葛斯龍
微積分題解

著作者

龔晨　王正鑴　江愛滋

發行者

周錫清

KEY TO
GRANVILLE SMITH AND LONGLEY

ELEMENTS OF THE DIFFERENTIAL AND INTEGRAL CALCULUS

"DIFFERENTIAL CALCULUS"

東華書社印行

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序

三氏微積分題解總於能與讀者見面了

我們原來計劃微分積分一次出全的，但因篇幅過巨，且人手不及，所以先出上册微分部份；其餘積分部份正在編纂中，一俟完成即問世。

本來三氏微積分原有徐任吾，周文，葉鴻瀛三位先生合編的題解；但因為三先生的題解根據的是三十三年前的舊版，現在新版問世已久，內容頗多增添，在習題方面改易，新增的頗不少，因時因勢，實有重新另編的必要。

關於讀者怎樣使用題解，我想無庸多嘵舌了。如果讀者當它作藍本，不加思索地抄襲一下，那便盡失了，編者編著本書的原意。對於好學深思的讀者，作為一個參窽比較的工具，這是本書編者所深深期望的，自修的讀者遇着難題時參看一下，如能詳細地辨別它的義意。當然也很好。

因為本書編輯的方針，是演繹原理，遵從邏輯規範，所以篇幅較多我們所要致意的是希望補充題（Additional Problems）的解答對讀者有些幫助。

本書的編輯抄寫校對完全是我們三人渺小的力量所成，所以挂誤錯，漏之處定然不免，讀者如肯賜教，我們無不誠怨地接受，俟再版時校正，如對數學上有任何疑難要與我們商討的話我們更是歡迎；可逕寄上海（〇）七浦路怡興里六號王正鑣收轉自當敬覆（但請附郵）若能因學術的交換，而多交到許多誠樸的朋友那更是我們的願望哩！

最後本書蒙東華書社的協助承印，得以早日出版謹謝意

一九四八年九月廿五日序

CHAPTER II

VARIABLES, FUNCTIONS, AND LIMITS.

PROBLEMS P. 9.

1. $f(x) = x^3 - 5x^2 - 4x + 20.$

(a) $f(1) = 1 - 5 - 4 + 20 = 12,$

(b) $f(5) = 125 - 125 - 20 + 20 = 0,$

(c) $\because \begin{cases} f(0) = 0 - 0 - 0 + 20 = 20, \\ f(3) = 27 - 45 - 12 + 20 = -10, \end{cases} \quad \therefore f(0) = -2f(3).$

(d) $\because \begin{cases} f(7) = 343 - 245 - 28 + 20 = 90, \\ f(-1) = -1 - 5 + 4 + 20 = 18. \end{cases} \quad \therefore f(7) = 5f(-1).$

