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1

INTRODUCTION

1.1 HISTORY

The origin of the branch of mathematics known as *differential equations* dates back at least to 1671 and Newton's classification of first order ordinary differential equations into three classes. Sir Isaac Newton (1642–1727) called these equations fluxional equations instead of differential equations. He assumed that the solution of these equations could be expressed as an infinite series, and he successively determined the coefficients in a manner similar to the technique employed today. However, he did not consider the convergence of the series. The notation $\dot{y}$ for the derivative of y with respect to the independent variable was also introduced by Newton.

Gottfried Leibniz (1646–1716) invented the differential notation dy and the symbol $\int$ for integration. To our knowledge he first used these notations in conjunction in 1675, when he wrote:

$$\int y\,dy = \tfrac{1}{2}y^2.$$

In 1676 Leibniz used the term "differential equation" to denote a relationship between two differentials dx and dy. Thus was christened the branch of mathematics that deals with equations which involve differentials or derivatives.

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The word *integral* was introduced into mathematics in 1690 by Jacques Bernoulli (1654–1705). In 1691 Leibniz discovered the technique of separation of variables and in 1692 he reduced the linear homogeneous first order differential equation to quadratures. Jean Bernoulli (1667–1748) introduced the concept of an integrating factor in 1694 and the technique of changing the dependent variable. By the end of the seventeenth century the techniques which are usually employed when attempting to solve first order ordinary differential equations were known. As we shall discover, these techniques often prove to be inadequate.

However, early in the development of the study of differential equations, it was believed that elementary functions would be sufficient for the representation of solutions of differential equations arising from geometry and mechanics. Thus, early attempts at solving differential equations were directed toward finding explicit solutions or reducing the solution to a finite number of quadratures. By 1723 at the latest it was recognized that even some first order ordinary differential equations do not have solutions which can be expressed in terms of elementary functions. As a matter of fact, if an ordinary differential equation is written down at random, the probability of being able to write the solution in terms of known functions or their integrals is nearly zero. This emphasizes the necessity for developing methods for obtaining approximate solutions.

In 1739 Léonard Euler (1707–1783) introduced the method of variation of parameters. Jean Bernoulli had unsuccessfully attempted to solve the general linear homogeneous differential equations with constant coefficients. Euler gave a complete discussion of this problem in 1743. He also devised the classical method for solving nonhomogeneous linear differential equations.

No adequate discussion of differential equations as a unified topic existed prior to the lectures developed and presented by Augustin-Louis Cauchy (1789–1857) in the 1820s. In these lectures Cauchy developed the first existence and uniqueness theorems for first order differential equations. Cauchy extended his theory to a system in n first order differential equations in n dependent variables which was equivalent to a single nth order differential equation. Rudolf Lipschitz (1832–1903) generalized Cauchy's existence and uniqueness theorems in 1876. Émile Picard (1856–1941) improved upon the theorems of Cauchy and Lipschitz in 1893 by introducing the method of successive approximations.

1.2 CLASSIFICATION OF DIFFERENTIAL EQUATIONS

By a *differential equation* (DE) we shall mean any equation that involves derivatives or differentials of a function or functions. The *order* of a differential equation is the largest positive integer n for which the nth derivative or differential occurs in the differential equation. If a differential equation is written as a polynomial, then the highest power to which the highest derivative appears in the equation is called the *degree* of the equation.

In the study of differential equations, it is both advantageous and convenient to classify the equations into different categories—much as one classifies chemical compounds into organic and inorganic categories in the study of chemistry. The first and most obvious two categories into which differential equations are classified are those of ordinary differential equations and partial differential equations. This classification is based on the unknown function appearing in the differential equation. If the unknown function depends on only one independent variable, then the differential equation is called an *ordinary differential equation* (ODE). Whereas, if the unknown function depends on two or more independent variables, then the differential equation is called a *partial differential equation* (PDE).

