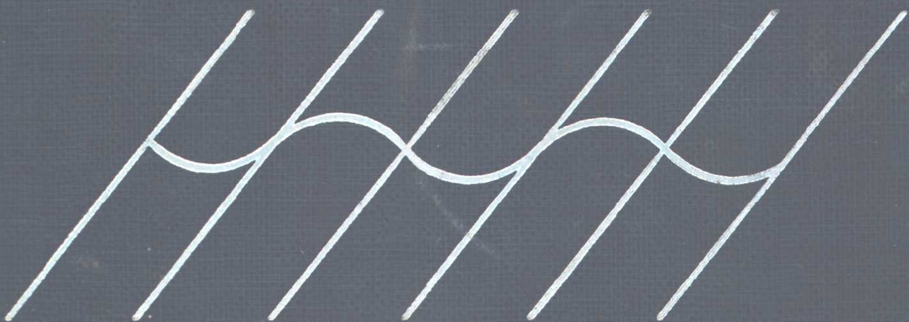
Sound and Structural Vibration

Radiation, Transmission and Response



Frank Fahy

Michael Brandstein · Darren Ward (Eds.)

Microphone Arrays

Signal Processing Techniques and Applications

With 149 Figures

Springer

ACADEMIC PRESS INC. (LONDON) LTD.
24–28 Oval Road
LONDON NW1 7DX

United States Edition published by
ACADEMIC PRESS, INC.
Orlando, Florida 32887

British Library Cataloguing in Publication Data

Fahy, F.J.
Sound and structural vibration : radiation, transmission and response.
1. Structural dynamics 2. Sound
I. Title
624.1'71 TA654

ISBN 0-12-247670-0

Library of Congress Cataloging in Publication Data

Fahy, Frank J.
Sound and structural vibration.

Includes index.
1. Vibration. 2. Sound-waves. I. Title.
TA355.F34 1985 620.2 84-20439
ISBN 0-12-247670-0 (alk. paper)

PRINTED IN THE UNITED STATES OF AMERICA

85 86 87 88 9 8 7 6 5 4 3 2 1

Preface

In writing this book my aim has been to present a unified qualitative and quantitative account of the physical mechanisms and characteristics of linear interaction between audio-frequency vibrational motion in compressible fluids and structures with which they are in contact. The primary purpose is to instruct the reader in theoretical approaches to the modelling and analysis of interactions, whilst simultaneously providing physical explanations of their dependence upon the parameters of the coupled systems. It is primarily to the engineering student that the book is addressed, in the firm belief that a good engineer remains a student throughout his professional life. A preoccupation with the relevance and validity of theoretical analyses in relation to practical problems is a hallmark of the engineer. For this reason there is a strong emphasis on the relationship of results obtained from theoretical analysis of idealised models and the behaviour of the less than ideal realities from which they are abstracted.

The teacher of analysis in any sphere of applied science is faced with a central dilemma: systems which can be modelled and analysed in a manner sufficiently explicit and direct to illustrate a principle are usually gross oversimplifications of the real world and are hence, to some extent, trivial; systems which are of practical concern are usually much too complex to offer suitable examples for didactic purposes. In attempting to grasp this nettle I hope I may be forgiven by any physicists and applied mathematicians who may pick up this book for sacrificing a certain amount of mathematical rigour for the sake of qualitative clarity.

In teaching mechanical engineering and engineering acoustics over a number of years it has struck me forcibly that an appreciation of structural vibration as a form of wave motion, a concept readily grasped by the student of physics, is often lacking in those reared on a diet of lumped elements and normal modes. One unfortunate effect is that the associated wave phenomena such as interference, scattering and diffraction are often believed to be the preserve of water and air, and the link between natural modes and frequencies of structures, and the component waves intrinsic to these phenomena, is not readily perceived. The subject of this book appeared to be the ideal vehicle for persuading students of the advantage to be gained by taking a dual view of vibrational motion in distributed elastic systems. Hence I have emphasized the wave "viewpoint" right from the start, in the hope of encouraging the reader to "think waves."

