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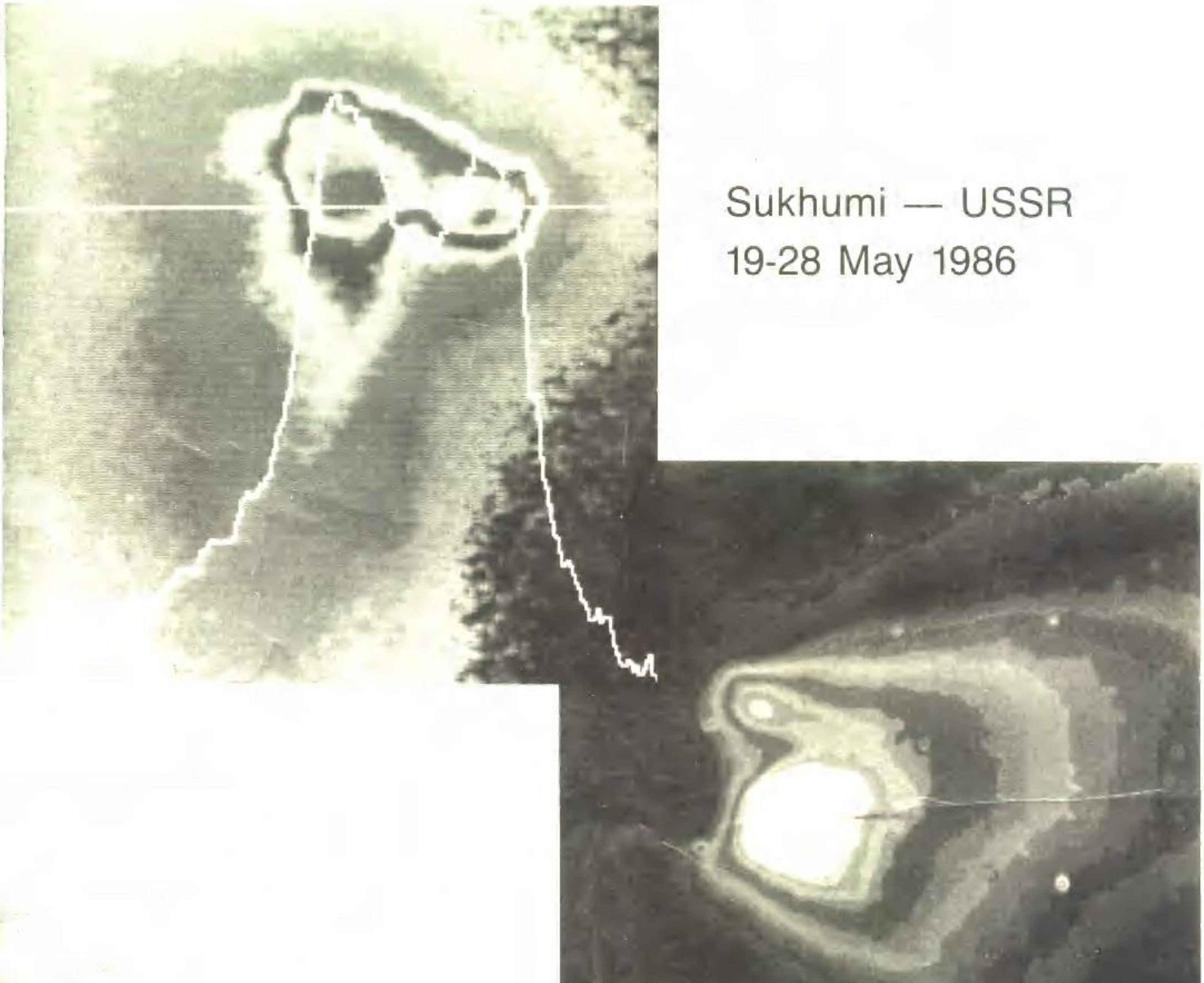

Sukhumi — USSR

19-28 May 1986

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PLASMA ASTROPHYSICS

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INTERNATIONAL SCHOOL & WORKSHOP ON PLASMA ASTROPHYSICS

Sukhumi, Georgia - USSR, 19-28 May 1986

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Cover shows pictures of comet Halley as seen by the Soviet VEGA-2 space probe on 9 March, and by the European satellite Giotto just 370 seconds before the historic encounter on 13/14 March 1986.