For example,

(1) $$y'' + x(y')^2 + xy = x^3$$

is a second order ordinary differential equation of degree one;

(2) $$(y''')^2 + yy'y'' + xy' + y^2 = \sin x$$

is a third order ordinary differential equation of degree two;

(3) $$yz_x + x^2z_y = xy$$

is a first order partial differential equation in two independent variables; and

(4) $$u_t = \alpha(u_{xx} + u_{yy} + u_{zz})$$

is a second order partial differential equation in four independent variables. It should be noted that order is defined for all differential equations, but degree is not defined for some. For instance, $y'' = \sin y$ is a second order differential equation. However, degree is not defined for this equation, since the equation is not a polynomial.

Throughout this text we shall concern ourselves primarily with the solution of ordinary differential equations. However, both ordinary differential equations and partial differential equations are subdivided into two large classes, according to whether they are linear or nonlinear.

The general form of an nth order ordinary differential equation is

(5) $$\phi(x, y, y^{(1)}, \ldots, y^{(n)}) = 0,$$

where $y^{(k)}$ denotes the kth derivative of y with respect to x. An nth order ordinary differential equation is *linear* if ϕ is a linear function in each of the variables $y, y^{(1)}, \ldots, y^{(n)}$. Hence, the general form of a linear nth order ordinary differential equation is

(6) $$a_0(x)y^{(n)} + a_1(x)y^{(n-1)} + \cdots + a_n(x)y = f(x).$$

An nth order ordinary differential equation that cannot be written in the form (6) is called a *nonlinear* nth order ordinary differential equation.

Hence, (1) and (2) are nonlinear ordinary differential equations while

$$x^2y''' + (x^2 - 2)y'' + (\sin x)y' + e^x y = x^2 - 1 \tag{7}$$

is a third order linear ordinary differential equation. Notice that every linear ordinary differential equation is of first degree but not every ordinary differential equation of first degree is linear.

An *explicit solution* of the nth order DE (5) on an interval I is a function $y(x)$ defined on I and satisfying $\phi(x, y(x), y^{(1)}(x), \ldots, y^{(n)}(x)) = 0$ for all x in I. Notice that this definition implies that an explicit solution y has n derivatives on I and, therefore, that $y, y^{(1)}, \ldots, y^{(n-1)}$ are all continuous on I. Generally, the interval I is not specified explicitly, but it is understood to be the largest interval on which y is defined and satisfies (5).

EXAMPLE Consider the differential equation

$$y' + y = 0. \tag{8}$$

The function $y_1(x) = e^{-x}$ is defined and continuous on the interval $(-\infty, \infty)$ and the derivative $y_1'(x) = -e^{-x}$ is defined on $(-\infty, \infty)$. Since

$$y_1'(x) + y_1(x) = -e^{-x} + e^{x} = 0 \quad \text{for all } x \text{ in } (-\infty, \infty);$$

that is, since $y_1(x) = e^{-x}$ satisfies the differential equation $y' + y = 0$ for all x in $(-\infty, \infty)$, $y_1(x) = e^{-x}$ is an explicit solution of the given differential equation for all real x.

The function

$$y_2(x) = \begin{cases} e^{-x}, & x < 0 \\ 2e^{-x}, & x \geq 0 \end{cases}$$

is not an explicit solution of the given differential equation on the interval $(-\infty, \infty)$, since $y_2(x)$ is not continuous and therefore not differentiable at $x = 0$. However, the function $y_2(x)$ is an explicit solution on any interval that does not contain the point $x = 0$.

A relation $f(x, y) = 0$ is said to be an *implicit solution* of the nth order DE (5) on an interval I if the relation defines at least one function $y_1(x)$ on I such that $y_1(x)$ is an explicit solution of (5) on I. We shall usually refer to both explicit and implicit solutions simply as solutions.

