Elementary
FLUID MECHANICS

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PREFACE TO THE SECOND EDITION

The overall plan and general philosophy of the first edition have been maintained in the second but much material has been added and some deleted. The principal change in the text has been the expansion of the original chapter entitled "Ideal Fluid Flow" to three chapters dealing with the incompressible fluid, the compressible fluid, and the impulse-momentum law. In the first of these the "energy-hydraulic-grade-line" approach has been emphasized more heavily; the second has been developed to introduce the beginner to the elements of a subject of increasing importance; the third consists of application of the impulse-momentum and energy principles to a larger variety of problems, particularly those of fluid machines. Other changes include further modernization of the chapter on pipe flow, a longer discussion of practical pipe-line problems, a different approach to the boundary layer, a discussion of the effects of velocity distribution on velocity head and momentum, and the development of the method for the calculation of critical depth in non-rectangular channels. The problems and illustrative problems have been drastically revised with the result that the problems have been made more analytical and the illustrative problems less routine. Problems available for student solution now number 700, and illustrative problems accompany most of the quantitive articles of the text. The second edition is further characterized by condensation of mathematical developments and deletion of certain topics which experience indicated were beyond the capacity of the beginner, or were treated adequately in other fields, or dealt with obsolete devices, or seemed to disrupt the continuity of the book. The net result of the foregoing changes is a shorter book which covers more material than the first edition.

Criticisms from both students and teachers indicated that many of the problems of the first edition were too easy to produce the intended challenge. Accordingly, the problems of the second edition have been made more difficult, but it should not be inferred that the whole text has been raised to a higher level of difficulty. On the contrary, it has been my intention to make it more readable and understandable. It is still meant for the beginner in the field and has been rewritten (after further extensive experience) with his problems primarily in mind.

I should like to take this opportunity to say to the student that fluid mechanics will present you with certain difficulties not encountered in the parallel courses in mechanics of solids. In subjects in mechanics of solids, analysis of a problem usually leads to an engineering answer of satisfactory accuracy with the use of a minimum of experimentally determined quantities. In the engineering problems of fluid flow, however, this is usually not the case, and a variety of experimental factors must frequently be employed before an answer is obtained; this means that you must learn to work with, interpret, and understand many more experimental results than before and requires that you develop some ingenuity along these lines. Another inherent difficulty which you will meet springs from lack of casual experience with fluid phenomena; compared with your casual experience with simple structures, solid objects in motion, etc., this may be almost nil. This deficiency will require deeper concentration on descriptive material and new terminology, along with the mathematical and physical developments. Finally, do not forget that your studies of mechanics have the twofold purpose of developing analytical ability as well as conveying information; do not bypass the first of these aims by blind substitution in formulas —the development of analytical ability may well be more important than the subject matter!

The many constructive criticisms and suggestions, obtained mostly through the efforts of the publisher, from teachers using the first edition have been invaluable in appraising that edition and in improving the second. I greatly appreciate the efforts of John Wiley and Sons, Inc., and of those teachers, all unknown to me, who have rendered me so valuable a service.

JOHN K. VENNARD

PALO ALTO, CALIFORNIA
March, 1947

PREFACE TO THE FIRST EDITION

Fluid mechanics is the study of all fluids under all possible conditions of rest and motion. Its approach is analytical, rational, and mathematical rather than empirical; it concerns itself with those basic principles which lead to the solution of numerous diversified problems, and it seeks results which are widely applicable to similar fluid situations and not limited to isolated special cases. Fluid mechanics recognizes no arbitrary boundaries between fields of engineering knowledge but attempts to solve all fluid problems, irrespective of their occurrence or of the characteristics of the fluids involved.