2. $f(x) = 4 - 2x^2 + x^4,$

$f(0) = 4 - 0 + 0 = 4,$

$f(1) = 4 - 2 + 1 = 3,$

$f(-1) = 4 - 2 + 1 = 3,$

$f(2) = 4 - 8 + 16 = 12,$

$f(-2) = 4 - 2(-2)^2 + (-2)^4 = 4 - 8 + 16 = 12.$

3. $F(\theta) = \sin 2\theta + \cos\theta,$

$F(0) = \sin 0 + \cos 0 = 0 + 1 = 1.$

$F(\frac{1}{2}\pi) = \sin\pi + \cos\frac{\pi}{2} = 0 + 0 = 0.$

$F(\pi) = \sin 2\pi + \cos\pi = 0 - 1 = -1.$

4. $f(x) = x^3 - 5x^2 - 4x + 20$

$f(t+1) = (t+1)^3 - 5(t+1)^2 - 4(t+1) + 20 =$

$t^3 + 3t^2 + 3t + 1 - 5t^2 - 10t - 5 - 4t - 4 + 20 = t^3 - 2t^2 - 11t + 12.$

5. $f(y) = y^2 - 2y + 6$

$f(y+h) = (y+h)^2 - 2(y+h) + 6 = y^2 + 2yh + h^2 - 2y - 2h + 6$

$= y^2 - 2y + 6 + 2(y-1)h + h^2.$

6. $f(x) = x^3 + 3x.$

$\because f(x+h) = (x+h)^3 + 3(x+h) = x^3 + 3x^2h + 3hx^2 + h^3 + 3x + 3h$

$= (x^3 + 3x) + 3(x^2+1)h + 3xh^2 + h^3$

$= f(x) + 3(x^2+1)h + 3xh^2 + h^3$

$\therefore f(x+h) - f(x) = 3(x^2+1)h + 3xh^2 + h^3.$

7. $f(x) = \frac{1}{x}$, $f(x+h) = \frac{1}{x+h}$,

$\therefore f(x+h) - f(x) = \frac{1}{x+h} - \frac{1}{x} = \frac{x-(x+h)}{x^2+xh} = \frac{-h}{x^2+xh}.$

8. $\phi(z) = 4^z$, $\phi(z+1) = 4^{z+1} = 4 \cdot 4^z$,

$\therefore \phi(z+1) - \phi(z) = 3 \cdot 4^z = 3\phi(z).$

9. $\because \phi(x) = a^x$, $\therefore \phi(y) = a^y$, $\phi(z) = a^z$,

and $\phi(y+z) = a^{y+z}$. From law of index,

$a^y \cdot a^z = a^{y+z}$, So that $\phi(y) \cdot \phi(z) = \phi(y+z).$

10. $\phi(x) = \log\left(\frac{1-x}{1+x}\right)$, $\phi(y) = \log\left(\frac{1-y}{1+y}\right)$,

$\phi(z) = \log\left(\frac{1-z}{1+z}\right)$, $\phi(y) + \phi(z) = \log\left(\frac{1-y}{1+y}\right) + \log\left(\frac{1-z}{1+z}\right)$

$= \log\left(\frac{1-y}{1+y}\right)\left(\frac{1-z}{1+z}\right)$, but $\phi\left(\frac{y+z}{1+yz}\right) = \log\left[\frac{1-\frac{y+z}{1+yz}}{1+\frac{y+z}{1+yz}}\right]$

$$= \log \left[\frac{\frac{1-y-z+yz}{1+yz}}{\frac{1+y+z+yz}{1+yz}} \right] = \log \left[\frac{1-y-z+yz}{1+y+z+yz} \right]$$

$$= \log \frac{(1-y)(1-z)}{(1+y)(1+z)} = \log \left(\frac{1-y}{1+y}\right)\left(\frac{1-z}{1+z}\right)$$

So that $\phi(y) + \phi(z) = \phi\left(\frac{y+z}{1+yz}\right)$.

11. $\because f(x) = \sin x$

$f(x+2h) = \sin(x+2h)$

$\therefore f(x+2h) - f(x) = \sin(x+2h) - \sin x =$

$2 \sin \frac{1}{2}[x+2h-x] \cos \frac{1}{2}[x+2h+x] = 2\cos(x+h)\sin h.$

PROBLEMS P.15.

2. $\lim\limits_{x\to\infty} \frac{4x+5}{2x+3} = \lim\limits_{x\to\infty} \left(\frac{4+\frac{5}{x}}{2+\frac{3}{x}}\right) = \frac{4}{2} = 2.$

3. $\lim\limits_{t\to 0} \frac{4t^2+3t+2}{t^3+2t-6} = \frac{2}{-6} = -\frac{1}{3}.$

4. $\lim\limits_{h\to 0} \frac{x^2h+3xh^2+h^3}{2xh+5h^2} = \lim\limits_{h\to 0} \frac{x^2+3xh+h^2}{2x+5h} = \frac{x^2}{2x} = \frac{x}{2}.$