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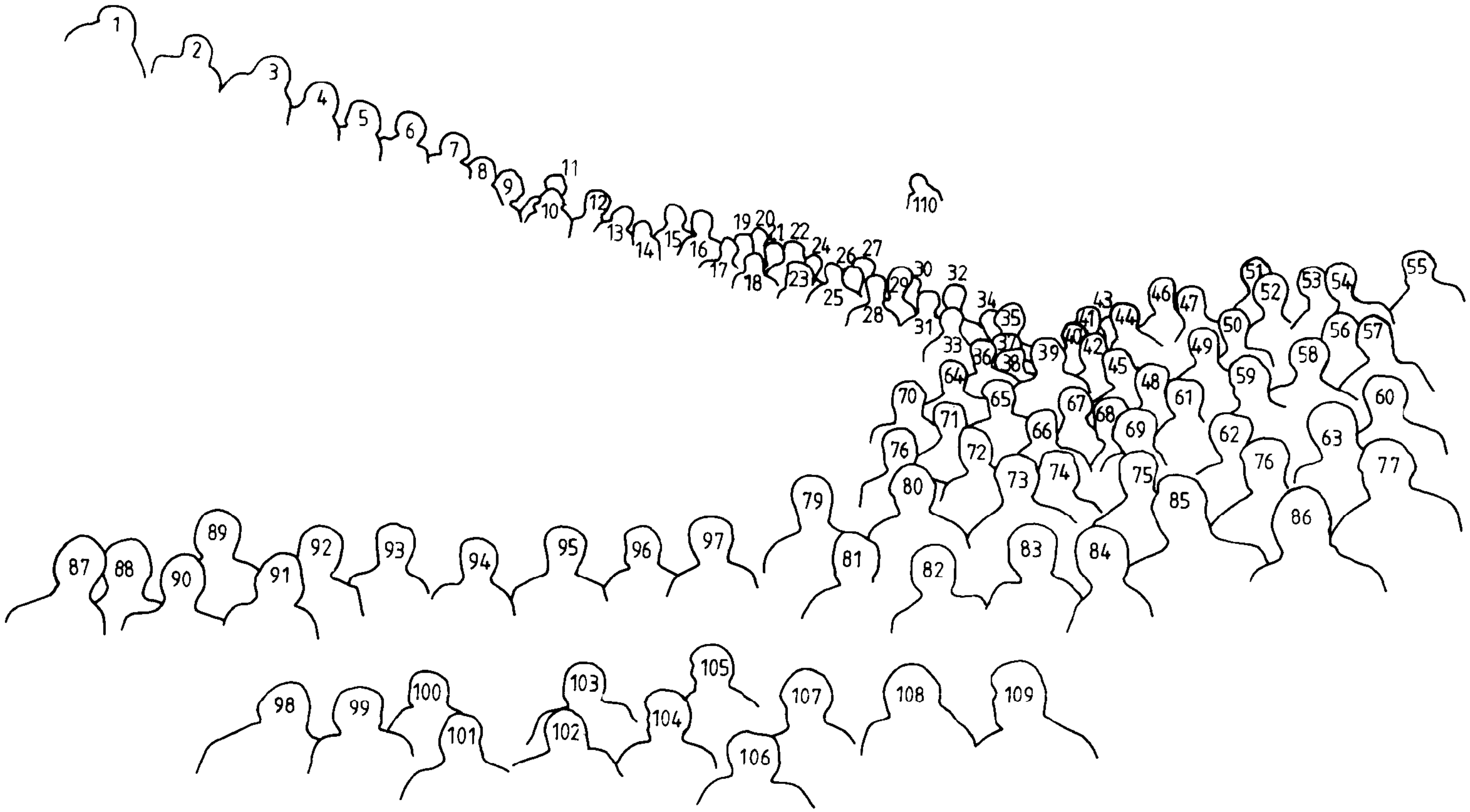

