Modern Mathematical Methods for Physicists and Engineers

C. D. Cantrell

MODERN MATHEMATICAL METHODS FOR PHYSICISTS AND ENGINEERS

C. D. CANTRELL

CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK   www.cup.cam.ac.uk
40 West 20th Street, New York, NY 10011-4211, USA   www.cup.org
10 Stamford Road, Oakleigh, Melbourne 3166, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain

First published 2000

Printed in the United States of America

Typeface Times Roman 10.5/13 pt.   *System* LaTeX 2ε   [TB]

A catalog record for this book is available from
the British Library.

Library of Congress Cataloging in Publication Data

Cantrell, C. D. (Cyrus D.), 1940–
Modern mathematical methods for physicists and engineers / C. D.
Cantrell.
p. cm.
Includes bibliographical references (p. –).
ISBN 0-521-59180-5 (hb). – ISBN 0-521-59827-3 (pbk.)
1. Mathematics. I. Title.
QA37.2.C24 1999
510 – dc21   98-24761
CIP

ISBN 0 521 59180 5 hardback
ISBN 0 521 59827 3 paperback

MODERN MATHEMATICAL METHODS FOR PHYSICISTS AND ENGINEERS

The advent of powerful desktop computers has revolutionized scientific analysis and engineering design in fields as disparate as particle physics and telecommunications. *Modern Mathematical Methods for Physicists and Engineers* provides an up-to-date mathematical and computational education for students, researchers, and practicing engineers.

The author begins with a review of computation and then deals with a range of key concepts including sets, fields, matrix theory, and vector spaces. He then goes on to cover more advanced subjects such as linear mappings, group theory, and special functions. Throughout, he concentrates exclusively on the most important topics for the working physical scientist or engineer, with the aim of helping them to make intelligent use of the latest computational and analytical methods.

The book contains well over 400 homework problems and covers many topics not dealt with in other textbooks. It will be an ideal textbook for senior undergraduate and graduate students in the physical sciences and engineering, as well as a valuable reference for working engineers.

C. D. Cantrell received his Ph.D. from Princeton University in 1968. He taught at Swarthmore College from 1967 until 1973 and was a staff member at the Los Alamos National Laboratory from 1973 until 1979. Since then he has been at the University of Texas at Dallas, where he is Professor of Physics and Electrical Engineering, and Director of the Photonic Technology and Engineering Center. Professor Cantrell is a consultant for Alcatel USA and Ericsson and is a Fellow of the American Physical Society, the Optical Society of America, and the IEEE.

To Lynn, Kate, and Sarah

PREFACE

The purpose of *Modern Mathematical Methods for Physicists and Engineers* is to help graduate and advanced undergraduate students of the physical sciences and engineering acquire a sufficient mathematical background to make intelligent use of modern computational and analytical methods. This book responds to my students' repeated requests for a mathematical methods text with a modern point of view and choice of topics.

For the past fifteen years I have taught graduate courses in computational and mathematical physics. Before introducing the course on which this book is based, I found it necessary, in courses ranging from numerical methods to the applications of group theory in physics, to summarize the rudiments of linear algebra and functional analysis before proceeding to the ostensible subjects of the course. The questions of the students who studied early drafts of this work have helped to shape the presentation. Some students working concurrently in nearby telecommunication, semiconductor, or aerospace, industries have contributed significantly to the substance of portions of the book.

The following is an example of the situations that motivated me to take the time to write a mathematical methods text that breaks significantly with the past: Every semester, students come to my office, puzzled over numerical models in which minor changes in the data produce drastic changes in the outputs. Unfortunately most of these students lack the mathematical background needed to conceptualize some of the most common problems of numerical computation. For an engineer, and for the increasingly large fraction of physics graduates who make careers in numerical modeling or electrical engineering, conceptual understanding of analytical and numerical models is an absolutely essential ingredient of successful designs. A computer can be a tool for understanding, and not merely a means for obtaining a numerical answer of unknown reliability and significance, only in the hands of those who understand the foundations and potential shortcomings of numerical methods. Yet the traditional mathematical methods taught to students in engineering and physics for most of the twentieth century do not provide a sufficient background even for introductory graduate texts on many important contemporary topics, of which numerical computation is only one.

What upper-level undergraduate and first-year graduate students in physics and engineering tend consistently to lack is an understanding of basic mathematical structures – groups, rings, fields, and vector spaces – and of mappings that preserve these structures. In times gone by, students learned mathematical structures though intensive practice with examples. However, in curricula that already are under fire for taking too many years, there simply is no time to learn the language of mathematics by example. Like adults who learn grammar in order to accelerate the acquisition of a foreign language, contemporary students in physics or engineering can more easily acquire a durable understanding of applied and numerical mathematics if they have been exposed to the most essential formal mathematical structures.

The core of *Modern Mathematical Methods for Physicists and Engineers* is linear algebra and basic functional analysis. Computation is the subject of two of the first three chapters because computational examples and exercises occur throughout the book. Chapters on sets and groups, rings and fields provide necessary background for subsequent chapters on vector spaces, inner-product spaces, linear mappings, and matrix representations of finite groups. Group-throry concepts provide an approach to partial differential equations and special functions based on algebra instead of complex analysis. Throughout the book, abstraction is not an end in itself, but a means for students to remember concepts and use them intelligently.

The exercises range in difficulty from simple applications of the definitions in the text to problems that may challenge strong students. In both the text and the exercises, asterisks indicate material that is unusually difficult, and that may be omitted on a first reading.

The manuscript for this book was created in LaTeX on a Macintosh Power Book® using the program Textures®. The illustrations were created using Adobe Illustrator®.

I thank all those who have contributed to this book, especially my students. Special thanks are due to Professors William J. Pervin and Poras Balsara, and to Dawn Hollenbeck, for their valuable comments on portions of the manuscript.

CONTENTS