5. $\lim\limits_{h\to\infty} \frac{3h+2xh^2+x^2h^3}{4-3xh-2x^3h^3} = \lim\limits_{h\to\infty} \frac{\frac{3}{h^2}+\frac{2x}{h}+x^2}{\frac{4}{h^3}-\frac{3x}{h^2}-2x^3} = \frac{x^2}{-2x^3} = -\frac{x}{2}.$

6. $\lim\limits_{k\to 0} \frac{(2z+3k)^3-4k^2z}{2z(2z-k)^2} = \lim\limits_{k\to 0}\left[\frac{8z^3+36z^2k+54zk^2+27k^3-4k^2z}{8z^3-8z^2k+2k^2z}\right]$

$= \frac{8z^3}{8z^3} = 1.$

7. $\lim_{y\to\infty} \frac{4y^2-3}{2y^3+3y^2} = \lim_{y\to\infty} \frac{\frac{4}{y}-\frac{3}{y^3}}{2+\frac{3}{y}} = 0.$

8. $\lim_{x\to\infty} \frac{6x^3-5x^2+3}{2x^3+4x-7} = \lim_{x\to\infty} \frac{6-\frac{5}{x}+\frac{3}{x^3}}{2+\frac{4}{x^2}-\frac{7}{x^3}} = \frac{6}{2} = 3.$

9. $\lim_{x\to\infty} \frac{a_0x^n+a_1x^{n-1}+\cdots+a_n}{b_0x^n+b_1x^{n-1}+\cdots+b_n} = \lim_{x\to\infty} \frac{a_0+\frac{a_1}{x}+\cdots+\frac{a_n}{x^n}}{b_0+\frac{b_1}{x}+\cdots+\frac{b_n}{x^n}}$

$= \frac{a_0}{b_0}.$

10. $\lim_{x\to 0} \frac{a_0x^n+a_1x^{n-1}+\cdots+a_n}{b_0x^n+b_1x^{n-1}+\cdots+b_n} = \frac{0+0+\cdots+a_n}{0+0+\cdots+b_n} = \frac{a_n}{b_n}.$

11. $\lim_{x\to\infty} \frac{ax^4+bx^2+c}{dx^5+ex^3+fx} = \lim_{x\to\infty} \frac{\frac{a}{x}+\frac{b}{x^3}+\frac{c}{x^5}}{d+\frac{e}{x^2}+\frac{f}{x^4}} = 0.$

12. $\lim_{x\to\infty} \frac{ax^4+bx^2+c}{dx^3+ex^2+fx+g} = \lim_{x\to\infty} \frac{ax+\frac{b}{x}+\frac{c}{x^3}}{d+\frac{e}{x}+\frac{f}{x^2}+\frac{g}{x^3}} = \infty.$

13. $\lim_{s\to a} \frac{s^4-a^4}{s^2-a^2} = \lim_{s\to a} \frac{(s^2-a^2)(s^2+a^2)}{(s^2-a^2)} = \lim_{s\to a}(s^2+a^2)$

$= a^2+a^2 = 2a^2.$

14. $\lim_{h\to 0} \frac{(x+h)^n-x^n}{h} = \lim_{h\to 0} \frac{x^n+nx^{n-1}h+\cdots+h^n-x^n}{h}.$

$= \lim_{h\to 0}(nx^{n-1}+\cdots+h^{n-1}) = nx^{n-1}$

15. $\lim_{x\to 2}\frac{x^2+x-6}{x^2-4}=\lim_{x\to 2}\frac{(x-2)(x+3)}{(x-2)(x+2)}=\lim_{x\to 2}\frac{x+3}{x+2}=\frac{5}{4}$

17. $\because f(x)=x^2$, and $f(x+h)=x^2+2hx+h^2$,

$\therefore \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{2hx+h^2}{h}=\lim_{h\to 0}(2x+h)=2x$.

