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**An Introduction to  
Dynamic Meteorology**

Second Edition

**James R. Holton**

# AN INTRODUCTION TO DYNAMIC METEOROLOGY

Second Edition

JAMES R. HOLTON

Department of Atmospheric Sciences  
University of Washington  
Seattle, Washington



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An Introduction to  
Dynamic Meteorology

SECOND EDITION

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**INTERNATIONAL GEOPHYSICS SERIES**

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## Preface

In preparing this revised version of “An Introduction to Dynamic Meteorology” I have been motivated by two primary objectives:

- (1) The incorporation of numerous pedagogical improvements based on suggestions from many colleagues as well as my own experience.
- (2) The inclusion of material based on advances in our knowledge which have occurred since the appearance of the first edition.

Nearly fifty percent of the text has been completely rewritten for this edition. Treatment of the fundamental fluid dynamics necessary for understanding large-scale atmospheric motions has been consolidated into the first five chapters. This group of chapters includes expanded treatments of atmospheric thermodynamics and of the planetary boundary layer. The basics of modern dynamical meteorology are presented in Chapters 7–12. This group of chapters includes an entirely new chapter on the dynamics of the stratosphere as well as extensive revision and updating of the remaining chapters.

In response to many requests, a number of additional problems have been included which span a wide range in difficulty. Answers to selected problems are included and a number of appendixes have been added for the aid of the student.

I am indebted to a large number of colleagues who have suggested improve-

ments in the text. I wish especially to thank Professor Jacques Van Mieghem for his thoughtful critique of the first six chapters of the first edition, and Dr. Duane Stevens and Mr. John Knox for their helpful comments on the manuscript of the present edition. Finally, I wish to thank Ilze Schubert for her expert help in preparing the manuscript.

## Preface to First Edition

During the past decade the rapid advances in the science of dynamic meteorology made during the 1940s and 1950s have been consolidated. There now exists a reasonably coherent theory for the development of midlatitude storms, as well as for the overall general circulation of the atmosphere. Therefore, the subject can now be organized and presented in textbook form.

In this book dynamic meteorology is presented as a cohesive subject with a central unifying body of theory—namely the quasi-geostrophic system. Quasi-geostrophic theory is used to develop the principles of diagnostic analysis, numerical forecasting, baroclinic wave theory, energy transformations, and the theory of the general circulation.

Throughout the book the emphasis is on physical principles rather than mathematical elegance. It is assumed that the reader has mastered the fundamentals of classical physics, and that he has a thorough knowledge of elementary differential and integral calculus. Some use is made of vector calculus. However, in most cases the vector operations are elementary in nature so that the reader with little background in vector operations should not experience undue difficulties.

Much of the material included in this text is based on a two-quarter course sequence for seniors majoring in atmospheric sciences at the University of



Washington. It would also be suitable for first-year graduate students with no previous background in meteorology.

The actual text may be divided into two main sections. The first section, consisting of Chapters 1–6, introduces those fundamentals of fluid dynamics most relevant for understanding atmospheric motions, primarily through consideration of a number of idealized types of atmospheric flow. I have found that nearly all the material in these six chapters can be presented successfully at the senior level in about 30 lectures. The second main section, consisting of Chapters 7–11, contains the core of modern dynamic meteorology with emphasis on the central unifying role of the quasi-geostrophic theory. These chapters contain more material than can easily be covered in a senior level course. However, I have attempted to arrange the material so that sections containing more advanced results can be omitted in elementary courses. On the other hand, for more intensive graduate level courses the material presented here might be supplemented by readings from original sources.

In addition to these two main sections, the book contains a concluding chapter which stands somewhat alone. In this final chapter I have attempted to review the current status of the dynamics of the tropical atmosphere. This chapter is by necessity somewhat speculative; however, the field of tropical meteorology is too important to omit entirely from a textbook on dynamic meteorology.

A short annotated list of suggested references for further reading is presented at the end of most of the chapters. I have limited the references in most cases to books and review articles which I have found to be particularly useful. No attempt has been made to provide extensive bibliographies of original sources. In the reference lists books are referred to by author and title, papers by author and date of publication. The complete references are listed in the bibliography at the end of the book.