EXAMPLE Consider the differential equation

$$yy' + x = 0. \tag{9}$$

We shall show that the relation

$$f(x, y) = y^2 + x^2 - 16 = 0 \tag{10}$$

is an implicit solution on the interval $(-4, 4)$. The graph of equation (10) is a circle of radius 4 with center at the origin. See Figure 1.1(a). Solving equation (10) for y in terms of x, we get

$$y(x) = \pm\sqrt{16 - x^2}.$$

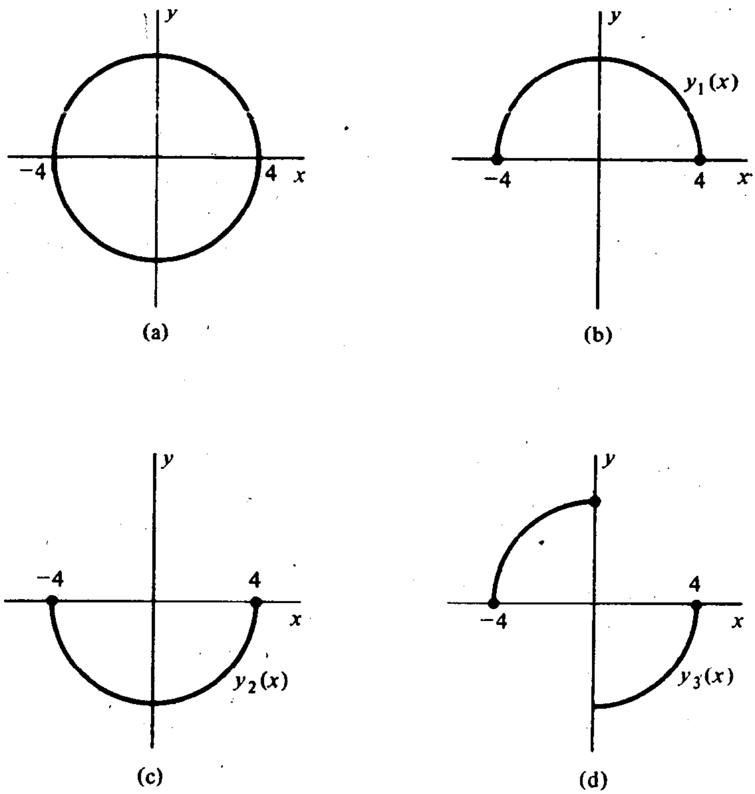

Figure 1.1

The functions

$$y_1(x) = \sqrt{16 - x^2} \tag{11a}$$

and

$$y_2(x) = -\sqrt{16 - x^2} \tag{11b}$$

are both defined and real for x in $[-4, 4]$. The graph of $y_1(x)$ is shown in

Figure 1.1(b) and the graph of $y_2(x)$ is shown in Figure 1.1(c). Differentiating equations (11a) and (11b), we obtain

$$y_1'(x) = -\frac{x}{\sqrt{16 - x^2}} \tag{12a}$$

and

$$y_2'(x) = \frac{x}{\sqrt{16 - x^2}}. \tag{12b}$$

Both y_1' and y_2' are defined and real for x in $(-4, 4)$. Substituting y_1 and y_1' into the differential equation (9), we find that

$$\sqrt{16 - x^2}\left(-\frac{x}{\sqrt{16 - x^2}}\right) + x = -x + x = 0.$$

So $y_1(x)$ is an explicit solution of (9) on the interval $(-4, 4)$, and therefore equation (10) is an implicit solution of (9). Likewise, $y_2(x)$ can be shown to be an explicit soiution of (9) on the interval $(-4, 4)$. Thus, the implicit solution (10) defines at least two explicit solutions of (9) on the interval $(-4, 4)$. The function

$$y_3(x) = \begin{cases} \sqrt{16 - x^2}, & -4 \le x \le 0 \\ -\sqrt{16 - x^2}, & 0 < x \le 4 \end{cases}$$

shown in Figure 1.1(d) satisfies relation (10), $y^2 + x^2 - 16 = 0$; however, $y_3(x)$ is not an explicit solution of (9) on the interval $(-4, 4)$, since $y_3(x)$ is not continuous and therefore not differentiable at $x = 0$.

