# The Few Body Problem

M. J. Valtonen
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# THE FEW BODY PROBLEM

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INTERNATIONAL ASTRONOMICAL UNION
HELD IN TURKU, FINLAND,
JUNE 14–19, 1987

Edited by

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PREFACE

Coinciding with the $300^{th}$ anniversary of the publication of Newton's Principia The International Astronomical Union organized the colloquium No. 96 "The Few Body Problem" in Turku, Finland, June 14.-19.1987. It provided an opportunity to review the progress in the very field which caused Newton a headache, as Victor Szebehely reminded the audience in his introductory remarks. It is a measure of the difficulty and complication of the few body problem that even after 300 years so many aspects of the problem are still unsolved. To quote Szebehely again, "Sir Isaac established the rules, Poincaré presented the challenges". Many of these challenges are reviewed in the present proceedings.

The gravitational few body problem cuts across the borders of established disciplines. The participants of the colloquium came from departments as different as Aerospace Engineering, Astronomy, Theoretical Physics, Physics, Mathematics, Applied Mathematics, Computer Science, Planetology, Geodesy, Celestial Mechanics and Space Science. The few body problem is a problem of practical significance in many fields and the main aim of the colloquium was to bring together people with research interests in this area, many of whom normally attend different conferences.

The scientific Organizing Committee was: V.A. Brumberg (Chairman, USSR), G. Contopoulos (Greece), M.G. Fracastoro (Italy), R.S. Harrington (U.S.A.), D.C. Heggie (U.K.), J. Kovalevsky (France), J.J. Monaghan (Australia), M. Scholl (F.R.G.), V.G. Szebehely (U.S.A.) and M. Valtonen (Finland). The colloquium was sponsored by the IAU Commission No 7 (Celestial Mechanics) together with Commissions No 20 (Minor Planets, Comets and Satellites), No 33 (Structure of the Galactic System) and No 37 (Star Clusters and Associations). The local arrangements were carried out by the Department of Physical Sciences of the University Turku, with financial assistance from the Ministry of Education of Finland, Turku University Foundation, NORDITA (Copenhagen), the city of Turku as well as the IAU. The bulk of the work in the Local Organizing Committee was carried out by Dr. Aimo Niemi, Dr. Esko Valtaoja, Mrs. Leena Valtaoja and Mr. Mats-Olof Snåre, to whom thanks are due. Dr. Tian-yi Huang and Dr. Jarmo Hietarinta brought valuable know-how to the LOC.

As the following pages testify, one of the major topics of the conference was the question of chaotic motions in Newtonian mechanics. A round table discussion on the matter was organized and is briefly reported in this volume. Jack Wisdom showed a movie illustrating the chaotic and ordered motions of minor planets. There were also movies by Dick Miller and Joanna Anosova, showing the evolution of a galaxy and a three-body system, respectively. The opportunity to see a "live" show of the Newtonian mechanics in action was much appreciated by the participants.

The other topic of much interest was the long-term evolution of the solar system. The time span of the calculations of outer planets has now reached the 200 million year mark, and it is expected that in near future many aspects of the evolution will be revealed over the whole age of the system. There are still many mysteries in the solar system, such as the origin of asteroid families. The present stage of understanding these questions is reviewed in this volume.

Many papers were given on the stellar many body problem. This, unlike the solar system problem, deals with bodies of equal or comparable mass. The three-body problem is fundamental in evolutionary studies of small stellar systems and in recent years also the four-body problem has been extensively studied. More and more quantitative data is now becoming available with the improvements in numerical integration methods.

Finally, in the last part of the conference, few body methods were applied to galactic systems. The interaction between galaxies has been recognized for a long time, but only recently its high frequency of occurence and relevance to many astrophysical processes have been realized. The basic physics of the interaction is exhaustively summarized in this volume and the usefulness of the few-body method is evaluated. This has been the major question mark in many earlier studies: how much could one learn about the interaction of two systems of (say) $10^{11}$ stars each by applying three-body methods. The answer appears to be, surprisingly, that we learn quite a lot. This is borne out by studies with fully interacting N-body systems, with $N \simeq 10^5$, which are also extensively reported in this volume.