18. $f(x)=ax^2+bx+c$

$f(x+h)=a(x+h)^2+b(x+h)+c$

$f(x+h)-f(x)=ax^2+2ahx+ah^2+bx+bh+c$
$-ax^2-bx-c=2ahx+bh+ah^2$

$=(2ax+b)h+ah^2$

$\therefore \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}[2ax+b+ah]=2ax+b$.

19. $f(x)=\frac{1}{x}$, $f(x+h)=\frac{1}{x+h}$,

$f(x+h)-f(x)=\frac{1}{x+h}-\frac{1}{x}=\frac{x-(x+h)}{x(x+h)}=\frac{-h}{x^2+hx}$,

$\frac{f(x+h)-f(x)}{h}=\frac{-1}{x^2+hx}$

$\therefore \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{-1}{x^2+hx}=-\frac{1}{x^2}$.

20. $f(x)=x^3$ $f(x+h)=(x+h)^3$

$f(x+h)-f(x)=3hx^2+3h^2x+h^3$

$$\frac{f(x+h)-f(x)}{h} = 3x^2+3hx+h^2.$$

hence $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(3x^2+3hx+h^2) = 3x^2.$

CHAPTER III
DIFFERENTIATION.

PROBLEMS P.25

1. $y = 2-3x.$

(1) $y+\Delta y = 2-3(x+\Delta x) = 2-3x-3\Delta x$

(2) $y = 2-3x$

$\Delta y = -3\Delta x$

(3) $\frac{\Delta y}{\Delta x} = -3,$ (4) $\frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = -3.$

2. $y = mx+b.$

(1) $y+\Delta y = m(x+\Delta x)+b = mx+b+m\Delta x$

(2) $y = mx+b$

$\Delta y = m\Delta x$

(3) $\frac{\Delta y}{\Delta x} = m$

(4) $\frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = m.$

3. $y = ax^2$

(1) $y + \Delta y = a(x+\Delta x)^2 = ax^2 + 2ax\Delta x + a\Delta x^2$

(2) $y = ax^2$

$\Delta y = 2ax\Delta x + a\Delta x^2$

(3) $\frac{\Delta y}{\Delta x} = 2ax + a\Delta x$

(4) $\frac{dy}{dx} = \lim_{\Delta x \to 0} (2ax + a\Delta x) = 2ax.$

4. $S = 2t - t^2$

(1) $S + \Delta S = 2(t+\Delta t) - (t+\Delta t)^2 = 2t + 2\Delta t - t^2 - 2t\Delta t - \Delta t^2$

(2) $S = 2t - t^2$

(3) $\Delta S = 2\Delta t - 2t\Delta t - \Delta t^2$

$\frac{\Delta S}{\Delta t} = 2 - 2t - \Delta t$

(4) $\frac{dS}{dt} = \lim_{\Delta t \to 0} (2 - 2t - \Delta t) = 2 - 2t.$

5. $y = cx^3$

(1) $y + \Delta y = c(x+\Delta x)^3 = cx^3 + 3cx^2\Delta x + 3cx\Delta x^2 + c\Delta x^3$

(2) $y = cx^3$

$\Delta y = 3cx^2\Delta x + 3cx\Delta x^2 + c\Delta x^3$

(3) $\frac{\Delta y}{\Delta x} = 3cx^2 + 3cx\Delta x + c\Delta x^2$

(4) $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} (3cx^2 + 3cx\Delta x + c\Delta x^2) = 3cx^2.$

6. $y=3x-x^3$

(1) $y+\Delta y=3(x+\Delta x)-(x+\Delta x)^3=3x-x^3+3\Delta x-3x^2\Delta x-3x\Delta x^2-\Delta x^3$

(2) $y \quad = \quad 3x-x^3$

$\Delta y = \quad 3\Delta x-3x^2\Delta x-3x\Delta x^2-\Delta x^3$

(3) $\frac{\Delta y}{\Delta x}=3-3x^2-3x\Delta x-\Delta x^2$

(4) $\frac{dy}{dx}=\lim_{\Delta x\to 0}(3-3x^2-3x\Delta x-\Delta x^2)=3-3x^2.$

7. $u=4v^2+2v^3$

(1) $u+\Delta u=4(v+\Delta v)^2+2(v+\Delta v)^3=4v^2+8v\Delta v+4\Delta v^2+2v^3+6v^2\Delta v+6v\Delta v^2+2\Delta v^3$

