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Shanghai University Doctoral Dissertation (2005)

Localized Excitations and Related Fractal and Chaotic Behaviors in (2+1)-Dimensional Nonlinear Systems

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摘　　要①

孤子、分形和混沌是非线性科学的三个重要方面.传统的学术研究,这三部分是彼此分开独立讨论的,因为人们一般认为孤子是可积系统的基本激发模式而分形和混沌是不可积系统的基本行为.也就是说,人们不会去考虑孤子系统中存在分形和混沌行为.但是,上述这些传统观点可能不全面,乃至有待修正,特别是在高维可积系统中的情形.

论文围绕一些具有广泛物理背景的(2+1)维非线性系统的局域激发模式及其相关非线性特性——分形特征和混沌行为展开讨论,这些(2+1)维非线性系统源于流体、等离子体、场论、凝聚态物理、力学和光学等实际问题.首先借鉴线性物理中的分离变量理论和非线性物理的对称约化思想,论文对处理非线性问题的多线性分离变量法和直接代数法进行研究和推广,对形变映射理论进行创新,得到了一些新的结果.然后,根据非线性系统的多线性分离变量解和广义映射解,分别讨论了(2+1)维局域激发模式及其相关的非线性动力学行为.本文研究表明,多线性分离变量方法与广义映射方法甚至Charkson-Kruskal约化方法蕴藏着内在的有机联系.另外,论文所得结果说明混沌和分形存在于高维可积非线性系统是相当普遍的现

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象.现将本文的主要内容概述如下:

第一章简要回顾了孤波的发现与研究历史,总结了当前研究的状况,并概述了孤子、混沌和分形三者之间的传统学术关系,列举了一些新的或典型的(2+1)维非线性系统,最后给出了本论文的研究工作安排.

第二章将多线性分离变量法推广应用到若干(2+1)维非线性系统,如:广义 Broer-Kaup 系统、广义 Ablowitz-Kaup-Newell-Segur 系统、广义 Nizhnik-Novikov-Vesselov 系统、广义非线性 Schrödinger 扰动系统及 Boiti-Leon-Pempinelli 系统等,并得到一个相当广义的多线性分离变量解,可以用来描述系统场量或相应势函数,进而讨论基于多线性分离变量解引起的(2+1)维系统局域激发及其相关非线性特性.文中报导了一些典型的局域激发模式,如:平面相干孤子 dromions 为所有方向都呈指数衰减的相干局域结构,可以由直线孤子,也可以由曲线孤子形成,不仅局域在直线或曲线的交点,也可以存在于曲线的近邻点上.而 dromions 格子则为多 dromions 点阵,振荡型 dromions 在空间某一方向上产生振荡.环孤子为非点状的局域激发,在闭合曲线的内部不为零,闭合曲线外部指数衰减.呼吸子则是孤子的幅度、形状、峰间的距离及峰的数目可能进行"呼吸".瞬子的幅度随时间的变化而快速衰减.周期性孤子在时间或空间上呈现周期性特征.峰孤子在波峰处有一个尖点,其一阶导数不连续.紧致子是在某有限区域上幅值不为零,而在这个有限区域之外幅值一致为零的一类特殊孤波.折叠子是在各个方向同时褶皱的多值孤波.混沌孤子和分形孤子展示出孤波形态中的分形特征和混沌行为.

第三章将双曲函数法、椭圆函数法和直接代数法推广到非线性离散系统及变系数系统，如：Ablowitz-Ladik-Lattice 系统、Hybrid-Lattice 系统、Toda Lattices 系统、相对论 Toda Lattices 系统、离散 mKdV 系统和变系数 KdV 系统等，得到这些非线性系统的精确行波解，如离散系统的孤波解、Jacobian 双周期波、变系数系统的周期波解、孤波解、Jacobian 双周期波、Weierstrass 双周期波解、有理函数和指数函数解等.

