{11}$$

the solution becomes

$$u_R(L,t) = (\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} A(k) e^{-j(w-w_0)[t-L/(dw/dk)|_{w_0}]} dk) e^{-j[(w_0(t-L)/v_p]} \tag{12}$$

Equation (12) shows that the wave packet propagates fron S to R without being distorted and that the propagation velocity of the packet is

$$u_g = \frac{d\omega}{dk} \tag{13}$$

This is the so-called group velocity which in the case of the ionosphere with $K = w / v_p$ and $v_p = c[1 + (f_p^2 / 2f_0^2)]$ becomes

$$v_g = c\left(1 - \frac{f_p^2}{2f_0^2}\right) \tag{14}$$

From equation (14) we define the group delay and the group refractive index n_g as

$$Gt_g = \frac{L}{v_g}; \qquad n_g = \frac{c}{v_g} = 1 + \frac{f_p^2}{2f_0^2} \tag{15}$$

We should point out that when a radio wave travels in the ionosphere, in addition to the change of the propagation velocity, also the bending from a straight propagation path causes extra delay. This extra delay can be expressed by

$$Dt_b = 67.7\left(\frac{dN_e}{dh}\right)^2 D^3 \frac{\sin^2 i}{\cos^3 i} \frac{1}{f_0^4} \tag{16}$$

where dN_e / dh is the rate of the electron density change N_e with height above 350 km; D is the thickness of the ionosphere; i is the incident angle of a radio wave; f_0 is the carrier frequency of the radio wave.

If f_0, dN_e / dh, D, i are assumed to be 1.5 GHz, $-3 \times 10^6 / m^4$, 1000 km, and 60 ° , respectively, then Dt_b will be equal to 0.00024 ns. Therefore,if f_0 is as high as 1.5 GHz, the delay will be small enough to be neglected.

4 The apparent distance

The apparent distance as estimated from the measured delay of a radio signal propagating a geometric distance L in a homogeneous ionosphere is

$$L_p = Dt_p c = L\left(1 - \frac{f_p^2}{2f_0^2}\right); \qquad L_g = Dt_g c = L\left(1 - \frac{f_p^2}{2f_0^2}\right) \tag{17}$$

where Dt_p is the phase delay; L_p and L_g are estimated from the phase and the envelope of the signal, respectively; and $f_0 >> f_p$. Usually we refer to L_p and L_g as the apparent distances. Substituting the plasma frequency $f_p = (80.616N_e)^{0.5}$ into (17) we obtain

$$L_p = L\left(1 - \frac{40.308N_e}{f_0^2}\right); \qquad L_g = L\left(1 - \frac{40.308N_e}{f_0^2}\right) \tag{18}$$

Now let us assume that a radio wave transmitted by a satellite S penetrates the ionosphere before reaching the receiver R. We assume that the thickness of the ionosphere is L and that the electron density N_e is a function of the position in the ionosphere. Then the apparent phase and group distance between the satellite and the receiver will be

$$r_p = r - \frac{40.308}{f_0^2}\int_L N_c dL = r - \frac{40.308TEC}{f_0^2} \tag{19}$$

$$r_g = r + \frac{40.308}{f_0^2} \int_L N_e dL = r + \frac{40.308 TEC}{f_0^2} \tag{20}$$

Here r is the geometric distance between the satellite and the receiver and TEC is the total electron content along the path L.

$$TEC = \int_L N_e dL \tag{21}$$

From (19) and (20) we see that $r_p - r$ and $r_g - r$ are opposite in sign and identical in value and that the values are directly proportional to TEC and inversely proportional to f_0^2. However, notice the restriction on f_h and f_c given in Chapter 2.

5 The apparent Doppler frequency shift

The frequency f_r observed by the GPS–receiver will be Doppler–shifted due to the radial velocity between the receiver and the satellite. With

$$f_r = f_0 + f_d \tag{22}$$

where f_0 is the carrier frequency transmitted by the satellite, and f_d is the Doppler frequency shift, the value of the doppler frequency shift f_d becomes

$$f_d = -f_0 \frac{\frac{dr}{dt}}{c} \tag{23}$$

where higher order terms have been ignored. The signal transmitted by the satellite must penetrate the ionosphere before reaching the receiver at the Earth's surface. Taking the ionospheric phase advance into account, the Doppler frequency shift as measured from the carrier becomes

$$f_{di} = -f_0 \frac{\frac{dr_p}{dt}}{c} \tag{24}$$

where f_{di} is called the apparent Doppler shift. By combining (24) with (19), we obtain

$$f_{di} = -f_0 \frac{\frac{dr}{dt}}{c} + \frac{40.308 \frac{dTEC}{dt}}{c f_0} \tag{25}$$

The effect of the ionosphere causes a extra Doppler frequency shift $40.308(dTEC / dt) / cf_0$. Its value is directly proportional to TEC / dt and inversely proportional to f_0. There are two reasons causing $dTEC / dt$ to change; the change of N_e versus time and the change of the ionospheric path L of the signal through the ionosphere.