In this case it was fairly easy to determine an explicit solution from the implicit solution and to determine the interval on which the solution exists. However, this will not generally be the case. Normally, we will not be able to solve a given relation in x and y explicitly for y. Therefore, we will usually obtain a relation in x and y by some means, verify that this relation formally satisfies the particular differential equation under consideration, and say that the relation is an implicit solution. For example, we will say that the relation

$$y^3 + 2xy - x^2 = c, \tag{13}$$

where c is a constant, is an implicit solution of the differential equation

$$y' = \frac{2x - 2y}{3y^2 + 2x}. \tag{14}$$

In order to verify that (13) formally satisfies the differential equation (14), we differentiate (13) with respect to x and solve for y', which gives us (14).

We shall soon discover that, in theory, we will be able to explicitly solve linear differential equations and, in theory, determine the interval on which the solution exists directly from the differential equation itself. However, the best that we will usually be able to accomplish for nonlinear differential equations is to obtain a series or implicit solution. This is one of the primary differences between the kinds of results that we can expect to obtain for linear differential equation versus nonlinear differential equations.

An n-parameter family of functions

$$f(x, y, c_1, c_2, \ldots, c_n) = 0 \tag{15}$$

is called *the general solution* of the nth order DE (5), if (15) is a solution of (5) (explicit or implicit) for every choice of the parameters $c_1, c_2, \ldots, c_n$, and the set of parameters cannot be replaced by another set of parameters with fewer elements and still represent the same set of solutions.

Although the two-parameter family of functions $y = c_1e^{x+c_2}$ satisfies the second order differential equation

$$y'' - y = 0 \tag{16}$$

for every choice of the parameters c_1 and c_2, it is not the general solution. The set of solutions represented by $y = c_1e^{x+c_2}$ can also be represented by the one-parameter family $y = ke^x$, since $y = c_1e^{x+c_2}$ may be rewritten as $y = c_1e^{c_2}e^x = ke^x$. Furthermore, there may be more than one function of the form (15) which is the general solution of (5). For example, $y_1 = c_1e^{-x} + c_2e^x$ and $y_2 = k_1 \sinh x + k_2 \cosh x$ are both general solutions of equation (16). (The reader is asked to verify this fact in Exercise 12 at the end of this section.) So one might well argue that the terminology *a* general solution should be used instead of *the* general solution. However, y_1 and y_2 are just two different representations of the same set of solutions—the general solution. Therefore, we shall follow the customary practice of calling any function of the form (15) which satisfies (5) the general solution. Any solution that is obtained by assigning definite values to the n parameters $c_1, c_2, \ldots, c_n$ of the general solution is called a *particular solution.*

The general solution of the first order differential equation (8) $y' + y = 0$ is the one-parameter family of functions $y = ce^{-x}$. Notice that this is an explicit solution. To verify that this is the general solution, we differentiate and obtain $y' = -ce^{-x}$. Substituting into the differential equation, we see that $y' + y = -ce^{-x} + ce^{-x} = 0$ for all x and all c. So $y = ce^{-x}$ is the general solution on the interval $(-\infty, \infty)$. The function $y_1(x) = e^{-x}$ is the particular solution which is obtained from the general solution by choosing $c = 1$.

The general solution of the first order differential equation (9) $yy' + x = 0$ is

$$y^2 + x^2 = c^2. \tag{17}$$

Notice that this is an implicit solution with one parameter, c. Differentiating

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(17), we obtain $2yy' + 2x = 0$ for any c, and dividing by 2 we get equation (9). So we have formally verified that (17) satisfies (9) for any choice of c. Choosing $c = 4$ or $c = -4$, we get the particular implicit solution (10) $y^2 + x^2 - 16 = 0$.

EXAMPLE Show that

$$y = c_1e^{-2x} + c_2e^x + x \tag{18}$$

is the general solution of the second order linear differential equation

$$y'' + y' - 2y = 1 - 2x. \tag{19}$$

Differentiating equation (18) twice, we obtain

$$y' = -2c_1e^{-2x} + c_2e^x + 1 \tag{20}$$

and

$$y'' = 4c_1e^{-2x} + c_2e^x. \tag{21}$$

Substituting equations (18), (20), and (21) into equation (19), we get

$$(4c_1e^{-2x} + c_2e^x) + (-2c_1e^{-2x} + c_2e^x + 1) - 2(c_1e^{-2x} + c_2e^x + x) = 1 - 2x$$

for all constants c_1 and c_2 and all real x. So equation (18) is the general solution of (19) for all real x.