第四章利用对称约化思想，提出了一种广义映射方法，突破了现有映射理论只能求解系统行波解的约束，并成功地运用若干(2+1)维非线性系统中，如：Broer-Kaup-Kupershmidt 系统、Boiti-Leon-Pempinelli 系统、广义 Broer-Kaup 系统和色散长波系统等，得到了新型的分离变量解，也称为广义映射解. 然后对广义映射法作对称延拓，发现上述(2+1)维非线性系统丰富的对称映射解. 根据所求得的映射解，我们可以得到丰富的局域激发结构. 事实上，基于多线性分离变量解得到的所有局域激发，用广义映射理论同样可以得到.

第五章，依据第四章得到的(2+1)维非线性系统新的广义映射通解，分析了若干新的或典型的局域激发模式，如：传播孤子与不传播孤子、单值与多值复合的半折叠孤子、裂变孤子和聚合孤子及其演化行为特性等，讨论了一些典型孤子所蕴涵的分形特征和混沌动力学行为. 研究结果再次表明混沌、分形存在于高维可积非线性系统是相当普遍的现象，其根源在于可积系统的初始状态或边界条件具有“不可积”的分形特性或混沌行为，修正了人们长期认为孤波产生于可积非线性系统而混沌、分形只存在于不可积非线性系统的认识局限性. 与此同时，

还分析并建立了(2+1)维非线性系统的广义映射解与多线性分离变量解的变换关系.理论分析表明,所有由多线性分离变量法得到(2+1)维非线性系统的局域激发,根据广义映射理论也可以找到.广义映射方法不仅突破了原映射理论只能求解非线性系统行波解的约束,而且有望进一步推广到其他(2+1)维非线性系统,这也发展和丰富了非线性科学的基本理论.

第六章,给出了论文的主要结果,提出了一些未来相关研究工作的设想.

关键词 (2+1)维非线性系统,局域激发,混沌,分形,孤子

Abstract①

Chaos, fractals and solitons are three important parts of nonlinearity. Conventionally, these three aspects are treated independently since one often considers solitons are basic excitations of an integrable model while chaos and fractals are elementary behaviors of nonintegrable systems. In other words, one does not analyze the possibility of existence of chaos and fractals in a soliton system. However, the above consideration may not be complete, or even should be modified, especially in some higher dimensions.

In this dissertation, author shall discuss the localized excitations and related fractal and chaotic behaviors in (2+1)-dimensional (two spatial-dimensions and one time dimension) nonlinear systems, which were originated from many natural sciences, such as fluid dynamics, plasma physics, field theory, condensed matter physics, mechanical and optical problems. With help of variable separation approach in linear physics and symmetry reduction theory in nonlinear physics, the multilinear variable separation approach and the direct algebra method were extended to nonlinear physics

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successfully, then a new algorithm, a general extended mapping approach, was proposed and applied to various (2+1)-dimensional nonlinear systems. Based on multilinear variable separation solutions and general mapping solutions respectively, abundant localized excitations and related fractal and chaotic behaviors for (2+1)-dimensional nonlinear models are investigated as well as rich evolution properties for these localized structures are discussed. The research results indicate that fractals and chaos in higher-dimensional soliton systems are quite universal phenomena. Meanwhile, it is also shown that one can establish the relationship between multilinear variable separation approach and extended mapping approach, and even Charkson-Kruskal reduction method. The main contents are summarized as follows.

In the first chapter, author outline a brief history and the current state on studying solitary waves and solitons, as well as review the traditional theoretical relations among solitons, chaos and fractals and list some new or typical (2+1)-dimensional nonlinear systems. The research arrangements of the dissertation are also given out in the end of the chapter.