This textbook is intended primarily for the beginner who knows the principles of mathematics and mechanics but has had no previous experience with fluid phenomena. The abilities of the average beginner and the tremendous scope of fluid mechanics appear to be in conflict, and the former obviously determine limits beyond which it is not feasible to go; these practical limits represent the boundaries of the subject which I have chosen to call *elementary fluid mechanics*. The apparent conflict between scope of subject and beginner's ability is only along mathematical lines, however, and the *physical ideas* of fluid mechanics are well within the reach of the beginner in the field. Holding to the belief that physical concepts are the *sine qua non* of mechanics, I have sacrificed mathematical rigor and detail in developing physical pictures and in many cases have stated general laws only (without numerous exceptions and limitations) in order to convey basic ideas; such oversimplification is necessary in introducing a new subject to the beginner.

Like other courses in mechanics, fluid mechanics must include disciplinary features as well as factual information—the beginner must follow theoretical developments, develop imagination in visualizing physical phenomena, and be forced to think his way through problems of theory and application. The text attempts to attain these objectives in the following ways: omission of subsidiary conclusions is designed to encourage the student to come to some conclusions by himself; application of bare principles to specific problems should develop ingenuity; illustrative problems are included to assist in overcoming numerical difficulties; and many numerical problems for the student to solve are intended not only to develop ingenuity but to show practical applications as well.

Presentation of the subject begins with a discussion of fundamentals, physical properties and fluid statics. Frictionless flow is then discussed to bring out the applications of the principles of conservation of mass and energy, and of impulse-momentum law, to fluid motion. The principles of similarity and dimensional analysis are next taken up so that these principles may be used as tools in later developments. Frictional processes are discussed in a semi-quantitative fashion, and the text proceeds to pipe and open-channel flow. A chapter is devoted to the principles and apparatus for fluid measurements, and the text ends with an elementary treatment of flow about immersed objects. Throughout the text, the foot-pound-second system of dimensions has been used, and problems of conversion from the metric system, which so frequently divert the beginner's attention from the physical ideas, have been avoided; justifications for experimental results and empirical formulas have been presented except at points where the student should discover them for himself; bibliographies have been included to guide the inquiring reader to more exhaustive treatments of the subject.

For criticism of my *Notes on Elementary Fluid Mechanics* which have been expanded into the present text, I wish to extend my appreciation to many of my colleagues at New York University, Professor Boris A. Bakhmeteff of Columbia University, and Professor William Allan of the College of the City of New York.

I am deeply indebted to Mr. William H. Peters of the Curtiss-Wright Corporation for carefully reviewing the first eight chapters of the manuscript, and to Professor Frederick K. Teichmann of New York University for critical comments on the last chapter. I also wish to thank Mr. J. Charles Morgan for general comments and assistance in reading proof and Miss Katherine Williams for her care and patience in typing the manuscript.

JOHN K. VENNARD

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CHAPTER I

FUNDAMENTALS

1. Development of Fluid Mechanics. Man's desire for knowledge of fluid phenomena began with his problems of water supply and disposal and the use of water for obtaining power. With only a rudimentary appreciation for the physics of fluid flow he dug wells, operated crude water wheels and pumping devices, and, as his cities increased in size, constructed ever larger aqueducts, which reached their greatest size and grandeur in the City of Rome. However, with the exception of the thoughts of Archimedes (250 B.C.) on the principles of buoyancy, little of the scant knowledge of the ancients appears in modern fluid mechanics. After the fall of the Roman Empire (A.D. 476) no progress was made in fluid mechanics until the time of Leonardo Da Vinci (1452–1519). This great genius designed and built the first chambered canal lock near Milan and ushered in a new era in hydraulic engineering; he also studied the flight of birds and developed some ideas on the origin of the forces which support them.

After the time of Da Vinci, the accumulation of hydraulic knowledge rapidly gained momentum, the contributions of Galileo, Torricelli, Newton, Pitot, D. Bernoulli, and D'Alembert to the fundamentals of the science being outstanding. Although the theories proposed by these scientists were in general confirmed by crude experiments, divergences between theory and fact led D'Alembert to observe in 1744 that, "The theory of fluids must necessarily be based upon experiment." D'Alembert showed that there is no resistance to motion when a body moves through an ideal (non-viscous) fluid, yet obviously this conclusion is not valid for bodies moving through real fluid. This discrepancy between theory and practice is called the "D'Alembert paradox" and serves to demonstrate the limitations of theory alone in solving fluid problems.