(2) $u \quad = \quad 4v^2+2v^3$

$\Delta u=(8v+6v^2)\Delta v+4\Delta v^2+6v\Delta v^2+2\Delta v^3$

(3) $\frac{\Delta u}{\Delta v}=8v+6v^2+4\Delta v+6v\Delta v+2\Delta v^2$

(4) $\frac{du}{dv}=\lim_{\Delta v\to 0}\frac{\Delta u}{\Delta v}=\lim_{\Delta v\to 0}(8v+6v^2+4\Delta v+6v\Delta v+2\Delta v^2)$

$=8v+6v^2.$

8. $y=x^4$

(1) $y+\Delta y=(x+\Delta x)^4=x^4+4x^3\Delta x+6x^2\Delta x^2+4x\Delta x^3+\Delta x^4$

(2) $y \quad = \quad x^4$

$\Delta y=4x^3\Delta x+6x^2\Delta x^2+4x\Delta x^3+\Delta x^4$

(3) $\frac{\Delta y}{\Delta x} = 4x^3 + 6x^2\Delta x + 4x\Delta x^2 + \Delta x^3$

(4) $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0}(4x^3 + 6x^2\Delta x + 4x\Delta x^2 + \Delta x^3) = 4x^3.$

9. $\rho = \frac{2}{\theta+1}$

(1) $\rho + \Delta\rho = \frac{2}{\theta+\Delta\theta+1}$

(2) $\rho = \frac{2}{\theta+1}$

$$\Delta\rho = \frac{2}{\theta+\Delta\theta+1} - \frac{2}{\theta+1} = \frac{-2\Delta\theta}{(\theta+1)(\theta+\Delta\theta+1)}$$

(3) $\frac{\Delta\rho}{\Delta\theta} = \frac{-2}{(\theta+1)(\theta+1+\Delta\theta)}$

(4) $\frac{d\rho}{d\theta} = \lim_{\Delta\theta \to 0} \frac{\Delta\rho}{\Delta\theta} = \lim_{\Delta\theta \to 0} \frac{-2}{(\theta+1)(\theta+1+\Delta\theta)} = -\frac{2}{(\theta+1)^2}.$

10. $y = \frac{3}{x^2+2}$

$y + \Delta y = \frac{3}{(x+\Delta x)^2+2}$ $\therefore \Delta y = \frac{-6x\Delta x - 3\Delta x^2}{(x^2+2)[(x+\Delta x)^2+2]}$

$\frac{\Delta y}{\Delta x} = \frac{-6x - 3\Delta x}{(x^2+2)[(x+\Delta x)^2+2]}$ hence

$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = -\frac{6x}{(x^2+2)^2}.$

11. $S=\frac{t+4}{t}=1+\frac{4}{t}$ $\quad S+\Delta S=1+\frac{4}{t+\Delta t}$

$$\Delta S=\frac{4}{t+\Delta t}-\frac{4}{t}=\frac{-4\Delta t}{t^2+t\Delta t}$$

$$\frac{\Delta S}{\Delta t}=\frac{-4}{t^2+t\Delta t} \qquad \frac{ds}{dt}=\lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t}=-\frac{4}{t^2}.$$

12. $y=\frac{1}{1-2x}$ $\quad y+\Delta y=\frac{1}{1-2(x+\Delta x)}$

$$\Delta y=\frac{2\Delta x}{(1-2x)[1-2(x+\Delta x)]}$$

$$\frac{\Delta y}{\Delta x}=\frac{2}{(1-2x)(1-2x-2\Delta x)}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\frac{2}{(1-2x)^2}$$