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PLASMA TRANSPORT AND BUBBLING MODEL FOR JUPITER'S MAGNETOSPHERE

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ABSTRACT

A series of plasma voids ('drop-outs') in the low energy particle population and of associated magnetic field perturbations has been observed in Jupiter's magnetosphere during the Voyager-2 spacecraft encounter. We suggest that these drop-outs are evidence of a state of bubbling of the Jovian magnetosphere that alternates with 'laminar' states where, as in the case of the encounter with Voyager-1, voids are not present, and that these states correspond to difference processes by which plasma is transported out. The nature of these states is related to the relative value upstream solar wind ram pressure. In the bubbling state this pressure is higher than in the laminar state and drives an intermittent instability whose onset can be described by the ideal MHD approximation, involving the frozen-in condition. We consider a particular type of ballooning modes that is appropriate for the high-β plasmas, characterizing the considered region of the Jovian magnetosphere, and that have a double peak about the magnetic equatorial plane. This is in fact consistent with the observation of drop-outs both northerly and southerly of this plane. At times of low solar wind ram pressure, as observed during the Voyager-1 encounter, the plasma pressure gradient that is assumed to be lower than the instability threshold of the relevant ideal MHD modes while the cold plasma population is seen to cool further. In this case cross field plasma transport is attributed to a different class of plasma modes, that depend on microscopic processes involving a violation of the frozen-in law, and unload, at a nearly steady state, the plasma injected by Io into the system. The development of this type of interpretation has been motivated by a re-examination of the Voyager-2 data that has led to conclude that the drop-outs phenomenon cannot be a manifestation of a geometrical wake produced by Ganymede as, given the relative proximinity of this satellite during the Voyager-2 encounter, had been originally proposed.[1]

REFERENCE

1. L.F. Burlaga, J.W. Belcher and N.F. Ness, Geophys. Res. Lett. 7, 21 (1980).

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RELATIVISTIC PLASMAS IN ACTIVE GALACTIC NUCLEI

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ABSTRACT

I discuss the processes and properties of relativistic astrophysical plasmas in connection with active galactic nuclei and quasars. Such plasmas are expected to be produced as a result of accretion onto massive black holes. Firstly important elementary processes such as Comptonization and electron-positron pair production are reviewed. Then the properties of optically thin pair equilibrium plasmas are summarized and it is shown that pairs constrain the attainable states of the plasmas. Finally applications to accretion plasmas are attemted and the importance of the study of time-dependent problems is emphasized. Some preliminary numerical results are also presented.

Keywords: Relativistic Plasmas, Electron-Positron Pairs, Radiation Processes, Comptonization, Accretion onto Black Holes, Active Galactic Nuclei, Quasars

1. INTRODUCTION

The most prevailing models of active galactic nuclei and quasars assume that accretion onto massive black holes powers their various activities on the bases of enormous luminosities and rapid time variabilities (Refs. 1-3). However, processes leading to their activities have not been clarified well in this picture. The overall observed spectrum from radio to X-rays suggests that a major part of the spectrum should be due to nonthermal mechanisms such as the synchrotron radiation and the inverse Compton scattering, which presuppose the existence of relativistic electrons with a power law energy distribution. The conversion mechanism of accretion energy to nonthermal particles remains to be investigated. Of course, some portion of the spectrum may be due to thermal mechanisms. For example, ultraviolet radiation from some AGNs may be a superposition of black body radiation from accretion disks (Refs. 4,5). The unsaturated Comptonization in a hot plasma is another thermal mechanism proposed to explain the X-ray emission (Refs. 6-11).

Some radio galaxies and quasars have long radio jets emanating from their nuclei and reaching out to their extended radio lobes. Clearly jets are supplying energy from nuclei to lobes. The mechanism of the jet formation is not well understood. Thus at present we have no established models for the formation of emergent spectra and jet structure. The standard accretion disk model, which was originally invented for galactic binary X-ray sources, has not been so successful in explaining these features (Ref. 2) and many variants of accretion models have been proposed such as two-temperature disks (Refs. 7,8,12), thick radiation tori (Refs. 13,14) and spherical accretion with shock formation (Refs. 15,16).

In this lecture, instead of describing those models, I concentrate on the basic physical properties of relativistic and mildly relativistic plasmas. When applying to accretion plasmas, I assume that optically thin plasmas are produced by the rapid dissipation of infalling kinetic energy near the central black holes. Of course such plasmas may be produced in other places such as accretion disk coronas through magnetic mechanisms etc.

In section 2, I describe the expected properties of accretion plasmas near the black holes and estimate the relevant time scales of various physical processes. Among important processes, the unsaturated Comptonization is discussed in section 3 and electron-positron pair production is discussed in sections 4 and 5. In section 4, elementary processes in and properties of pair equilibrium plasmas are treated and section 5 is devoted to applications to accretion plasmas and attempts to treat the time-dependent problems. Finally in section 6 several future problems are remarked.