The proceedings contain a few papers which were sent to the colloquium by authors who could not come to the meeting. These papers were displayed during the conference. A few participants who gave talks in the colloquium, could not prepare their contributions in written form within the time limits of these proceedings. Discussions are recorded only when the parties involved wrote down their questions and answers. Therefore much of the lively discussions are unfortunately not included in the proceedings.

A final quote from Szebehely: "After studying the papers, the reader will feel like the playmate of Newton playing on the seashore and finding a smoother pebble or prettier shell. The totality of the contributions presented at this meeting allows us not only to enjoy viewing the pretty shells but also brings us closer to the discovery of the great ocean of Truth, paraphrasing once again Sir Isaac."

Mauri Valtonen

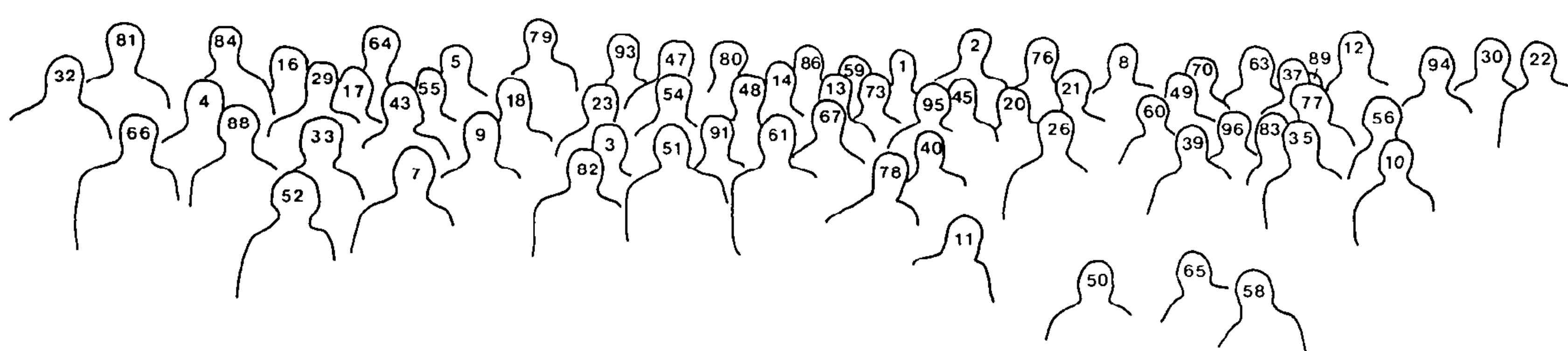

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TABLE OF CONTENTS

Preface xi
List of Participants xv

V.G. Szebehely: Introductory remarks. 1

Part I. General theory 3

C. Marchal: Qualitative analysis in the few-body problem 5

J.P. Anosova: A classification of motion types in the general three-body problem 27

J.D. Hadjidemetriou: Periodic orbits and stability 31

C. Ni and X. Zheng: The effect of perturbing potentials on Hill's stable range in restricted three-body problem 49

R. Gonczi: Fractal dimensions and integrability of Hamiltonian systems: a discretization method 55

H. Yoshida: Non-integrability of three body problems with homogeneous potential 61

B. Elmabsout: On the existence of some configurations of relative equilibrium in the N-body problem 65

A.J. Maciejewski: Rotational motion of a rigid body in an orbit of the three body problem 69

E. Koda: Possibility of exchange of a rectilinear three-body system with zero energy 73

J. Yoshida: Escape in the three-body problem with two degrees of freedom - Rectilinear case and isosceles case - 77

A. Ollongren: On a restricted five-body problem: An analysis with Computer Algebra 81

J. Henrard: A perturbative method for problems with two critical arguments 85

L. Cao and F. Tong: Generalized Hamilton's principle and its application 87

C. Marchal: Round table discussion on chaotic motions 91

Part II. Solar system 99

Cl. Froeschle, P. Farinella, M. Carpino, Ch. Froeschle, R. Gonczi, P. Paolicchi, V. Zappala and Z. Knezevic: Asteroid families 101

M. Šidlichovský: On the origin of 5/2 Kirkwood gap 117

Ch. Froeschle and H. Scholl: A possible source for highly inclined Apollo-Armor asteroids: the secular resonance $\nu_{16}$ 123