EXAMPLE Verify that

$$y = (x^2 + c)^2 \tag{22}$$

is the general solution of

$$(y')^2 - 16x^2y = 0. \tag{23}$$

Differentiating (22), we get

$$y' = 4x(x^2 + c) \quad \text{for all real } x \text{ and any constant } c. \tag{24}$$

Squaring equation (24) and substituting for the factor $(x^2 + c)^2$ from equation (22), we find that

$$(y')^2 = 16x^2y \quad \text{for all real } x \text{ and any constant } c.$$

So (22) is the general solution of (23) for all real x.

Let us consider for the moment the function $y = x^2 + 1$. The graph of this function is a parabola with axis the y-axis, with vertex at (0, 1), and which opens upward. The derivative of this function is $y' = 2x$. Given the function y we are able to calculate the derivative y'. The inverse problem is: given the function y', how do we obtain the original function y? The differential equation $y' = 2x = f(x, y)$ defines a real value for each point (x, y)

of the xy-plane. The value at the point $(x, y), f(x, y) = 2x$ in this case, represents the slope of the tangent line to the solution of the differential equation which passes through (x, y). A small segment of the tangent line at various points of the xy-plane are shown in Figure 1.2. The one-parameter

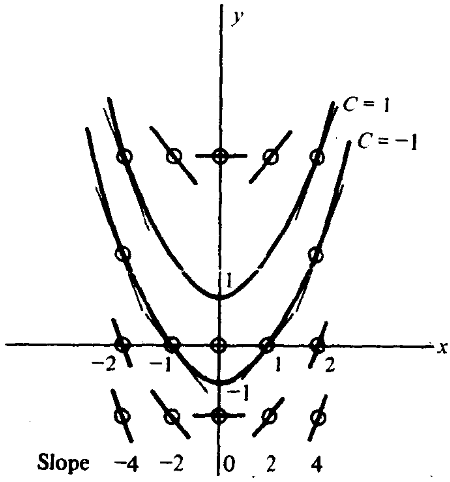

Figure 1.2 Integral curves for the differential equation $y' = 2x$.

family of curves $y(x) = x^2 + C$—parabolas with axis the y-axis, with vertex at $(0, C)$, and which open upward—where C is an arbitrary real number, constitute the set of indefinite integrals of the differential equation $y' = 2x$. The members of this one-parameter family of curves are called the *integral curves* of the differential equation. The curve $y = x^2 + 1$ is a member of this family. Hence, to obtain the original function we must specify, in addition to the differential equation, a point through which the curve is to pass. We might specify, for example, that y is to satisfy $y' = 2x$ and to pass through the point $(1, 2)$—that is, $y(1) = 2$.

Proceeding one step further we see that $y'' = 2$. The set of indefinite integrals of this differential equation is the two-parameter family of curves $y(x) = x^2 + Ax + B$, where A and B are arbitrary real constants. To obtain the original function $y = x^2 + 1$ in this instance, we must specify a combination of conditions that will require us to choose $A = 0$ and $B = 1$. Specifying that (i) $y(1) = 2$ and $y'(1) = 2$ or (ii) $y(1) = 2$ and $y(3) = 10$ will accomplish the desired result. The problem of determining a function that satisfies the differential equation $y'' = 2$ subject to the conditions in (i), called *initial conditions*, is called an *initial value problem*; while the problem of solving the differential equation subject to the conditions in (ii), called *boundary conditions*, is called a *boundary value problem*.

Thus, in the study of ordinary differential equations we are confronted with two large classes of problems—initial value problems and boundary value problems. A precise statement of these two types of problems for nth order ordinary differential equations follows.