2. UNDERLYING MODEL

The underlying model discussed here assumes that the infalling kinetic energy is dissipated rapidly by some mechanisms to produce a mildly relativistic plasma near the central black hole. The following analysis was made in Ref. 17. We assume that the infall of a plasma into the hole is approximated by a radial flow with a velocity of a factor $\underline{a}$ of the free fall. The slowing down of the infall velocity may be due to a small amount of angular momentum or large pressure gradient and the factor $\underline{a}$ is supposed to take on a value of $10^{-2} \sim 1$. This

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approximation is made only for the rough estimation of physical quantities below.

Then the velocity v, proton density N, and the optical thickness to the Thomson scattering τ_{th} are estimated as

$$v=v_{ff}\underline{a}=1.7\times10^{10}r_*^{-1/2}\underline{a}\ \text{cm sec}^{-1}, \quad (1)$$

$$N=2.2\times10^{10}\dot{M}_*M_8^{-2}r_*^{-3/2}\underline{a}^{-1}\ \text{cm}^{-3} \quad (2)$$

and

$$\tau_{th}=N\sigma_{th}r=1.3\dot{M}_*M_8^{-1}r_*^{-1/2}\underline{a}^{-1}, \quad (3)$$

respectively, where $\dot{M}_*$ and M_8 are the accretion rate and the mass of the central black hole in units of 1 $M_\odot yr^{-1}$ and $10^8 M_\odot$, respectively, and $r_*\equiv r/(3r_g)$ with $r_g\equiv 2GM/c^2$; σ_{th} denotes the Thomson cross section. Here the effect of pair production is not taken into account. If all the dissipated energy is converted to thermal energy of protons, protons attain the maximum temperature $T_{p,max}$, which is given by

$$T_{p,max}=1.2\times10^{12}r_*^{-1}\ \text{K}. \quad (4)$$

2-1 Time scales

Here we enumerate several relevant time scales. The infall time scale is estimated as

$$t_{fall}=r/v=5.1\times10^3M_8r_*^{3/2}\underline{a}^{-1}\ \text{sec} \quad (5)$$

and the Thomson scattering time and escape time for a photon are

$$t_{th}=1/n_e\sigma_{th}c=2.3\times10^3\dot{M}_*^{-1}M_8^{2}r_*^{3/2}\underline{a}\ \text{sec}, \quad (6)$$

and

$$t_{esc}=r(1+\tau_{th})/c=3.0\times10^3M_8r_*(1+\tau_{th})\ \text{sec}, \quad (7)$$

respectively.

Thus if $\underline{a}$ is as small as 0.1 and $\dot{M}_*/M_8$ is larger than 0.1, we obtain $t_{th}<t_{esc}<t_{fall}$ at $r_*=1$, then steady states for photon distribution are expected. On the other hand for a smaller ratio of accretion rate to mass and $\underline{a}$ close to one, photons are swallowed into the hole or escape from a plasma before they are reprocessed in a plasma. Hereafter we confine our concern to a range of $\dot{M}_*/M_8<$ a few because higher value of $\dot{M}_*/M_8$ leads to super-Eddington luminosity if the conversion efficiency of rest mass to radiation is an order of 0.1.

2-2 Relaxation time scales

Next we examine relaxation processes. The two body processes in relativistic plasmas have recently been investigated by several authors (Refs. 18-20). Here we mainly use nonrelativistic approximations. First, the proton-proton relaxation time is estimated as

$$t_{p-p}=t_{th}\frac{4\sqrt{\pi}}{\ln\Lambda}\left(\frac{m_p}{m_e}\right)^2T_{p*}^{3/2}, \quad (8)$$

where $T_{p*}=kT_p/m_pc^2$ and $\ln\Lambda$ is the Coulomb logarithm. It is shown that t_{p-p} is fairly long compared to t_{fall} for plausible parameters and protons do not take a Maxwellian distribution and T_p represents only the mean kinetic energies of random motion.