The three main categories of practical problems to which the material of this book is relevant are sound radiation from vibrating structures, sound transmission between adjacent regions of fluid media separated by an intervening solid partition, and the response of structures to excitation by incident sound fields. Much of the source material is only available (in English at least) in articles scattered throughout the learned journals of the world. In particular, fundamental analyses in acoustics textbooks of sound transmission through partitions tend to be restricted to highly idealised cases, and the complicating effects of finite panel size, non-homogeneous structures, cavity absorption and frames, and panel curvature are at best briefly and only qualitatively described. This is why Chapter 4 is the longest in the book.

Although the aim of the book is instructional, it is different from many textbooks in that it is not divided into neat, self-contained sections of analysis, which can be concluded with Q.E.D.; it also contains a large amount of descriptive text. The first feature is connected with the "dilemma" previously mentioned; the second stems from a desire to provide a text from which the reader can learn in the absence of a formal lecture course, although it is hoped that my prolixity will not deter a lecturer from using the book to complement his course.

The arrangement of questions in the book does not generally follow the conventional pattern of formalised quantitative examples at the end of chapters. The reader is challenged at various places within the text to think about the material which he is currently reading, while it is fresh in his mind. I hope, in this way, to solicit more active cooperation in the learning process, and to stimulate a questioning approach to the material, rather than passive acceptance. The questions at the ends of chapters are linked to specific sections in the text and range from straightforward numerical evaluation of quantities, intended to encourage a "physical feel" for their orders of magnitude, to rather open-ended questions, which can only be answered in qualitative terms. The absence of a large number of formal calculation exercises reflects both the nature of the subject and the fact that the readership is expected to have developed previously the facility for performing formal analyses of fundamental vibrational and acoustical problems.

Numerous references to other books, research publications, and reports are provided in the text. The list is clearly not comprehensive, but it is hoped that it will provide the reader with jumping-off points for further and deeper study. The omission of any particular relevant reference in no way constitutes a reflection of its value, any more than the inclusion of a reference implies that it is to be considered uniformly meritorious and correct.

It is my hope that this book, for all the faults which will no doubt emerge, will help at least a few people to understand more fully the fascinating interplay between sound and structural vibration, and thereby serve to increase their ability to control whatever aspect of the subject commands their attention.

Acknowledgements

This book is the product not only of my knowledge, research and teaching experience, but of numerous discussions, debates and joint endeavours with many colleagues and students over a period of twenty years. In particular, I would like to acknowledge the help given to me by Professor Phil Doak and Dr. Denys Mead, Dr. Maurice Petyt, Dr. Stewart Glegg, Dr. Philip Nelson and Dr. Chris Morfey of the University of Southampton. For her vital contribution in translating my vile scribble into an excellent typescript, I am greatly indebted to Jan Ward, and for the skillful conversion of my original figures into reproducible form I wish to thank Georgina Allan. For the unenviable task of proofreading I am indebted to my youngest son Tom and my wife Beryl. I also wish to acknowledge the help and guidance provided by the editorial and production staff of Academic Press in bringing this offspring of my labour into the world in the fine form you have before you. Finally, I have great pleasure in acknowledging the loving patience and practical assistance, accompanied by innumerable cups of coffee, with which my wife Beryl supported me during the three-year gestation period.

Introduction

> I think I shall never undertake to write a book again. If one were a scamp, the work would be easy enough, but for an honest man it is dreadful.
>
> John Tyndall, 1859

As you read these words you are almost certainly experiencing various manifestations of the process of vibrational interaction between fluids and solid structures. In all probability traffic noise is being transmitted through the windows of the room, the plumbing system may be announcing its operation, or perhaps the radio is providing background music for your pleasure. The first two examples represent the undesirable aspect of the phenomenon and suggest that study of its qualitative and quantitative aspects is of importance to those concerned with the control and reduction of noise. The third example shows that the process may be put to good use; vibrations of musical instruments, microphone diaphragms, and loudspeakers act as intermediaries in the creation of sound which is, at least for some listeners, the very antithesis of noise. The function of this book is to explain the physical process of interaction and to introduce the reader to various mathematical models and analyses of the behaviour of coupled fluid–structural systems.