An initial value problem (IVP) is a differential equation of the form (5) together with a set of n constraints, the initial conditions (IC), of the form

$$y(x_0) = c_0; \qquad y^{(1)}(x_0) = c_1; \qquad \ldots; \qquad y^{(n-1)}(x_0) = c_{n-1},$$

where $x_0, c_0, c_1, \ldots, c_{n-1}$ are real constants.

A boundary value problem (BVP) is a differential equation of the form (5) together with a set of n constraints, the boundary conditions (BC), specifying values of the function y and/or its derivatives at two or more distinct values of the independent variable x.

Given an algebraic equation such as the polynomial equation $2x^4 - 3x^3 + 3x^2 - 3x + 1 = 0$, we seek the solution set—those values of x which when substituted into the equation yield a true statement. The solution set is often restricted to be a subset of a given set, such as the integers, rationals, reals, or complex numbers. For the example given, the solution set for the integers is $\{1\}$, the solution set for the reals is $\{1, \frac{1}{2}\}$, and the solution set for the complex numbers is $\{1, \frac{1}{2}, +i, -i\}$. The Fundamental Theorem of Algebra states that: "Every polynomial of degree $n \geq 1$ with complex coefficients—$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$, where $a_n, a_{n-1}, \ldots, a_0$ are complex numbers, $n \geq 1$, and $a_n \neq 0$—has n (not necessarily distinct) roots among the complex numbers." Hence, the Fundamental Theorem of Algebra tells us two things concerning the roots of a polynomial with complex coefficients of degree $n \geq 1$. First, the polynomial has n roots, and second, all the roots can be found in the set of complex numbers.

One would like theorems of this nature for both initial and boundary value problems. That is, one would like to have a Fundamental Theorem for Initial Value Problems and a Fundamental Theorem for Boundary Value Problems which state conditions under which a solution to the problem is guaranteed to exist and which also state conditions under which a solution is guaranteed to be unique. The theory for initial value problems is well established and relatively simple. In Chapter 4 we shall state a Fundamental Theorem for an Initial Value Problem and sketch the proof of the theorem. On the other hand, the theory for boundary value problems is very complex and consequently not as well developed. Therefore, we shall not present any general theory for boundary value problems. The complexities inherent in boundary value problems can be attributed at least partially to the interaction of the boundary conditions with the differential equation. The following example illustrates this interaction.

Consider the relatively simple boundary value problem

$$y'' + y = 0; \qquad y(0) = 0, \quad y(a) = \alpha. \tag{25}$$

The general solution of the differential equation is: $y = A \sin x + B \cos x$, where A and B are arbitrary constants. Imposing the first boundary condition, $y(0) = 0$, results in the equation $0 = B \cos 0$. From which we conclude that $B = 0$. Hence, any solution of the BVP (25) must have the form $y = A \sin x$. We now try to satisfy the second boundary condition $y(a) = \alpha$. If $a \neq n\pi$, where n is an integer, then the BVP (25) has a unique solution, namely, $y = \alpha \sin x/\sin a$. If $a = n\pi$ for some integer n and $\alpha \neq 0$, then there is no solution, since imposing the boundary condition results in the equation —and contradition—

$$\alpha = y(a) = y(n\pi) = A \sin n\pi = 0.$$

If $a = n\pi$ for some integer n and $\alpha = 0$, then there are infinitely many solutions, since any value of A satisfies the equation

$$A \sin n\pi = 0,$$

which results from imposing the second boundary condition.

Because of the inherent complexities, we shall defer the study of the theory of boundary value problems until much later in the text, and then we shall only consider special types of boundary value problems.

EXERCISES

1. For each of the following, state whether the ordinary differential equation is linear or nonlinear, and determine its order and degree.
   (a) $y' = a(x)y + b(x)$
   (b) $y' = a(x)y + b(x)y^n \quad (n \neq 0, n \neq 1)$
   (c) $(y')^2 + xy' = x^3$
   (d) $y'' + k^2y = 0$
   (e) $x\,dx + 2y\,dy = 0$
   (f) $2y\,dx + x\,dy = 0$
   (g) $y(y'')^2 + x(y')^3 + y \sin x = 1$
   (h) $x^2y^{(4)} + y = \tan x$

2. Is $y(x) = 1/x$ a solution of the differential equation $y' = -y^2$
   (a) on the interval $[-1, 1]$? Why?
   (b) on the interval $(0, \infty)$? Why?