The proton-electron relaxation time is given by

$$t_{p-e}=t_{th}\sqrt{\frac{\pi}{2}}\frac{1}{\ln\Lambda}\frac{m_p}{m_e}(T_*+T_{p*})^{3/2}, \quad (9)$$

where $T_*=kT_e/m_ec^2$. Comparing t_{p-e} and t_{fall}, we find that t_{p-e} is shorter than t_{fall} if $g\equiv\dot{M}_*/M_8\underline{a}^2$ is greater than $50(T_*+T_{p*})^{3/2}$. If this condition is satisfied, significant energy transfer from protons to electrons occurs; protons are cooled, while electrons are heated since T_e is supposed to be much less than T_p.

It is to be noted that electron-electron relaxation time is rather short as

$$t_{e-e}=t_{th}\frac{4\sqrt{\pi}}{\ln\Lambda}T_*^{3/2}, \quad (10)$$

so that electrons are safely assumed to take a Maxwellian distribution.

2.3 Electron temperature

When we examine the radiative properties of a plasma, it is essential to estimate the electron temperature. If the infall velocity of electrons is thermalized, it is about 6×10^8K, but they can be heated by collisions with hotter protons. As electrons are heated, t_{p-e} becomes longer and cooling due to radiative and pair processes comes into play.

The attainable electron temperature can be estimated by balancing the two body heating rate with the cooling rate and by taking a finite life time t_{fall} into account. If we take account of only bremsstrahlung as cooling mechanism, the resultant electron temperature is obtained as a function of $g\equiv\dot{M}_*/M_8\underline{a}^2$ (Ref. 17). The cooling time of electrons due to bremsstrahrung is given by

$$t_{br}=t_{th}\frac{3\pi}{8\alpha}\sqrt{\frac{\pi}{2}}T_*^{1/2}, \quad (11)$$

for $T_*<1$, where $\alpha=1/137$ is the fine structure constant.

The results are tabulated in Table 1.

Table 1

$g\equiv\dot{M}_*/M_8\underline{a}^2$	T_*
0.25∿0.4	$1.7g^{2/5}<1$
	$1.6g^{1/2}>1$
13	5.6
450	$T_*\ln(5T_*)=8500g^{-1}$, $5.6>T_*>1$
5300	$T_*<1$

Here for $T_*>1$ relativistic expressions for t_{p-e} and t_{br} are adopted properly, but effects of Comptonization and pairs are not taken into account. As is seen, T_* turns out to less than 5.6 in any case. For $g>450$, t_{p-e} is shorter than t_{fall} and proton temperature tends to be equal to the electron temperature. In reality other cooling mechanisms through electron-positron pair production and Comptonization will much reduce the attainable electron temperature, which results in the decrease of the critical value of g for $t_{p-e}<t_{fall}$.

3. COMPTONIZATION

As was discussed in the previous section, various cooling processes should be properly taken into account to determine the electron temperature and also to obtain the emergent spectrum from plasmas. Optically thin tenuous hot plasmas are not very efficient radiator. As is seen in Eq.(11), t_{br} is fairly long, which implies that bremsstrahlung is not an efficient radiation mechanism. But for $g>450$, which is realized for nearly Eddington accretion rate and fairly slow infall velocity, the calculated bremsstrahlung luminosity becomes as high as $\sim 0.1\dot{M}c^2$.

As one of the efficient mechanisms to radiate a significant fraction of the dissipated energy, the unsaturated Comptonization of soft photons has been discussed these ten years.

3.1 Unsaturated Comptonization

If a plasma contains soft photon sources other than bremsstrahlung, repeated scattering of photons off hot electrons produces high energy photons. The efficiency of this process is determined by the Comptonization parameter $y_c=4T_*\mathrm{Max}(\tau_{th},\tau_{th}^2)$. For each scattering a photon has a fractional increase of its energy $4T_*$ on average and $\mathrm{Max}(\tau_{th},\tau_{th}^2)$ represents the number of scattering before a photon escapes from a plasma. For y_c of less than a few, only a small fraction of injected photons gain high energy to reach the Wien region. A major portion of injected soft photons escape from a plasma with an energy proportional to the number of scattering before escape. This situation is similar to that in Fermi acceleration process of galactic cosmic rays and a power law spectrum is formed. For larger value of y_c a significant portion of injected photons are scattered into the Wien region to form a Wien spectrum.

3.1.1 Methods of treating Comptonization

Comptonization may be treated by various methods. Kompaneets equation is useful for relatively low temperature and large optical thickness since it assumes the Fokker-Planck approximation on the bases of Thomson cross section and isotropic distribution of photons (Refs. 21,22). However for $T_*>0.1$ and $\tau_{th}<$ a few, deviations from Thomson cross section and isotropy become large and different treatments are needed.

