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# **DIRECT AND INVERSE PROBLEMS OF ELECTROMAGNETIC AND ACOUSTIC WAVE THEORY**

# **DIPED-2000**

ational Seminar/Workshop



**Tbilisi State University, Georgia  
Institute for Applied Problems of Mechanics and Mathematics of  
NASU, Ukraine**

**IEEE MTT/ED/AP/EMC Republic of Georgia Chapter  
IEEE MTT/ED/AP/CPMT/SSC West Ukraine Chapter**

## ***DIPED - 2000***

**Proceedings of V<sup>th</sup> International Seminar/Workshop  
on**

# **DIRECT AND INVERSE PROBLEMS OF ELECTROMAGNETIC AND ACOUSTIC WAVE THEORY**

**Tbilisi, October 3-6, 2000**



**Lviv-Tbilisi, 2000**

*Organized and sponsored by*

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**in Cooperation with**

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# *Plenary Session*



## ON THE THEORY OF HALLEN INTEGRAL EQUATION

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**Abstract.** A strict approach to solution of the Hallen integral equation for current density of a symmetric active dipole is proposed. It is shown that the nucleus of the equation has a logarithmic characteristic. The solution, which at dipole edges, satisfies the Meiksner's condition of the  $\rho^{1/2} \rightarrow 0$  type (at  $\rho \rightarrow 0$ ) was found, that assures uniqueness of the desired solution.

As it is known, the Hallen integral equation for a symmetric dipole has the form [1]:

$$\frac{r_0}{4\pi\omega\epsilon_0} \int_{-h/2}^{h/2} I(z') K(|z-z'|) dz' = A \cos kz + B \sin(k|z|) \quad (1)$$

$$(-h/2 \leq z \leq h/2)$$

where  $r_0$  is the dipole radius;  $h$  is the dipole length;  $\omega$  is the cyclic frequency;  $\epsilon_0 = (1/36\pi) \cdot 10^{-9}$ ,  $I(z')$  is the desired current density;  $A$  is the unknown coefficient, and  $B$  - the specified one;  $k = 2\pi/\lambda$ ,  $\lambda$  is the wavelength in vacuum;  $k(|z-z'|)$  is the nucleus of the integral equation, and

$$K(|z-z'|) = \int_0^{2\pi} \frac{e^{-ikr}}{r} d\psi \quad (2)$$

where

$$r = \sqrt{2r_0^2(1 - \cos\psi) + (z-z')^2} \quad (3)$$

It is convenient to reduce equation (1) to the dimensionless form:

$$\int_{-1}^1 \omega(\xi) K_0(|\eta - \xi|) d\xi = A \cos \beta\xi + B \sin(\beta|\xi|) \quad (4)$$

where

$$\xi = 2z'/h, \quad \eta = 2z/h, \quad \omega(\xi) = \frac{r_0}{4\pi\omega\epsilon_0} I(z'), \quad \left( z' = \frac{h}{2}\xi \right)$$

$$K_0(|\xi - \eta|) = 4 \int_0^{\pi/2} \frac{e^{-i\beta\sqrt{\gamma^2 \sin^2\psi + (\xi - \eta)^2}}}{\sqrt{\gamma^2 \sin^2\psi + (\xi - \eta)^2}} d\psi \quad (5)$$

and  $\gamma = \frac{4r_0}{h}, \quad \beta = \frac{\pi h}{\lambda}.$

The coefficient  $A$  in the right part of Eq. (4) should be found from the condition

$$\omega(\pm 1) = 0 \quad (6)$$

We have proved that nucleus (5) has a logarithmic characteristic in the following form:

$$K_0(|\xi - \eta|) = K_0^*(|\xi - \eta|) - \frac{4}{\gamma} \ln(|\xi - \eta|) \quad (7)$$



where  $K_0^*(|\xi-\eta|)$  is the regular function,  $|K_0^*(0)| < \infty$ .

Basing on Eq. (7), it is proved that, at  $\gamma \ll 1$  (a case of a thin dipole  $4r_0/h \ll 1$ ), the solution of Eq. (4) has the form:

$$\omega(\xi) = \frac{2}{\pi} \sum_{m=0}^{\infty} \frac{X_{2m} \cos(2m \arccos \xi)}{\sigma_m \sqrt{1-\xi^2}} \quad (8)$$

$$(\sigma_m=1 \text{ when } m=0 \text{ and } \sigma_m=1/2 \text{ when } m \neq 0)$$

The  $X_{2m}$  coefficients have the following structure

$$X_0 = \frac{\gamma}{2\pi} q(Aa_0 + Bb_0) \quad (q=1/2 \ln 2) \quad (9)$$

$$X_{2m} = \frac{\gamma}{2\pi} (Aa_{2m} + Bb_{2m}) \quad (10)$$

where as

$$a_{2m} = \pi(-1)^m J_{2m}(\beta), \quad b_{2m} = 2(-1)^m \sum_{s=0}^{\infty} \frac{(2s+1)J_{(2s+1)}(\beta)}{(2s+1)^2 - 4m^2}$$

Substituting Eqs.(9) and (10) into Eq.(8) and using condition (6), we find for the unknown coefficient  $A$

$$A = -BQ, \quad Q = \left( b_0 q + 2 \sum_{m=1}^{\infty} m b_{2m} \right) / \left( a_0 q + 2 \sum_{m=1}^{\infty} m a_{2m} \right)$$

where as

$$\sum_{m=1}^{\infty} m a_{2m} = -\pi \frac{\beta}{4} J_1(\beta), \quad \sum_{m=1}^{\infty} m b_{2m} = -\frac{1}{2} \sum_{s=1}^{\infty} J_{2s+1}(\beta)$$

Finally Eq.(8) is reduced to the following form

$$\omega(\xi) = \frac{4\gamma B}{\pi^2 G} \left\{ q \sum_{m=1}^{\infty} m C_{2m} \sin^2(m \arccos \xi) + \sum_{m=1}^{\infty} m D_{2m} \cos(2m \arccos \xi) \right\} \frac{1}{\sqrt{1-\xi^2}} \quad (11)$$

where

$$C_{2m} = a_0 b_{2m} - b_0 a_{2m}, \quad D_{2m} = p_1 a_{2m} - p_2 b_{2m}$$

$$p_1 \sum_{s=1}^{\infty} s b_{2s}, \quad p_2 \sum_{s=1}^{\infty} s a_{2s}, \quad G = a_0 q + 2 \sum_{m=1}^{\infty} m a_{2m}$$

At  $\xi \rightarrow \pm 1$ , Eq.(11) degenerates into the indeterminate form 0/0. Evaluating it, we convince ourselves that

$$\lim_{\xi \rightarrow \pm 1} \omega(\xi) = \text{const} \cdot \lim_{\xi \rightarrow \pm 1} \sqrt{1-\xi^2} = 0 \quad (12)$$

Hence, the dipole current density  $I(z')$  at  $z' \rightarrow \pm h/2$  decreases as

$$\lim_{z' \rightarrow \pm h/2} I(z') = \text{const} \cdot \lim_{z' \rightarrow \pm h/2} \sqrt{\left(\frac{h}{2}\right)^2 - z'^2} = 0 \quad (13)$$

Eqs.(12) and (13) are the Meiksner's condition for the case when the dipole is a hollow thin tube with sharp edges. They assure the uniqueness of obtained solution (11).

As the dipole radiation characteristic  $F(\theta)$  is proportional to the integral