Because of the conflict between theory and practice, two schools of thought arose in the treatment of fluid problems, one dealing with the theoretical and the other with the practical aspects of fluid flow, and in a sense these two schools of thought have persisted down to the present day, resulting in the theoretical field of "hydrodynamics" and the practical one of "hydraulics." Notable contributions to theoretical hydrodynamics have been made by Euler, La Grange, Helmholtz, Kirchhoff, Lord Rayleigh, Rankine, Lord Kelvin, and Lamb. In a broad sense, experimental hydraulics became a study of the laws of fluid resistance, mainly in pipes and open channels. Among the many scientists who devoted their energies to this field were Brahms, Bossut, Chezy, Dubuat, Fabre, Coulomb, Eytelwein, Belanger, Dupuit, d'Aubisson, Hagen, and Poisseuille.

Toward the middle of the last century, Navier and Stokes succeeded in modifying the general equations for ideal fluid motion to fit that of a viscous fluid and in so doing showed the possibilities of adjusting the differences between hydraulics and hydrodynamics. At about the same time, theoretical and experimental work on vortex motion by Helmholtz was aiding in explaining away many of the divergent results of theory and practice.

Meanwhile, hydraulic research went on apace, and large quantities of excellent data were collected or formulas proposed for fluid resistance, notably by Darcy, Bazin, Weisbach, Fanning, Ganguillet, Kutter, and Manning; among researchers on other hydraulic problems were Thomson, Fteley, Stearns, and H. Smith. Unfortunately, researches led frequently to empirical formulas obtained by fitting curves to experimental data or by merely presenting the results in tabular form, and in many instances the relationship between the physical facts and the resulting formula was not apparent.

Toward the end of the last century, new industries arose which demanded data on the flow of fluids other than water; this fact and many significant advances in knowledge tended to arrest the increasing empiricism of hydraulics. These advances were: (1) the theoretical and experimental researches of Reynolds; (2) the development of dimensional analysis by Lord Rayleigh; (3) the use of models by Froude, Reynolds, Fargue, and Engels in the solution of fluid problems; and (4) the rapid progress of theoretical and experimental aeronautics in the work of Lanchester, Lilienthal, Kutta, Joukowski, and Prandtl. These advances allowed new tools to be applied to the solution of fluid problems and gave birth to modern fluid mechanics.

Since the beginning of the present century, empiricism has waned and fluid problems have been solved by increasingly rational methods; these methods have produced so many fruitful results and have aided so materially in increasing our knowledge of the details of fluid phenomena that the trend appears likely to continue into the future. Among the foremost contributors to modern fluid mechanics are Prandtl, Blasius, Kármán, Stanton, Nikuradse, Bakhmeteff, Koch, Buckingham, Gibson, Rehbock, Durand, and Taylor.

2. Physical Characteristics of the Fluid State. Matter exists in two states—the solid and the fluid, the fluid state being commonly divided into the liquid and gaseous states.

Solids differ from liquids and liquids from gases in the spacing and latitude of motion of their molecules, these variables being large in a gas, smaller in a liquid, and extremely small in a solid. It follows that intermolecular cohesive forces are large in a solid, smaller in a liquid, and extremely small in a gas. These fundamental facts account for the familiar compactness and rigidity of form possessed by solids, the ability of liquid molecules to move freely within a liquid mass, and the capacity of gases to fill completely the containers in which they are placed.

A more fruitful and rigorous mechanical definition of the solid and fluid states may be made on the basis of their actions under the various types of stress. Application of tension, compression, or shear stresses to a solid results first in elastic deformation, and later, if these stresses exceed the elastic limits, in permanent distortion of the material. Fluids, however, possess elastic properties under compression stress, but application of infinitesimal shear stress results in continual and permanent distortion. This inability to resist shear stress gives fluids their characteristic ability to "flow." Fluids will support tension stress to the extent of the cohesive forces between their molecules. Since such forces are extremely small, it is customary in engineering problems to assume that fluids can support no tension stress.