In the second chapter, the multilinear variable separation approach is extended and applied to several (2+1)-dimensional nonlinear models, such as generalized Ablowitz-Kaup-Newell-Segur system, generalized Broer-Kaup system, generalized Nizhnik-Novikov-Veselov model, general perturbed nonlinear Schrödinger equation, Boiti-Leon-Pempinelli system, and new dispersive long water wave system *etc*. A quite "universal" variable separation formula with several arbitrary function which is valid for a large classes of (2+1)-dimensional nonlinear models is obtained. In terms of the "universal" formula, various localized excitations, such as multi-dromion solutions, multi-lump solutions, multi-compacton solutions, multi-peakon solutions, multi-foldon solutions, lattice dromion solutions, oscillating dromion solutions, ring-soliton solutions, motiving or static breather solutions, instanton solutions, periodic wave solutions, chaotic pattern structures and fractal pattern structures for (2+1)-dimensional nonlinear systems are revealed by selecting appropriate initial and/or boundary conditions. Based on the plots and theoretical analysis, we explored some typical localized excitaions. Dromions are localized solutions decaying exponentially in all directions, which can be driven not only by straight line solitons but also driven by curved line solitons and can be located not only at the cross points of the lines but also at the closed points of the curves. Dromion lattice is a special type of multi-dromion solution. The oscillating dromion solution is a dromion oscillating in special dimensional direction. Ring solitons are not the point-like localized excitations, which are not equal to zero identically at some closed curves and decay exponentially away from the closed curves. The breathers may breath in their amplitudes, shapes, distances among the peaks and even the number of the peaks. The amplitudes of instantons will change fleetly with the time. Peakons have peak points at their wave peaks in which one-order derivatives are not continuous. Compactons with finite wavelengths are a class of solitary waves with compact supports. Foldons are a class of multi-valued solitary waves, which can be folded in all directions. The fractal solitions and chaotic solitons reveal fractal characteristic and chaotic dynamic behaviors in solitary waves, respectively.

In the third chapter, the direct algebraic method based on traveling wave reduction is generalized to solve nonlinear partial differential systems and (2+1)-dimensional nonlinear models with constants and variable coefficients respectively. The tanh function approach, Jacobi elliptic function method and deformation mapping approach are introduced and extended respectively, then applied to several class of nonlinear models, such as Ablowitz-Ladik-Lattice system, Hybrid-Lattice system, Toda Lattices system, relativity Toda Lattices system, discrete mKdV system and variable coefficient KdV system *etc*. by making use of computer algebra. Rich solitary wave solutions and Jacobian doubly periodic wave structures for the above mentioned nonlinear partial differential systems are obtained, as well as abundant solitary waves, periodic waves, Jacobian doubly periodic waves and Weierstrass doubly periodic waves, rational function solutions and exponential function solutions to (2+1)-dimensional nonlinear models with constants and variable coefficients are derived.

In the forth chapter, a new algorithm, i. e. a general extended mapping approach, was proposed and applied to various (2+1)-dimensional nonlinear systems, such as Broer-Kaup-Kupershmidt system, Boiti-Leon-Pempinelli system, generalized Broer-Kaup system and dispersive long water-wave model. A new type of variable separation solution (also named extended mapping solution) with two arbitrary functions, which is valid for all the above-mentioned nonlinear systems, is derived. Then making further the new mapping approach in a symmetric form, we find abundant mapping solutions to above-mentioned (2+1)-dimensional nonlinear systems. In terms of the new type of mapping solution, we can find rich localized excitations. Actually, all the localized excitations based on the multilinear variable separation solutions can be re-derived from the new mapping solutions.

Based on a new universal extended mapping solution derived from (2+1)-dimensional nonlinear systems in chapter 4, chapter 5 is devoted to revealing some new or typical localized coherent excitations and their evolution properties contained in (2+1)-dimensional nonlinear systems. By introducing suitably these arbitrary functions, we constructed considerably novel localized structures, such as solitons with and without propagating properties, some semifolded localized structures with and without phase shafts, and certain localized excitations with fission and fusion behaviors. Some typical localized excitations with fractal properties and chaotic behaviors are also discussed. Why the localized excitations possess such kinds of chaotic behaviors and fractal properties? If one considers the boundary or initial conditions of the chaotic and fractal solutions obtained here, one can straightforwardly find that the initial or boundary conditions possess chaotic and fractal properties. These chaotic and fractal properties of the localized excitations for an integrable model essentially come from certain "nonintegrable" chaotic and fractal boundary or initial conditions. From these theoretical results, one may interpret that chaos and fractals in higher-dimensional integrable physical models would be a quite universal phenomenon. Meanwhile, we have established a simple relation between the multilinear variable separation solutions and the universal extended mapping solutions, which are essentially equivalent by taking certain variable transformation. Therefore, all the localized excitations based on the multilinear variable separation solution can be re-derived by the universal extended mapping solution. The general extended mapping approach not only outbreaks its original limitations merely searching for traveling wave solutions to nonlinear systems, but also be extended to many (2 + 1)-dimensional nonlinear dynamical systems, which means the mapping approach has been developed and richened to the basic theory of nonlinearity.