3. Is $y(x) = |x|$ a solution of the differential equation $(y')^2 = 1$
   (a) on the interval $[-1, 1]$? Why?
   (b) on the interval $(0, \infty)$? Why?

4. For each of the following differential equations, verify that the given function or functions is an explicit solution and specify the interval or intervals on which the solution exists.
   (a) $y' - y^2 = 1$; $y_1 = \tan x$
   (b) $y' + 3y = 1 + 3x$; $y_1 = x$, $y_2 = 2e^{-3x} + x$

(c) $y'' - 4y = 0$; $y_1 = e^{2x}$, $y_2 = 3 \sinh 2x$
(d) $x^2y'' + xy' - y = 0$; $y_1 = x$, $y_2 = 1/x$
(e) $2x^2y'' + xy' - y = 0$; $y_1 = x$, $y_2 = 1/\sqrt{x}$
(f) $y' = \sin x^2$; $y_1 = \int_0^x \sin t^2\, dt$, $y_2 = -\int_x^\pi \sin t^2\, dt$

5. Verify that $y^2 - x = 1$ is an implicit solution of the differential equation $2yy' = 1$ on the interval $(-1, \infty)$.

6. Verify that $xy^2 + x = 1$ is an implicit solution of the differential equation $2xyy' + y^2 = -1$ on the interval $(0, 1)$.

7. Verify that $x = e^{xy}$ is an implicit solution of the differential equation $y' = (1 - xy)/x^2$ on the interval $(0, \infty)$.

8. Verify that the relation $xy^2 + yx^2 = 1$ formally satisfies the differential equation $y' = -y(2x + y)/x(2y + x)$.

9. Verify that the relation $y = e^{xy}$ formally satisfies the differential equation $y' = y^2/(1 - xy)$.

10. Show that $y = ce^{2x} + xe^{2x}$ is the general solution of the differential equation $y' - 2y = e^{2x}$.

11. Show that $y = c_1 \sin x + c_2 \cos x + x$ is the general solution of the differential equation $y'' + y = x$.

12. (a) Show that $y = c_1e^{-x} + c_2e^x$ is the general solution of the differential equation $y'' - y = 0$.
(b) Show that $y = k_1 \sinh x + k_2 \cosh x$ is also the general solution of the differential equation $y'' - y = 0$.

13. Show that $y = c(x + c)$ is the general solution of the differential equation $(y')^2 + xy' - y = 0$.

14. Given that $y = ce^{x^2}$ is the general solution of the differential equation (*) $y' = 2xy$, solve the initial value problem consisting of the DE(*) and the following initial conditions:
(a) $y(0) = 0$ (b) $y(0) = 2$
(c) $y(1) = e^2$

15. Given that $y = c_1e^x + c_2e^{-x}$ is the general solution of the differential equation (0) $y'' - y = 0$, solve the initial value problem consisting of the DE (0) and the following initial conditions:
(a) $y(0) = 1$, $y'(0) = 0$ (b) $y(0) = 0$, $y'(0) = 1$
Solve the boundary value problem consisting of the DE (0) and the following boundary conditions:
(c) $y(0) = 1$, $y(1) = (e^2 + 1)/2e$ (d) $y(0) = 0$, $y(1) = (e^2 - 1)/2e$

16. Given that $y = c_1 \sin x + c_2 \cos x$ is the general solution of the differential equation (+) $y'' + y = 0$, solve the boundary value problem consisting of the DE(+) and the following boundary conditions:

(a) $y(0) = 0,\ y(\pi/2) = 1$ (b) $y(0) = 0,\ y(\pi) = 0$

(c) $y(0) = 0,\ y(\pi/2) = 0$ (d) $y(0) = 0,\ y(\pi) = 1$

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