Acoustic vibrations in fluids and solid structures essentially involve the propagation of wave motion throughout the supporting media, although explicit recognition of this fact is not always apparent in textbooks on mechanical vibration. Indeed, an emphasis on the work–energy approach to vibration analysis, which is fundamental to many modern computational techniques, tends to obscure the wave nature of the processes under analysis. In dealing with audio-frequency vibrations of systems involving coupling of compressible fluids with plate and shell structures, it is important to possess an appreciation of the "wave view" of vibration. There are three main reasons for this requirement: the first concerns the three-dimensional nature, and often very great extent, of fluid volumes, which effectively rules out the assumption of a limited number of degrees of freedom in describing the vibrational state of the medium; the second reason relates to the description of the interaction of sound waves with structural boundaries of diverse geometric and dynamic form, which is most appropriately framed in terms of the wave field phenomena of reflection, diffraction, and scattering; and the third reason is associated with the fact that frequencies of practical concern are usually far above the fundamental natural frequencies of the structures involved, and discrete modal models are not appropriate because of uncertainties in the modelling of detail and the very large number of degrees of freedom involved. Hence, vibration wave field models, analogous to those used in room acoustics, are more useful and effective.

For these reasons Chapter 1 introduces the reader to a unified mathematical description of temporal and spatial distributions of wave field variables and presents a partly qualitative account of the characteristics of waves in beam, plate, and shell structures. In particular, it shows how the concept of a dispersion relationship between wave speed and frequency forms a basis for categorising the regimes of interaction between waves of various type travelling in contiguous media. The phenomena of natural frequencies and characteristic modes of bounded elastic systems and the related phenomenon of resonance are explained qualitatively in terms of wave reflection and interference; and the roles of outgoing and returning waves in determining the input impedances of distributed elastic systems are illustrated by one-dimensional examples.

In Chapter 2 the mechanics of sound radiation from vibrating surfaces is explained in terms of the distribution of normal surface acceleration. Analyses of sound radiation from planar surfaces by means of far-field evaluation of the Rayleigh integral and in terms of travelling wave Fourier component synthesis are presented in such a way that the equivalence of these dual approaches can be appreciated. The utility of the latter approach is illustrated by application to the evaluation of the contributions to radiated power made by locally applied forces and by reaction forces arising from the presence of local constraints. The chapter closes with brief treatments of sound radiation from orthotropic and sandwich plates and from circular cylindrical shells.

The problem of evaluating the reaction forces applied by a fluid to a vibrating structure is addressed in Chapter 3. The Kirchhoff–Helmholtz integral equation is introduced and the concept of complex acoustic radiation impedance illustrated by some elementary examples. The value of the concept of wave impedance in analysing wave propagation in coupled fluid–structure systems is demonstrated in the case of bounded and unbounded uniform plates; and the effect of fluid loading on plate natural frequencies is discussed. An elementary, one-dimensional example of fluid loading of an elastic structure by an enclosed volume of fluid is presented in order to demonstrate the existence of coupled system modes; this section is essentially a forerunner to a more comprehensive analysis of closed coupled systems presented in Chapter 6. Finally, brief mention is made of the effects of heavy fluid loading on the radiation of plates produced by locally applied forces.

Chapter 4 presents a detailed account of sound transmission through plane partitions of various forms, including single-leaf, double-leaf, and non-homogeneous constructions. A considerable amount of analytical detail is felt to be justified by the apparent lack of a unified, comprehensive treatment of the general problem in other English language textbooks on acoustics, much of the material appearing only in specialised papers and technical reports. The reader is led from elementary analyses of highly idealised systems, through a discussion of the relative importance of resonant and non-resonant transmission mechanisms, to an appreciation of the current state of the art with regard to understanding the behaviour of the more complex structures used in practice. A brief review is presented of the performance of enclosure structures and of stiffened, composite, and non-uniform panels. The final section consists of a fairly detailed treatment of the problem of sound transmission through thin-walled, circular cylindrical shells; unfortunately, this does not extend as far as the presentation of practical formulae because research has not yet produced definitive theoretical results and formulae.

Analysis of the vibrational response of thin-plate and shell structures to incident sound is the subject of Chapter 5. In practice, this topic is of importance in cases where the integrity of structures or attached systems is at risk because of excitation by very intense sound fields produced, for instance, by aircraft engines, rockets, or industrial plant components. The relationship between radiational and response characteristics of structures is strongly emphasized, since this property has considerable experimental significance; it is employed, for instance, in the underwater noise reduction of ships and is now being used in the study of the acoustics of stringed musical instruments. Among other practical applications of the theory are the optimisation of the performance of low-frequency panel absorbers and estimates of the absorbing properties of non-load-bearing structures in buildings.