Monte Carlo simulation is a suitable method to calculate the emergent spectrum and photon distribution function for such cases. Several groups have performed simulations to obtain steady state spectra for a given electron temperature and geometry (Refs. 23-27). An example of such simulations is shown in Fig.1.

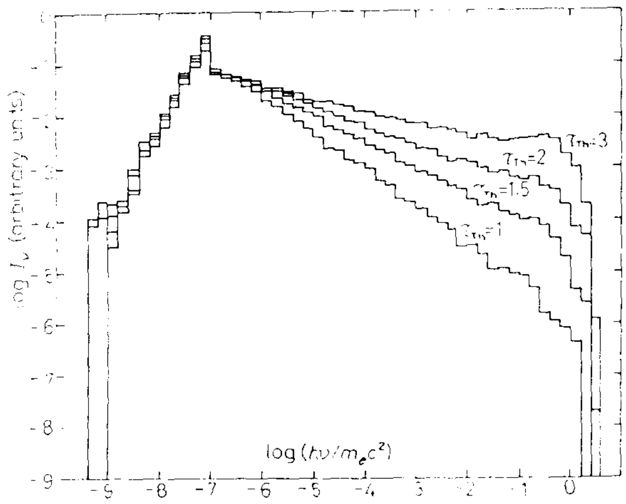

Fig.1 Emergent spectra from a uniform sphere through Comptonization of soft photons by electrons with $T_*=0.25$. Soft photons are injected uniformly within the sphere below $x\equiv h\nu/m_ec^2\leq 10^{-7}$ with Rayleigh-Jeans spectrum mimicing the self-absorbed cyclotron higher harmonics. (taken from Ref. 10).

Another method is to solve the transport equation directly, where calculation of differential scattering rate is needed. Lightman and Rybicki proposed a method by making approximations about scattering rate and assuming the separability between spatial transport and energy change (Refs. 28-30). This method has not been extended to regimes of mildly relativistic temperatures. Guilbert proposed a useful method, which expresses the differential scattering rate in tabular forms and interpolation formula and can be incooporated in finite difference equations (Ref. 31). This method has been applied to calculations of time development of pair concentration by Guilbert and Stepney (Ref. 32) and Kusunose (Ref. 33). It is to be noted that the original formula contain an error when evaluating an integral containing δ function which represents the conservation of energy and momentum, as was pointed out by Kusunose.

3.2 Problems of the unsaturated Comptonization

The unsaturated Comptonization is particularly interesting in producing power law emergent spectra. X-ray observations of Seyfert galaxies show power law spectrum with a small range of index centered on $\alpha=0.7$ (Ref. 34). This poses a difficult problem on theoretical side; the predicted index strongly depends on T_* and τ_{th} and we naturally expect a broad range of α among various sources and during time variabilities, which contradicts the observational facts. There may be unknown mechanisms to produce this universal power law spectrum or X-ray emission is due to other mechanisms.

Another problem is the source of soft photons. It may be supplied either externally outside from a plasma or through cyclotron higher harmonics. If a plasma has magnetic fields with roughly equipartition with proton pressure, self-absorbed cyclotron higher harmonics can supply a sufficient amount of soft photons for $T_*>0.25$ (Refs. 9,10,35, 36). However, in this estimation very high energy tail of a Maxwellian distribution function contributes to the highest harmonics utilized for soft photons and a more careful examination of distribution function is needed.

It is to be noted that the power law portion of the spectrum cannot explain observed optical radiation, which has also a power law form, but a slightly steeper one than the X-rays. The extrapolation from X-rays to optical bands lies an order of magnitude below the observed flux, so that optical emission is due to other mechanisms, probably synchrotron radiation by nonthermal electrons. With these reservations, however, Comptonization itself surely plays an important and decicive role in an optically thin mildly relativistic plasma.

4. ELECTRON-POSITRON PAIRS

Another important role is played by electron-positron pair production and annihilation. The most relevant process is photon-photon pair production, the cross section of which is an order of Thomson cross section if the product of energies of two photons is $\sim 2(m_e c^2)^2$. Then this process is important for

$$\tau_{\gamma\gamma}=n_\gamma \sigma_{th} r=\frac{L_\gamma \sigma_{th} r}{4\pi r^2 m_e c^3} > 1, \tag{12}$$

in other words, if

$$L_\gamma / r > 4\pi m_e c^3/\sigma_{th}=5\times 10^{29} \text{ erg cm}^{-1}. \tag{13}$$

Here n_γ and L_γ are the number density and luminosity with photons with energy around $m_e c^2$. This condition is satisfied for high luminosity sources even for sub-relativistic teneratures and in reality may have important implications for observed properties of compact X-ray sources (Refs. 37-39).