Finally, some main and important results as well as future research topics are given in the last chapter.

Key words (2 + 1)-dimensional nonlinear system, localized excitation, chaos, fractal, soliton

目　录

第一章　绪　　论

20 世纪初量子力学和相对论的创立，是物理学，或更确切地说是科学的两次革命. 因为它们提出了突破人们传统思维的新概念，将人类的世界观推进到超越经典的领域. 牛顿创立的经典力学被发现并不始终是正确的，当深入到微观尺度 ($L < 10^{-10}$ m) 时，应该代之以量子力学；而物体的速度接近光速 ($v \sim c = 10^8$ m/s) 时，则应该代之以相对论.

现在对线性系统问题，人们已经有了深入的了解和研究. 但是，这些线性系统对于复杂客观世界只是近似的线性抽象和描述. 自然界中错综复杂的现象激发人们去进一步探索其本质，这使得非线性科学得以产生且蓬勃发展. 因为相比于线性系统，非线性模型能更好更准确地描述自然现象从而更接近现象的本质. 由此，很自然地，若干描述真实世界的非线性系统大量涌现，从而使研究这些非线性系统自然地成为现代前沿科学研究领域的重要任务之一.

非线性科学作为现代科学的一个新分支，如同量子力学和相对论一样，将我们引向全新的思想，给予我们惊人的结果. 非线性科学的诞生，进一步宣布牛顿经典决定论的局限性. 正如它指出，即使是通常的宏观尺度和一般物体的运动速度条件下，经典决定论也不适用于非线性动力学系统的混沌轨道的动力学行为分析.

近三十年来，随着人类认识的深入和科学技术研究水平的提高，人们越来越多地发现自然科学和工程技术中普遍存在的非线性问题，其非线性效应可以产生本质上全新的物理现象，称之为非线性现象. 对这些现象的恰当描述，用线性化模型已不能完全反映客观的真实世界，应该取而代之为各种不同的非线性模型，即非线性演化方程[1-7]. 通常非线性演化方程包含非线性常微分方程(对未知函数及

其导数都不全是线性的或一次式的常微分方程)、非线性偏微分方程(对未知函数及其偏导数都不全是线性的或一次式的常微分方程)、非线性差分方程(又称为非线性映射或非线性迭代,它通常是非线性常微分方程或偏微分方程的离散形式,它对未知函数的 n 次迭代值都不全是线性的或一次式的)和函数方程(一个函数自身或多个函数之间满足的一个代数关系式).与线性模型不同,非线性模型不服从迭加原理,不能或者至少不能明显地把非线性问题分解成一些小的子问题而把它们的解迭加起来,而必须整体地考虑非线性方程.在一般情况下,人们不能靠直觉和简单计算来判断非线性系统的运动特征,特别是当动力学系统的维数越高,耦合程度越强,问题的研究就越复杂和更困难.非线性科学就是近三十多年来在综合各门以非线性为基本特征的科学研究基础上逐步形成和发展的,旨在揭示非线性系统的共同特征和运动规律的一门跨学科的综合性科学.继牛顿力学和量子力学之后发展起来的非线性科学正在改变人们对世界的看法,形成一种新的科学观点,促进了一大类新兴学科的诞生和发展,极大地影响着现代科学的逻辑体系.非线性科学研究的主要范畴是混沌、分形、孤子、斑图,还包括神经网络、元胞自动机和复杂系统[8-19].

特别近十多年来,随着计算机技术的快速发展和计算机代数成熟应用,非线性科学已被深入研究并被广泛应用到诸多自然学科如生物学、化学、数学、通讯和几乎所有的物理分支如凝聚态物理、场论、低温物理、流体力学、等离子物理、光学等等.这之中涌现了大量的非线性系统.为此,人们很自然地考虑到:如何求解描述非线性系统的非线性偏微分方程呢?非线性系统的解具有什么样的特性呢?……本论文的内容之一将围绕着这些问题展开讨论.

1.1 孤波的发现和研究回顾

在非线性科学中,孤子理论在自然科学的各个领域里起着非常