13. $\rho=\frac{\theta}{\theta+2}=1-\frac{2}{\theta+2}$ $\quad \rho+\Delta\rho=1-\frac{2}{\theta+\Delta\theta+2}$

$$\Delta\rho=\frac{2}{\theta+2}-\frac{2}{\theta+\Delta\theta+2}=\frac{2\,\Delta\theta}{(\theta+2)(\theta+2+\Delta\theta)}$$

$$\frac{\Delta\rho}{\Delta\theta}=\frac{2}{(\theta+2)(\theta+2+\Delta\theta)} \qquad \frac{d\rho}{d\theta}=\lim_{\Delta\theta\to 0}\frac{\Delta\rho}{\Delta\theta}=\frac{2}{(\theta+2)^2}.$$

14. $S=\frac{At+B}{Ct+D}$ $\quad S+\Delta S=\frac{A(t+\Delta t)+B}{C(t+\Delta t)+D}$

$$\Delta S=\frac{A(t+\Delta t)+B}{C(t+\Delta t)+D}-\frac{At+B}{Ct+D}=$$

$$\frac{A(Ct+D)\Delta t - C(At+B)\Delta t}{(Ct+D)(Ct+D+C\Delta t)} = \frac{(AD-BC)\Delta t}{(Ct+D)(Ct+D+C\Delta t)}$$

$$\frac{\Delta S}{\Delta t} = \frac{AD-BC}{(Ct+D)(Ct+D+C\Delta t)}$$

$$\frac{dS}{dt} = \lim_{\Delta t\to 0}\frac{\Delta S}{\Delta t} = \frac{AD-BC}{(Ct+D)^2}.$$

5. $y = \frac{x^3+1}{x} = x^2 + \frac{1}{x}$ $\quad$ $y+\Delta y = (x+\Delta x)^2 + \frac{1}{x+\Delta x}$

$$\Delta y = 2x\Delta x + \Delta x^2 - \frac{\Delta x}{x^2+x\Delta x}$$

$$\frac{\Delta y}{\Delta x} = 2x + \Delta x - \frac{1}{x^2+x\Delta x} \qquad \frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = 2x - \frac{1}{x^2}.$$

16. $y = \frac{1}{x^2+a^2}$ $\quad$ $y+\Delta y = \frac{1}{(x+\Delta x)^2+a^2}$

$$\Delta y = \frac{-2x\Delta x - \Delta x^2}{(x^2+a^2)[(x+\Delta x)^2+a^2]}$$

$$\frac{\Delta y}{\Delta x} = \frac{-2x-\Delta x}{(x^2+a^2)[(x+\Delta x)^2+a^2]}$$

$$\frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \frac{-2x}{(x^2+a^2)^2}.$$

17. $y = \frac{x}{x^2+1}$ $\quad$ $y+\Delta y = \frac{x+\Delta x}{(x+\Delta x)^2+1}$

$$\Delta y = \frac{x+\Delta x}{(x+\Delta x)^2+1} - \frac{x}{x^2+1} = \frac{(x^2+1)\Delta x - x(2x+\Delta x)\Delta x}{(x^2+1)[(x+\Delta x)^2+1]}$$

$$= \frac{(1-x^2-x\Delta x)\Delta x}{(x^2+1)[(x+\Delta x)^2+1]}$$

$$\frac{\Delta y}{\Delta x} = \frac{1-x^2-x\Delta x}{(x^2+1)[(x+\Delta x)^2+1]}$$

$$\therefore \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{1-x^2}{(x^2+1)^2}$$

18. $$y = \frac{x^2}{4+x^2} = 1 - \frac{4}{4+x^2}$$

$$y+\Delta y = 1 - \frac{4}{4+(x+\Delta x)^2}$$

$$\Delta y = \frac{4}{4+x^2} - \frac{4}{4+(x+\Delta x)^2} = \frac{4(2x\Delta x+\Delta x^2)}{(4+x^2)[4+(x+\Delta x)^2]}$$

$$\frac{\Delta y}{\Delta x} = \frac{8x+4\Delta x}{(4+x^2)(4+x^2+2x\Delta x+\Delta x^2)}$$

$$\therefore \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{8x}{(4+x^2)^2}$$