Now before I discuss their role in accretion plasmas, it is better to describe here elementary processes and pair equilibrium states.

4.1 Elementary pair processes

There are several pair processes important in astrophysical plasmas enumerated below.

$$p+e \rightarrow p+e+e^+ + e^-, \tag{14}$$

$$e+e \rightarrow e+e+e^+ + e^-, \tag{15}$$

$$p+\gamma \rightarrow p+e^+ + e^-, \tag{16}$$

$$e+\gamma \rightarrow e+e^+ + e^-, \tag{17}$$

$$\gamma+\gamma \rightarrow e^+ + e^- \tag{18}$$

and

$$e^+ + e^- \rightarrow \gamma+\gamma. \tag{19}$$

Here e denotes an electron or a positron. As for details and other processes which have comparatively minor importance, see Svensson (Ref. 40) and Stepney and Guilbert(Ref. 19).

4.1.1 Particle-particle processes

The processes (14) and (15) are collisions of two particles and have cross sections of an order of $\alpha^2 \sigma_{th}$. If we assume that thermal motions of protons are neglected and that electrons take a Maxwellian distribution, the pair production rates due to these processes are given by

$$(\dot{n}_+)_{p-e}=\frac{N n_e c}{T_* K_2(1/T_*)}\int_3^\infty (\gamma^2-1) e^{-\gamma/T_*} \sigma_{p-e}(\gamma) d\gamma, \tag{20}$$

and

$$(\dot{n}_+)_{e-e}=\frac{n_e^2 c}{2(T_* K_2(1/T_*))^2}\int_7^\infty \frac{(\gamma^2-1)}{\gamma_1} K_1(\gamma_1/T_*) \sigma_{e-e}(\gamma) d\gamma, \tag{21}$$

where $\gamma_1=\sqrt{2(\gamma+1)}$ and $n_e=n_- + n_+$. In the relativistic limit of $T_* >> 1$, the sum of Eqs.(20) and (21) gives rise to

$$(\dot{n})_{pt-pt}=\frac{7}{18\pi^2} \alpha^2 N^2 \sigma_{th} c(2y-1)(8y-3)(\ln T_*)^3, \tag{22}$$

where $y \equiv n_-/N$, the ratio of number density of electrons to that of protons, is a measure of pair concentration.

4.1.2 Particle-photon process

The cross sections of processes (16) and (17) are an order of $\alpha\sigma_{th}$ and the production rates are given by

$$(\dot{n}_+)_{e-\gamma}=\frac{n_e c}{2K_2(1/T_*)}\int_0^\infty dx \frac{n(x)}{x^2} \times \int_4^\infty dy \sigma_{e-\gamma}(y) y \exp\{-(\frac{x}{y}+\frac{y}{x})\frac{1}{2T_*}\} \tag{23}$$

and

$$(\dot{n}_+)_{p-\gamma}=Nc\int_2^\infty dx n(x) \sigma_{p-\gamma}(x), \tag{24}$$

where $n(x)$ is the photon number density per unit x with $x \equiv h\nu/m_e c^2$.

4.1.3 Photon-photon processes

This process has an order of Thomson cross section and become important for high energy photon sources. The rate is calculated as

$$(\dot{n}_+)_{\gamma-\gamma}=\frac{c}{2}\int dx_1 d\Omega_1 dx_2 d\Omega_2 n(x_1,\vec{n}_1) n(x_2,\vec{n}_2)(1-\mu)\sigma_{\gamma-\gamma}, \tag{25}$$

where $n(x,\vec{n})$ is the photon distribution function per unit x and per unit solid angle around $\vec{n}$ and $\mu=\vec{n}_1 \vec{n}_2$. The rate for anisotropic case was calculated by Stepney and Guilbert (Ref. 19), but a factor 2 should be multiplied in their equations (B4) and (B5).

4.1.4 Pair annihilation

The pair annihilation cross section is an order of Thomson cross section and the rate is approximated by