6 Dual–frequency observations

The prime observables in satellite navigation systems are the signal propagation time and the Doppler frequency shift .The propagation time Dt_g is measured from the envelope of the signal, while the Doppler shift F_{di} is measured from the carrier. From the obtained values of Dt_g and f_{di} the apparent distance and radial velocity are

$$r_g = r + \frac{40.308TEC}{f_0^2} \tag{26}$$

$$\frac{dr_p}{dt} = \frac{dr}{dt} - \frac{40.308\frac{dTEC}{dt}}{f_0^2} \tag{27}$$

In order to obtain the real distance r and the real radial velocity we must remove the extra apparent distance due to the ionosphere, $40.308TEC / f_0^2$, and the extra velocity $-40.308(dTEC / dt) / f_0^2$ from (26)and (27). Vice versa,to obtain the ionospheric parameters we have to remove r and dr / dt.

In the case of GPS each satellite transmits two carrier frequencies, f_1 and f_2, with the same envelope and the GPS–receiver measures the two envelope delays.In this way we obtain two independent equations.

$$r_{g1} = r + \frac{40.308TEC}{f_1^2}; \quad r_{g2} = r + \frac{40.308TEC}{f_2^2} \tag{28}$$

where r_{g1} and r_{g2} are the two apparent distances associated with the two measured envelope delays. From equations (28), we obtain r and TEC simultaneously.

$$r = \frac{r_{g1}f_1^2 - r_{g2}f_2^2}{f_1^2 - f_2^2} \tag{29}$$

$$TEC = \frac{(r_{g2} - r_{g1})(f_1 f_2)^2}{40.308(f_1^2 - f_2^2)} \tag{30}$$

If the receiver also measures the Doppler frequency shifts of the two carrier frequencies, we obtain another set of equations

$$\frac{dr_{p1}}{dt} = \frac{dr}{dt} - \frac{40.308\frac{dTEC}{dt}}{f_1^2}; \quad \frac{dr_{p2}}{dt} = \frac{dr}{dt} - \frac{40.308\frac{dTEC}{dt}}{f_2^2} \tag{31}$$

where dr_{p1} / dt and dr_{p2} / dt are the two apparent radial velocities corresponding to the measured apparent Doppler frequency shifts. We obtain from equations (31)

$$\frac{dr}{dt} = \frac{\frac{dr_{p1}}{dt}f_1^2 - \frac{dr_{p2}}{dt}f_2^2}{f_1^2 - f_2^2} \tag{32}$$

$$\frac{dTEC}{dt} = \frac{\left(\frac{dr_{p1}}{dt} - \frac{dr_{p2}}{dt}\right)(f_1 f_2)^2}{40.308(f_1^2 - f_2^2)} \tag{33}$$

This method enable the receiver to estimate the ionospheric correction in real time, which is very important for navigation. Both (30)and (33) are also the fundamental equations for the ionosphere research.

When the radio wave passes through the atmosphere from the satillite to a receiver, the measured group delay includes not only geometric distance delay and ionospheric extra delay but also the tropospheric extra delay Dt_a, the time difference Dt_t between the satillite and receiver clocks, and the instrumental delay Dt_e [1,2]. The measured group delay could be written as

$$Dt_g = \frac{r}{c} + Dt_i + Dt_a + Dt_t + Dt_e \tag{34}$$

By using dual–frequency observation, we have

$$Dt_{g1} = \frac{r}{c} + Dt_{i1} + Dt_a + Dt_t + Dt_{e1}; \quad Dt_{g2} = \frac{r}{c} + Dt_{i2} + Dt_a + Dt_t + Dt_{e2} \tag{35}$$