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# 1 Introduction

## 1.1 The Atmospheric Continuum

Dynamic meteorology is the study of those motions of the atmosphere which are associated with weather and climate. For all such motions the discrete molecular nature of the atmosphere can be ignored, and the atmosphere can be regarded as a continuous fluid medium, or *continuum*. The various physical quantities which characterize the state of the atmosphere—pressure, density, temperature, and velocity—are assumed to have unique values at each point in the atmospheric continuum. Moreover, these *field variables* and their derivatives are assumed to be continuous functions of space and time. The fundamental laws of fluid mechanics and thermodynamics which govern the motions of the atmosphere may then be expressed in terms of partial differential equations involving the field variables.

The general set of partial differential equations governing the motions of the atmosphere is extremely complex, and no general solutions are known to exist. To acquire an understanding of the physical role of atmospheric motions in determining the observed weather and climate it is necessary to develop models based on systematic simplification of the fundamental governing equations. As we shall see in later chapters the

development of models appropriate to particular atmospheric motion systems requires careful consideration of the scales of motion involved.

## 1.2 Physical Dimensions and Units

The fundamental laws which govern the motions of the atmosphere are expressed in terms of physical quantities (field variables and coordinates) which depend on four dimensionally independent *properties*: length, time, mass, and thermodynamic temperature. The dimensions of all atmospheric field variables may be expressed in terms of multiples and ratios of these four fundamental properties. To measure and compare the scales of atmospheric motions a set of units of measure must be defined for the four fundamental properties.

In this text the international system of units (SI) will be used almost exclusively. The four fundamental properties are measured in terms of the *SI base units* shown in Table 1.1. All other properties are measured in terms of *SI derived units* which are units formed from products and/or ratios of the base units. For example, velocity has the derived units of meter per second ( $\text{m s}^{-1}$ ). A number of important derived units have special names and symbols. Those which are commonly used in dynamic meteorology are indicated in Table 1.2. In addition, the *supplementary unit* designating a plane angle—the radian (rad)—is required for expressing angular velocity ( $\text{rad s}^{-1}$ ) in the SI system.<sup>1</sup>

**Table 1.1** *SI Base Units*

Property	Name	Symbol
Length	Meter (metre)	m
Mass	Kilogram (kilogramme)	kg
Time	Second	s
Temperature	Kelvin	K

In order to keep numerical values within convenient limits it is conventional to use decimal multiples and submultiples of SI units. Prefixes used to indicate such multiples and submultiples are given in Table 1.3. The prefixes of Table 1.3 may be affixed to any of the basic or derived SI units except the kilogram. Since the kilogram already is a prefixed unit decimal multiples and submultiples of mass are formed by prefixing the gram (g), not the kilogram (kg).

<sup>1</sup> Note that the *hertz* measures frequency in *cycles* per second not in radians per second.



**Table 1.2** *SI Derived Units with Special Names*

Property	Name	Symbol
Frequency	Hertz	Hz ( $\text{s}^{-1}$ )
Force	Newton	N ( $\text{kg m s}^{-2}$ )
Pressure	Pascal	Pa ( $\text{N m}^{-2}$ )
Energy	Joule	J ( $\text{N m}$ )
Power	Watt	W ( $\text{J s}^{-1}$ )

Although the use of non-SI units will generally be avoided in this text there are a few exceptions worth mentioning:

- (1) In some contexts the time units minute (min), hour (h), and day (d) may be used in preference to the second in order to express quantities in convenient numerical values.
- (2) For work in dynamic meteorology the kilopascal (kPa) is the preferred SI unit for pressure. However, most meteorologists are accustomed to using the millibar (mb), which is equal to 100 Pa or 0.1 kPa. Thus for the reader's convenience we will generally give pressures in kilopascals followed by the equivalent in millibars—e.g., standard sea level pressure equals 101.325 kPa (1013.25 mb). However, to conform with conventional meteorological practice upper level maps will be referred to using the millibar (e.g., the 500-mb surface).
- (3) In discussing observed temperatures we will generally use the Celsius temperature scale, which is related to the thermodynamic temperature scale as follows:

$$T_c = T - T_0$$

where  $T_c$  is expressed in degrees Celsius ( $^{\circ}\text{C}$ ),  $T$  is the thermodynamic temperature in kelvins (K), and  $T_0 = 273.15 \text{ K}$  is the freezing point of water

**Table 1.3** *Prefixes for Decimal Multiples and Submultiples of SI Units*

Multiple	Prefix	Symbol
$10^6$	Mega	M
$10^3$	Kilo	k
$10^2$	Hecto	h
$10^1$	Deka	da
$10^{-1}$	Deci	d
$10^{-2}$	Centi	c
$10^{-3}$	Milli	m
$10^{-6}$	Micro	$\mu$