In many cases of practical concern, structures totally or partially enclose fluid volumes. In such cases, the vibrational behaviour of two coupled energy storage systems is of interest; examples include vehicle cabins and fluid transport ducts. In Chapter 6, theoretical approaches to the analysis of coupled system behaviour are presented, and the conditions under which the coupling drastically modifies the vibrational characteristics of the components from those in the uncoupled state are explained. The final section analyses an elementary case of wave propagation in a waveguide comprising coupled fluid and structural components.

The development of efficient computational procedures for the analysis of the vibrational behaviour of systems described by large numbers of degrees of freedom has naturally led to their application to acoustic problems. Although such analyses are generally limited to the lower end of the audio-frequency range, they can be of considerable value, particularly where fluid-loading effects are strong, for instance, in the case of a sonar transducer arrays in which vibrations of the transducer support structure can significantly affect the array performance. Chapter 7 presents a necessarily brief introduction to such basic techniques as finite element analysis, the details of the modelling and computational techniques being too diverse to allow more than a superficial review to be presented herein. However, I trust that the brevity of the treatment will not discourage the reader from looking more closely into this fast-developing area of the subject, with the aid of the cited references.

F. J. Fahy

Contents

Preface xi
Acknowledgements xv
Introduction xvii

1. Waves in Fluids and Solid Structures

1.1 Frequency and Wavenumber 1
1.2 Sound Waves in Fluids 6
1.3 Longitudinal Waves in Solids 8
1.4 Quasi-Longitudinal Waves in Solids 10
1.5 Transverse Shear Waves in Solids 13
1.6 Bending Waves in Bars 16
1.7 Bending Waves in Thin Plates 23
1.8 Dispersion Curves 24
1.9 Waves in Thin-Wall Circular Cylindrical Shells 25
1.10 Natural Frequencies and Modes of Vibration 27
1.11 Forced Vibration and Resonance 36
1.12 The Concept of Impedance 39
1.13 Point Force Impedance of an Elastic Structure 42
Problems 50

2. Sound Radiation by Vibrating Structures

2.1 The Importance of Sound Radiation 53
2.2 The Volume Source 56
2.3 The Baffled Piston 58
2.4 Sound Radiation by Flexural Modes of Plates 60
2.5 Sound Radiation by Flexural Waves in Plates 72
2.6 The Frequency-Average Radiation Efficiency of Plates 81
2.7 Sound Radiation Due to Concentrated Forces and Displacements 90
2.8 Sound Radiation from Non-Uniform Plate Structures 98
2.9 Sound Radiation from Curved Shells 101
2.10 Sound Radiation from Irregularly Shaped Bodies 109
Problems 109

3. Fluid Loading of Vibrating Structures

3.1 Practical Aspects of Fluid Loading 113
3.2 Pressure Fields on Vibrating Surfaces 115
3.3 Wave Impedances of Structures and Fluids 126
3.4 Fluid Loading of Vibrating Plates 130
3.5 Natural Frequencies of Fluid-Loaded Plates and Shells 136
3.6 Effects of Fluid Loading on Vibration of and Sound Radiation from Point-Excited Plates 140
Problems 142

4. Transmission of Sound through Partitions

4.1 Practical Aspects of Sound Transmission through Partitions 143
4.2 Transmission of Normally Incident Plane Waves through an Unbounded Partition 144
4.3 Transmission of Sound through an Unbounded Flexible Partition 149
4.4 Transmission of Diffuse Sound through a Bounded Partition in a Baffle 159
4.5 Transmission on Sound through a Partition between Two Rooms 163
4.6 Double-Leaf Partitions 166
4.7 Transmission of Normally Incident Plane Waves through an Unbounded Double-leaf Partition 167
4.8 The Effect of Cavity Absorption 173
4.9 Transmission of Obliquely Incident Plane Waves through an Unbounded Double-Leaf Partition 175
4.10 Mechanical Coupling of Double Partition Leaves 181
4.11 Close-Fitting Enclosures 186
4.12 Transmission of Sound through Stiffened, Composite and Non-Uniform Panels 191
4.13 Transmission of Sound through Thin-Walled Circular Cylindrical Shells 197

4.14 Flexural Wave Propagation in a Circular Cylindrical Shell 200
4.15 Coupling between Shell Modes and Acoustic Duct Modes 205
4.16 Transmission Characteristics 210
Problems 210