19. $$y = 3x^2-4x-5$$

$$y+\Delta y = 3(x+\Delta x)^2-4(x+\Delta x)-5 = 3x^2+6x\Delta x+3\Delta x^2$$

$$-4x-4\Delta x-5 = 3x^2-4x-5+(6x-4)\Delta x+3\Delta x^2$$

$$\Delta y = (6x-4)\Delta x + 3\Delta x^2.$$

$$\frac{\Delta y}{\Delta x} = 6x-4+3\Delta x \qquad \therefore \frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = 6x-4.$$

20. $s = at^2+bt+c$

$$s+\Delta s = a(t+\Delta t)^2 + b(t+\Delta t)+c$$
$$= at^2+bt+c+(2at+b)\Delta t + a\Delta t^2$$
$$\therefore \Delta s = (2at+b)\Delta t + a\Delta t^2$$
$$\frac{\Delta s}{\Delta t} = 2at+b+a\Delta t$$

hence $\frac{ds}{dt} = \lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t} = \lim_{\Delta t\to 0}(2at+b+a\Delta t) = 2at+b.$

21. $u = 2v^3-3v^2$

$$u+\Delta u = 2(v+\Delta v)^3 - 3(v+\Delta v)^2$$
$$= 2v^3+6v^2\Delta v+6v\Delta v^2+2\Delta v^3-3v^2-6v\Delta v-3\Delta v^2$$
$$\Delta u = 6v^2\Delta v+6v\Delta v^2-6v\Delta v-3\Delta v^2+2\Delta v^3$$
$$\frac{\Delta u}{\Delta v} = 6v^2-6v+6v\Delta v-3\Delta v+2\Delta v^2$$
$$\frac{du}{dv} = \lim_{\Delta v\to 0}\frac{\Delta u}{\Delta v} = 6v^2-6v$$

22. $y = ax^3+bx^2+cx+d$

$$y+\Delta y = a(x+\Delta x)^3+b(x+\Delta x)^2+c(x+\Delta x)+d$$
$$= ax^3+bx^2+cx+d+(3ax^2+2bx+c)\Delta x+(3ax+b)\Delta x^2$$

$+ a\Delta x^3.$

$\therefore \Delta y = (3ax^2+2bx+c)\Delta x + (3ax+b)\Delta x^2 + a\Delta x^3.$

$$\frac{\Delta y}{\Delta x} = 3ax^2+2bx+C+(3ax+b)\Delta x + a\Delta x^2$$

$$\therefore \frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = 3ax^2+2bx+C.$$

23 $\rho = (a-b\theta)^2$

$\rho+\Delta\rho = [a-b(\theta+\Delta\theta)]^2$ $\therefore \Delta\rho = [a-b(\theta+\Delta\theta)]^2-(a-b\theta)^2$

$= (2a-2b\theta-b\Delta\theta)(-b\Delta\theta)$

$$\frac{\Delta\rho}{\Delta\theta} = -b(2a-2b\theta-b\Delta\theta)$$ hence

$$\frac{d\rho}{d\theta} = \lim_{\Delta\theta\to 0}\frac{\Delta\rho}{\Delta\theta} = -2b(a-b\theta).$$

24. $y = (2-x)(1-2x)$

$y+\Delta y = [(2-x)-\Delta x][(1-2x)-2\Delta x]$

$\therefore \Delta y = [(-1)(1-2x)-2(2-x)]\Delta x + 2\Delta x^2$

$= (-1+2x-4+2x)\Delta x + 2\Delta x^2$

$= (4x-5)\Delta x + 2\Delta x^2$

$$\frac{\Delta y}{\Delta x} = 4x-5+2\Delta x$$

hence $\frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = 4x-5.$