A. Lemaitre: The 2/1 Jovian resonance in the elliptic problem 129

S. Fernández and C. Beaugé: Asteroids in the 1:3:2 commensurability 131

L. Liu and X.-H. Liao: The problem of evolution on orbital resonance 135

K.A. Innanen, S. Mikkola and S.P. Zhang: Stability of the planetary triangular Lagrangian points 141

G. Hahn and C.-I. Lagerkvist: Long-term orbital evolution studies of Aten-Apollo-Amor objects 143

A.M. Nobili: Long-term dynamics of the outer solar system Review of LONGSTOP project 147

M. Carpino: Secular variations of the semimajor axes of the planets 165

D.L. Richardson and A.M. Segerman: Higher-order numerical secular planetary theory 171

J.M. Petit and M. Hénon: Numerical simulations of ring dynamics 175

H. Salo and C.F. Yoder: Dynamics of coorbital satellite rings 179

S.J. Goldstein, Jr. and H.R. Bailey: Orbital energy transfer in exactly commensurate satellites 185

D. Benest, R. Bien and H. Rickman: Capture of comets as temporary satellites of Jupiter 191

A. Carusi, E. Perozzi, E. Pittich and G.B. Valsecchi: Influence of close encounters on the determination of cometary orbits 195

M.S. Staniucha and M. Banaszkiewicz: Passage of the Sun through an interstellar nebula and the Oort cloud comets 201

S.Y. Zhu, E. Groten, R.S. Pan, H.J. Yan, Z.Y. Cheng, W.Y. Zhu, C. Huang and M. Yao: Motion of satellites - the choice of reference frames 207

Part III. Stellar systems 211

D.C. Heggie: The scattering problem 213

L.K. Babadzanjanz and V.A. Brumberg: Influence of a hyperbolic flyby of a small mass on the orbital evolution of a massive binary 231

V.A. Brumberg and T.V. Ivanova: Numerical investigation of the orbital evolution of a massive binary in the field of small masses 233

M. Valtonen and T.-Y. Huang: Energy exchange in a parabolic three-body encounter 239

A.A. Orlov and N.A. Solovaja: The stellar problem of three bodies and applications 243

P. Grujić and N. Simonović: The triple-escape function behaviour for small energy systems 249

J.P. Anosova and V.V. Orlov: Dynamical evolution of double and triple subsystems in the N-body systems 253

A. Duquennoy: Multiple stars in the solar neighbourhood 257

S. Mikkola: On the effects of unequal masses in the statistics of three- and four-body interactions 261

G. Contopoulos: Escapes from stellar systems 265

I.V. Petrovskaya: The force distribution for close encounters 275

C.D. Bailyn: Mass transferring binaries in hierarchical triples 279

V.G. Shkodrov, J.P. Anosova and V.V. Orlov: On some stability regions in the three-body problem 283

S.J. Aarsèth: Integration methods for small N-body systems 287

S. Mikkola and K.A. Innanen: A numerical experimenter's view of the few-body problem 309

S. McMillan, S. Casertano and P. Hut: Relaxation in small N-body systems 313

S. Inagaki and P. Hut: Long-term evolution of cores of globular clusters after core collapse 319

Part IV. Multiple galaxies

S.M. Alladin, K.S.V.S. Narasimhan and G.M. Ballabh: The merger time of interacting galaxies 327

T.S. Statler: Orbital decay in non-spherical galaxies 349

R.H. Miller and B.F. Smith: Cluster influences on the internal dynamics of a galaxy 353

J.C. Muzzio and M.M. Vergne: Tidal stripping and accretion in collapsing clusters of galaxies 361

A.D. Chernin, L.G. Kiseleva, J.P. Anosova and V.V. Orlov: Close approaches and coalescence in the triple systems of gravitating masses 367

G.G. Byrd: Tidal interaction of small satellite galaxies with spiral primaries 371

M. Thomasson: Inelastic collisions in models of disk galaxies 387

B. Basu, A. Saha and T. Bhattacharyya: Influence of gas motion on star formation in the central region of a disk galaxy 391

B. Sundelius: Numerical simulations of massive ejections from galactic nuclei 397

P.M.S. Namboodiri, R.K. Kochhar and S.M. Alladin: Formation of bridges and tails in interacting galaxies 401