5. Acoustically Induced Vibration of Structures

5.1 Practical Aspects of Acoustically Induced Vibration 217
5.2 Decomposition of a Sound Field 218
5.3 Response of a Baffled Plate to Plane Waves 221
5.4 Applications of the Principle of Reciprocity 227
5.5 Modal Reciprocity: Radiation and Response 228
5.6 Radiation Due to Point Forces and Response to Point Sources 232
5.7 Applications of Response Theory 236
Problems 239

6. Acoustic Coupling between Structures and Enclosed Volumes of Fluid

6.1 Practical Importance of the Problems 241
6.2 Fundamentals of Fluid–Structure Interaction 242
6.3 Interaction Analysis by Green's Function 246
6.4 Modal Interaction Model 249
6.5 Solutions of the Modal Interaction Model 252
6.6 Power Flow Analysis 256
6.7 Wave Propagation in Structures Loaded by Fluid Layers 259
Problems 268

7. Introduction to Numerically Based Analyses of Fluid–Structure Interaction

7.1 The Role of Numerical Analysis 271
7.2 Numerical Analysis of Sound Fields 272
7.3 Finite Element Methods 275
7.4 Integral Equation Analysis 285
Problems 289

Answers 295

References 297

Index *303*

1 Waves in Fluids and Solid Structures

1.1 Frequency and Wavenumber

In this book we shall confine our attention largely to audio-frequency vibrations of elastic structures that take the form of thin flat plates, or thin curved shells, of which the thickness dimension is very much less than those defining the extent of the surface. Such structures tend to vibrate in a manner in which the predominant motion occurs in a direction normal to the surface. This characteristic, together with the often substantial extent of the surface in contact with a surrounding fluid, provides a mechanism for displacing and compressing the fluid: hence such structures are able effectively to radiate, and to respond to sound. In order to understand the process of acoustic interaction between solid structures and fluids, it is essential to appreciate the wave nature of the responses of both media to time-dependent disturbances from equilibrium, whether these be transient or continuous. In this chapter we shall take a look at the phenomena of natural frequency and resonance, and the impedance of simple structures, from a wave point of view.

A mechanical wave may be defined as a phenomenon in which a physical quantity (e.g., energy or strain) propagates in a supporting medium, without net transport of the medium. It may be characterised kinematically by the form of relative displacements from their positions of equilibrium of the particles of the supporting medium, that is to say the form of distortion, together with the speed and direction of propagation of this distortion. Wave disturbances in nature rarely occur at a single frequency; however, it is mathematically and conceptually more convenient to study single-frequency characteristics, from which more complex time-dependent behaviour can be synthesised mathematically if required.

Before we consider wave motion in particular types of physical systems, we shall discuss the mathematical representation of relationships between variations in time and space, which are fundamental to the nature of wave motion in general. Simple harmonic variations in time are most conveniently described mathematically by means of a complex exponential representation, of which there are two forms: in one, only positive frequencies are recognised; in the other, which is more appropriate to analytical and numerical frequency analysis techniques, both positive and negative frequencies are considered (Randall, 1977). In this book the former representation will be employed, because it avoids the confusion that can arise between signs associated with variations in time and space.

The basis of the representation is that a simple harmonic variation of a quantity with time, which may be expressed as $g(t) = A\cos(\omega t + \phi)$, where A symbolises amplitude and ϕ symbolises phase, can also be expressed as $g(t) = \mathrm{Re}\{\tilde{B}\exp(j\omega t)\}$, where $\tilde{B}$ is a complex number, say $a + ib$, and Re{ } means real part of. $\tilde{B}$ may be termed the complex amplitude. It will be seen that

$$A = (a^2 + b^2)^{1/2} \qquad a = A\cos\phi, \qquad b = A\sin\phi, \qquad \phi = \arctan(b/a).$$

Hence $g(t)$ may be represented graphically in the complex plane by a rotating vector (phasor) as illustrated in Fig. 1.