The corresponding apparent ranges, pr_{g1} and pr_{g2}, which usually are called pseudoranges, are

$$pr_{g1} = cDt_{g1}; \qquad pr_{g2} = cDt_{g2} \tag{36}$$

As the tropospheric delay D_{ta} and the time difference between the clocks does not change with the carrier frequency and the instrumental delay can be calibrated,we have

$$pr_{g2} - pr_{g1} = r_{g2} - r_{g1}; \quad \frac{dpr_{g2}}{dt} - \frac{dpr_{g1}}{dt} = \frac{dr_{g2}}{dt} - \frac{dr_{g1}}{dt} \tag{37}$$

Thus (30) can be rewritten as

$$TEC = \frac{(pr_{g2} - pr_{g1})(f_1 f_2)^2}{40.308(f_1^2 - f_2^2)} \tag{38}$$

Furthermore we have

$$\frac{dTEC}{dt} = \frac{\left(\frac{dpr_{p1}}{dt} - \frac{dpr_{p2}}{dt}\right)(f_1 f_2)^2}{40.308(f_1^2 - f_2^2)} \tag{39}$$

7 *TEC* along the path and in the zenith direction

In equation (38) we gave a relation between measured pseudoranges and the total electron content along the path L defined as

$$TEC = \int_L N_e dL \tag{40}$$

It is important to remember that *TEC* is the estimated total electron contents in the direction of the satellite. In order to calculate *TEC* in the zenith direction, denoted TEC_z, we must assume an ionospheric model. Although the structure of the ionosphere is rather complex and time variable, as far as its effect on the observations is concerned, most of its charged particles can be considered to be concentrated into a thin layer which is near the peak electron density area of layer *F*. Its height varies with day and night, the activity of the sun, the geographic position, etc. Typical value is between 250 km and 450 km. We assume the thin layer to be at the constant height of 350 km from the Earth's surface. For convenience we also assume the earth and the ionosphere to be circular with the same centre. With the ionosphere as a shell with radius $r_e + h_i$, where r_e is the radius of the Earth, we obtain the mapping function $f(EL)$ as

$$f(EL) = \cos Z = \left[1 - \left(\frac{\cos EL}{1 + \frac{h_i}{r_e}}\right)^2\right]^{1/2} \tag{41}$$

where Z is given in Fig. 1. From Fig. 1 we also obtain a relation between EL, Z and α:

$$\sin\alpha = \cos(EL + Z) \tag{42}$$

where the earth centre angle α is defined in Fig. 1.

By means of equation (42) we thus map TEC into the zenith total electron content TEC_z[4], using the relation

$$TEC_z = f(EL)TEC \tag{43}$$

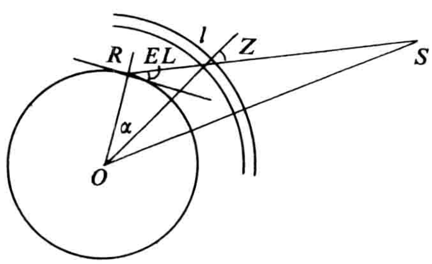

Figure 1. Earth, receiver (R), and satellite (S) configuration.

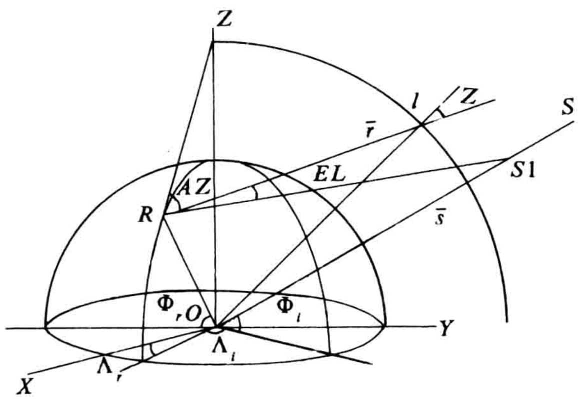

Figure 2. Geometry to derive the geographic coordinates of the ionosphere under investigation.

8 The geographic coordinates of TEC_z

The geographic coordinates of TEC_z, estimated from observations at point R in the direction of the satellite S, are obtained from Figure 2.

We easily derive the following relations by means of spherical geometry:

$$\sin\Phi_i = \sin\Phi_r\cos\alpha + \cos\Phi_r\sin\alpha\cos AZ \tag{44}$$

$$\cos(EC + \alpha) = \frac{r}{r + h_i}\cos EL \tag{45}$$

$$\cos\alpha = \sin\Phi_r\sin\Phi_i + \cos\Phi_r\cos\Phi_i\cos(\Lambda_i - \Lambda_r) \tag{46}$$