M. Noguchi: Bar formation in interacting galaxies 405

L.S. Sparke, R.A. Nicholson and K. Taylor: The polar ring galaxy NGC 4650A. 409

T. Weeks: A computational method for galaxy interactions 413

J. Wisdom: Some aspects of chaotic behavior in the Solar System 417

Index of names 421
Index of subjects 429

# INTRODUCTORY REMARKS

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ABSTRACT. As an introduction to the Proceedings of I.A.U. Colloquium 96 on "The Few Body Problem" the progress made since Sir Isaac Newton in celestial mechanics is shortly reviewed and the fundamental problems concerning predictability in dynamics are discussed.

## 1. BACKGROUND AND HISTORY

It is indeed a great honor to be invited to present these introductory remarks at our meeting on the problem of few bodies.

The basic ideas to which our meeting is dedicated were published 300 years ago in 1687 by the Royal Society of London and written by Sir Isaac Newton in his Principia. His laws of dynamics and his law of gravitation set the basic rules. It is unquestionable that his genius is one of the major contributors to our meeting. The major theoretical limitation of our field appeared 200 years later and it was offered by another of our heroes, Henry Poincaré. In 1890, he announced the non-integrability principle applicable to the differential problems of three and more bodies. We might say that while Sir Isaac established the rules, Poincaré presented the challenges. The contributions of both of these giants were essential in our field and, indeed, I doubt if we would have our meeting without their contributions.

The papers presented at this meeting and published in this volume show the impressive progress made in the past 300 years. It might be appropriate to quote Sir Isaac Newton's two humble statements concerning the progress displayed at our meeting. "If I have seen further it is by standing on ye shoulders of Giants" wrote Newton. This gives us a considerable advantage over Newton since we have many more giants than he had. This also gives us a serious responsibility to study the enormous literature published in our field. But Newton goes further and says:

> "I seem to have been only like a boy playing on the seashore and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary while the great ocean of Truth lay all undiscovered before me."



*M. J. Valtonen (ed.), The Few Body Problem, 1–2.*

## 2. SUBJECTS OF THE COLOQUIUM

Some of the major subjects discussed at our meeting included: satellite dynamics, stellar dynamics, galactic dynamics, several versions of the restricted and general problems of three or more bodies, planetary theories, periodic orbits, capture, escape, regularization, equilibrium solutions, quantitative approaches, numerical experiments, mappings, chaotic motion, K-systems, ergodicity, instability, entropy, Liapunov characteristic numbers, randomness, recurrence and many others. After studying the papers, the reader will feel like the playmate of Newton playing on the seashore and finding a smoother pebble or prettier shell. The totality of the contributions presented at this meeting allows us not only to enjoy viewing the pretty shells but also brings us closer to the discovery of the great ocean of Truth, paraphrasing once again Sir Isaac.

The non-deterministic nature of dynamics and of celestial mechanics became clear at the colloquium. The limitations imposed by not knowing exactly the initial conditions, by using only approximate physical models, by not having integrable systems and by encountering various types of instabilities (structural, numerical, etc.) represented challenges which were not ignored but were investigated in detail.

Newton and his present-day disciples were humble enough to accept the idea that celestial mechanics, stellar dynamics and dynamics in general are not deterministic sciences. Newton did not speak about the relations between chaotic motion, non-integrability, ergodicity, and general instability but he was clear about the concept which today might be referred to as the finite predictability horizon. Participants at the colloquium spent a considerable time attempting to define these concepts and definite progress might be reported along these lines, though no simple and clearly formulated results were arrived at. Some of the participants of our meeting occupying the "Citadel of Determinism" with Aristotle, Einstein, Galileo, Laplace, Leibnitz just to mention a few, enjoyed looking over the heads of the rest. Others joined the "Club of the Statistical Craze" (as Einstein called this group), with Bohr, Born, Heisenberg, Newton, Poincaré, etc., already in residence and completely submerged in chaos. During the 300 years since Newton, significant progress was made in our field and many new unsolved problems emerged. The list is very long and impressive. We all inherited Sir Isaac's headache which he contributed to the difficulties of the lunar problem, or in general, of the few body problem.

# PART I. GENERAL THEORY