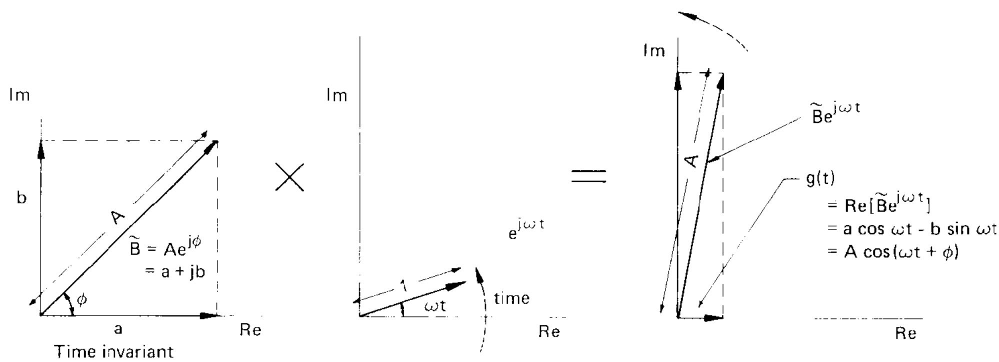


Fig. 1. Complex exponential representation of $g(t) = A\cos(\omega t + \phi)$.

In a wave propagating in only one space dimension x, of a type in which the speed of propagation of a disturbance is independent of its magnitude, a simple harmonic disturbance generated at one point in space will clearly propagate away in a form in which the spatial disturbance pattern, as observed at any one instant of time, is sinusoidal in space. This spatial pattern will travel at a speed c, which is determined by the kinematic form of the disturbance, the properties of the medium, and any external forces on the medium. As the wave progresses, the disturbance at any point in space will vary sinusoidally in time at the same frequency as that of the generator, provided the medium responds linearly to the disturbance. Suppose we represent the disturbance at the point of generation by $g(0,t) = \mathrm{Re}\{\tilde{B}\exp(j\omega t)\}$, as represented by Fig. 1. The phase of the disturbance at a point distance x_1 in the direction of propagation will *lag* the phase at 0 by an angle equal to the product of the circular frequency ω (phase change per unit time) of the generator and the time taken for the disturbance to travel the distance x_1: this time is equal to x_1/c. Hence, $g(x_1, t)$ may be represented by multiplying $\tilde{B}\exp(j\omega t)$ by $\exp(-j\omega x_1/c)$, i.e., $g(x_1, t) = \mathrm{Re}\{\tilde{B}\exp[j(\omega t - \omega x_1/c)]\}$. Thus we see that the quantity $-(\omega/c)$ represents phase change per unit increase of distance, in the same way as ω represents phase change per unit increase of time. Figure 2 illustrates the combined effects of space and time variations. The horizontal component of the rotating phasor represents the disturbance $g(x,t)$, which may or may not physically correspond to displacement in the x direction, depending upon the type of wave.

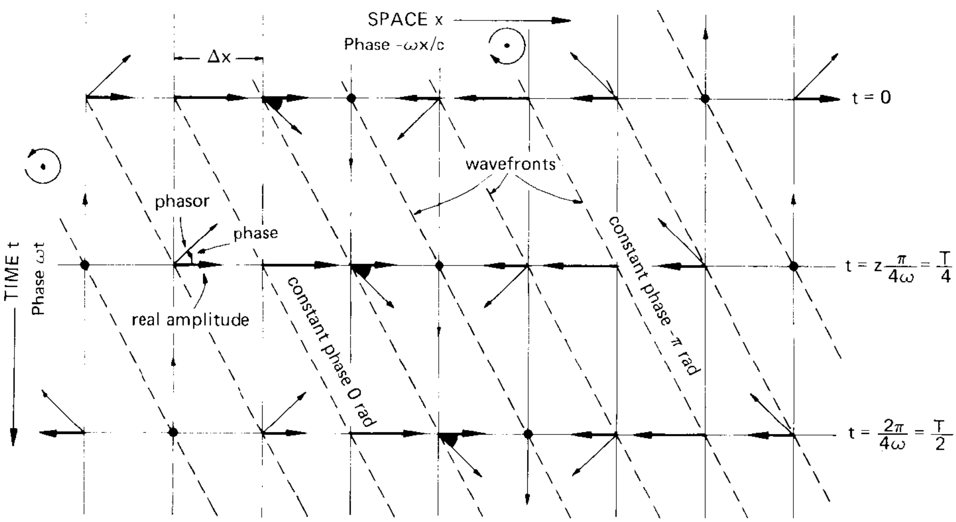


Fig. 2. Phasor diagram illustrating phase changes due to variation of t and x ($\omega\Delta x/c$ chosen to equal $\pi/4$ for clarity).