Since shear stress applied to fluids always results in distortion or "flow," it is evident that in fluids at rest no shear stresses can exist and compression stress, or "pressure," becomes the only stress to be considered.

Fluids being continuous media, it follows that pressures occurring or imposed at a point in a fluid will be *transmitted undiminished* to all other points in the fluid.

Since fluids cannot support tangential (shear) stress, no component of stress can exist in a fluid at rest *along* a solid boundary or *along* an arbitrary section passed through the fluid. This means that pressures must be transmitted to solid boundaries or across arbitrary sections *normal* to these boundaries or sections at every point. Furthermore if a "free body" of fluid is isolated as in Fig. 1, pressure must be shown acting *inward* (p_1) upon the free body (according to the usual conventions of mechanics for compression stress). Pressures exerted by fluid on container (p_2) will of course act *outward*, but their reactions (p_3) will act inward as before. Another property of fluid pressure is that at a point it has the same magnitude in all directions; this may be easily seen by reducing the arbitrary volume of Fig. 1 to infinitesimal size.

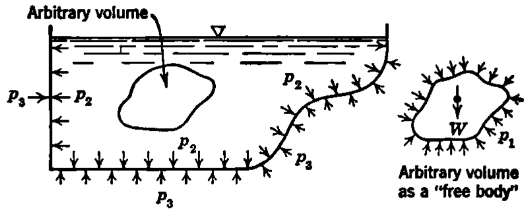

FIG. 1

PHYSICAL PROPERTIES OF FLUIDS

3. Density, Specific Weight, Specific Volume, and Specific Gravity. Density [1] is the *mass* of fluid contained in a unit of volume; specific weight,[1] the *weight* of fluid contained in a unit of volume. Both these terms are fundamentally measures of the number of molecules per unit of volume. Since molecular activity and spacing increase with temperature fewer molecules will exist in a given unit volume as temperature rises, thus causing density and specific weight to decrease with increasing temperature.[2] Since a larger number of molecules can be forced into a given volume by application of pressure, it will be found that density and specific weight will increase with increasing pressure.

Density, ρ (rho), will be expressed in the mass-length-time system of dimensions and will have the dimensions of mass units (slugs) per cubic foot (slugs/ft^3).

Specific weight, w, will be expressed in the force-length-time system of dimensions and will have the dimensions of pounds per cubic foot (lb/ft^3).

Since a mass, M, is related to its weight, W, by the equation

$$M = \frac{W}{g}$$

in which g is the acceleration due to gravity, density and specific weight (the mass and weight of a unit volume of fluid) will be related

[1] In American engineering practice, specific weight is frequently termed "density" and density "mass density."

[2] A variation in temperature from 32° F to 212° F will decrease the specific weight of water 4 per cent (Appendix II) and will decrease the density of gases 37 per cent (assuming no pressure variation).

by a similar equation

$$\rho = \frac{w}{g} \quad \text{or} \quad w = \rho g \tag{1}$$

Using the fact that physical equations are dimensionally homogeneous, the foot-pound-second dimensions of ρ (which are equivalent to slugs per cubic foot) may be calculated as follows:

$$\text{Dimensions of } \rho = \frac{\text{Dimensions of } w}{\text{Dimensions of } g} = \frac{\frac{\text{lb}}{\text{ft}^3}}{\frac{\text{ft}}{\text{sec}^2}} = \frac{\text{lb sec}^2}{\text{ft}^4}$$

This algebraic use of the dimensions of quantities in the equation expressing physical relationship will be employed extensively and will prove to be an invaluable check on engineering calculations.[3]

The specific volume, v, defined as volume per unit of weight, will have dimensions of cubic feet per pound (ft^3/lb). This definition identifies specific volume as the reciprocal of specific weight and introduces the equations

$$v = \frac{1}{w} \quad \text{or} \quad w = \frac{1}{v} \tag{2}$$

Specific gravity, S, is the ratio of specific weight or density of a substance to the specific weight or density of pure water. Since all these items vary with temperature, temperatures must be quoted when specific gravity is used in precise calculations of specific weight or density. Specific gravities of a few common liquids are presented in Table I,[4] from which the specific weights of liquids may be readily calculated by

$$w = S \times 62.45 \text{ lb/ft}^3$$

The specific weight of gases may be calculated by means of Boyle's law and Charles' law. Using the specific volume of a gas, Boyle's law may be stated as[5]

$$pv = \text{Constant}$$

[3] A summary of quantities and their dimensions is given in Appendix I.

[4] The reader should refer to physical tables if precise specific gravities at other temperatures are required. The student may use the tabulated values in problem solutions even though the temperatures are not exactly the same.

[5] p is the absolute pressure in pounds per square foot. T is the temperature in degrees F absolute (degrees F + 459.6), and the "constants" are constant if the gas is "perfect." Common gases in the ordinary engineering range of pressures and temperatures may be considered to be "perfect" for most engineering calculations.

which expresses the law of compression or expansion of a gas at constant temperature. Charles' law, expressing the variation of pressure with temperature in a constant volume of gas, is [5]

$$\frac{p}{T} = \text{Constant}$$

TABLE I *

SPECIFIC GRAVITIES, S, OF VARIOUS LIQUIDS AT 68° F †
(Referred to water at 39.2° F)

Ethyl alcohol	0.789
Turpentine (*d*-pinene)	0.862
Benzene	0.879
Olive oil	0.918 (59° F) ‡
Linseed oil	0.942 (59° F) ‡
Castor oil	0.960
Water	0.998
Glycerine	1.262
Carbon tetrachloride	1.594
Tetrabromethane	2.964
Mercury	13.546

* *International Critical Tables*, McGraw-Hill Book Co., 1933, except as noted.
† Except as noted.
‡ *Smithsonian Physical Tables*, Eighth edition, Smithsonian Institution, 1933.

Obviously the only combination of variables which will satisfy both Boyle's and Charles' laws simultaneously is

$$\frac{pv}{T} = R$$

which is called the "equation of state" of the gas in which the constant, R, is called the "gas constant" and has dimensions of feet/degree Fahrenheit absolute. Since $w = 1/v$, the above equation may be transformed into

$$w = \frac{p}{RT} \tag{3}$$

from which specific weights of gases may be readily calculated.

Application of Avogadro's law, that "all gases at the same pressures and temperatures have the same number of molecules per unit of volume," allows the calculation of a "universal gas constant." Consider two gases having constants R_1 and R_2, specific weights w_1 and w_2, and existing at the same pressure and temperature, p and T. Dividing their equations of state

$$\frac{\dfrac{p}{w_1 T} = R_1}{\dfrac{p}{w_2 T} = R_2}$$

results in

$$\frac{w_2}{w_1} = \frac{R_1}{R_2}$$

but, according to Avogadro's principle, the specific weight of a gas must be proportional to its molecular weight, giving $w_2/w_1 = m_2/m_1$, in which m_1 and m_2 are the respective molecular weights of the gases. Combining this equation with the preceding one gives $m_2/m_1 = R_1/R_2$, or

$$m_1 R_1 = m_2 R_2$$

In other words, the product of molecular weight and gas constant is the same[6] for all gases. This product mR is called the "universal

TABLE II

GAS CONSTANTS FOR COMMON GASES *

	R, ft/°F abs	mR
Sulphur dioxide	23.6	1512
Carbon dioxide	34.9	1536
Oxygen	48.3	1546
Air	53.3	1545
Nitrogen	55.1	1543
Ammonia	89.5	1516
Hydrogen	767.0	1546

* O. W. Eshbach, *Handbook of Engineering Fundamentals*, p. 7-16, John Wiley & Sons, 1936.

gas constant" and is preferred for general use by many engineers. Values of these gas constants are given in Table II.

[6] The constancy of mR is particularly true for the monatomic and diatomic gases. Gases having more than two atoms per molecule tend to deviate from the law mR = Constant. See Table II.